SM3 Theorems 2-4 (Polynomials)

Multiplicity: the number of times a zero is repeated for a polynomial. $y=(x-1)^2=(x-1)(x-1)$ x = 1 is a "zero" of the polynomial twice.

"x = 1 is a zero of the polynomial with multiplicity 2"

Fundamental Theorem of Algebra: If a polynomial has a degree of "n", then the polynomial has "n" zeroes (provided that repeat zeroes, called "multiplicities" are counted separately).

$y=6x^4+42x^3+96x^2+28x+48$ "4th Degree" → 4 zeroes

Linear Factorization Theorem: If a polynomial has a degree of "n", then the polynomial can be factored into "n" linear factors.

$y=6(x+4)(x+3)(x-2i)(x+2i)$

Since each linear factor has one zero, these two theorems are almost saying the same thing.

Quadratic Formula: gives zeroes of 2nd degree polynomials and proves two more theorems. $x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

1. **Complex Conjugates Theorem**: If f(x) is a function with real coefficients and if (a + bi) is a zero of f(x), then its complex conjugate (a - bi) is also a zero of f(x). Example: $y=x^2-4x+5$ Factorization: $y=(x-2-3i)(x-2+3i)$ Zeroes: $x=2-3i$, $x=2+3i$

2. **Irrational Roots (Zeroes) Theorem**: If an irrational number is the zero of a polynomial, then the conjugate of the irrational number is also a zero.

Example: $y=x^2+4x+1$ Factorization: $y=(x+2-\sqrt{3})(x+2+\sqrt{3})$ Zeroes: $x=2-\sqrt{3}$, $x=2+\sqrt{3}$

Example: $y=x^2+4$ Factorization: $y=(x+2i)(x-2i)$ Zeroes: $3-\sqrt{2}$, $3+\sqrt{2}$