Math-3

Lesson 3-4

Add and Subtract Rational Expressions

What are “like terms” ?

Like variables: $2x + 3y + 4x$

Multiples of the same variable

Like powers: $x^2 + y^2 + x^3 + x^2$

same ‘base’ and same exponent.

Like radicals: $\sqrt{2} + \sqrt{3} + 3\sqrt{2}$

same ‘radicand’ and same index number.

Like fractions: $\frac{2}{3}, \frac{4}{3}, \frac{3}{4}$

same denominator.

Adding Fractions

We can add “like fractions”

$$\frac{2}{3} + \frac{1}{3} + \frac{4}{3}$$

Combine the numerator over a <u>common</u> denominator.

$$= \frac{2+1+4}{3} \qquad = \frac{7}{3}$$

Identity Property of Multiplication

The numeral "one" multiplied by any number does not change the "identity" (value) of the number (the product will have an "equivalent" value).

Used for:

Obtaining Common Denominators:

$$\frac{2}{3} + \frac{3}{5} = \frac{3}{5} * \frac{3}{3} + \frac{2}{3} * \frac{5}{5} = \frac{10}{15} + \frac{9}{15}$$

Rationalizing Denominators:

$$\frac{2}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Least Common Multiple: The smallest number that two other numbers divide evenly.

Used for: Obtaining ***Least*** Common Denominators:

$$\frac{1}{30}+\frac{1}{42} = \frac{1}{6*5}+\frac{1}{6*7} = \frac{1}{6*5}*\frac{7}{7}+\frac{1}{6*7}*\frac{5}{5}$$

Both denominators have a common factor of 6.

Left denominator has an uncommon factor of 5

Multiply $\frac{1}{42}$ by “one” in the form of $\frac{5}{5}$

right denominator has an uncommon factor of 7

Multiply $\frac{1}{30}$ by “one” in the form of $\frac{7}{7}$

Find the LCD

$$\frac{2}{15}+\frac{3}{20}$$

$$15 = 3*5$$

$$20 = 4*5$$

$$LCD = 3*4*5$$

$$LCD = 60$$

$$\frac{1}{60}+\frac{1}{48}$$

$$60 = 2*6*5$$

$$48 = 6*8 = 6*2*4$$

$$LCD = 2*6*4*5$$

$$LCD = 240$$

A rational number

can be written as a ratio of integers.

$$\frac{2}{5}, \frac{3}{1}$$

A rational expression

can be written as a ratio of expressions.

$$\frac{x}{(x+1)}$$

We will be looking at ratios of polynomial expressions.

Excluded Value: the value for ‘x’ that results in division by zero

$$\frac{x}{(x+1)}, \quad x \neq -1$$

What type of rational expressions can you combine together using addition or subtraction?

"like rational expressions"

Which of these expressions are "like expressions"?

1. $\dfrac{x}{(x+1)}$ 2. $\dfrac{3x}{(x-1)}$ 3. $\dfrac{4}{(x-5)}$ 4. $\dfrac{2x^2}{(x+1)}$

Which of these expressions are "like expressions"?

5. $\dfrac{x}{(x+1)(x-1)}$ 7. $\dfrac{2x^2}{x^2-2x+1}$

6. $\dfrac{3x}{(x-1)^2}$ 8. $\dfrac{4}{(x^2-1)}$

What is the excluded value for each expression?

1. $\frac{x}{(x+1)}$

$x \neq -1$

2. $\frac{3x}{(x-1)}$

$x \neq 1$

3. $\frac{4}{(x-5)}$

$x \neq 5$

4. $\frac{2x^2}{(x+1)}$

$x \neq -1$

5. $\frac{x}{(x+1)(x-1)}$

$x \neq -1,+1$

6. $\frac{3x}{(x-1)^2}$

$x \neq 1$

7. $\frac{2x^2}{x^2-2x+1}$

$x \neq 1$

8. $\frac{4}{(x^2-1)}$

$x \neq -1,+1$

Simplifying Fractions

$$\frac{32}{44} = \frac{32 \div 4}{44 \div 4} = \frac{32 * \frac{1}{4}}{44 * \frac{1}{4}} = \frac{8}{11}$$

What property are you applying?

Identity Property Of Multiplication

"multiplying by '1' in the form of ¼ over ¼ "

Unfortunately this method ***WILL NOT WORK*** for simplification of ratios of polynomials.

$$\frac{x^2 - 4}{x^2 - 3x + 2}$$

Simplifying Fractions

<u>You must FACTOR the fractions.</u>

$$\frac{32}{44} = \frac{4*8}{4*11}$$

<u>Break them apart into the product of fractions.</u>

$$= \frac{4}{4} * \frac{8}{11}$$

<u>Notice the fractions that equal '1'</u>

$$= 1 * \frac{8}{11} \qquad = \frac{8}{11}$$

Reducing Rational Expressions to their Lowest form (simplification)

You must FACTOR the fractions.

$$\frac{32}{44} = \frac{4*8}{4*11} \qquad \frac{x^2-4}{x^2-3x+2} = \frac{(x-2)(x+2)}{(x-2)(x-1)}$$

Break them apart into the product of fractions.

$$= \frac{4}{4} * \frac{8}{11} \qquad = \frac{(x-2)}{(x-2)} * \frac{(x+2)}{(x-1)}$$

Notice the fractions that equal '1'

$$= 1 * \frac{8}{11} \quad = \frac{8}{11} \qquad = 1 * \frac{(x+2)}{(x-1)} = \frac{(x+2)}{(x-1)}$$

Adding/Subtraction Rational Expressions

The <u>easy</u> problem: $$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

Combine the numerator over a <u>common</u> denominator.

The <u>easy</u> problem: $$\frac{4}{(x-5)} + \frac{3x}{(x-5)} = \frac{4+3x}{(x-5)}$$

Combine the numerator over a <u>common</u> denominator.

Can you combine
4 and 3x ?

Why not?

Your turn: add/subtract

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$= \frac{\cancel{2}(x-1)}{\cancel{2}*x^2} = \frac{(x-1)}{x^2}$$

What property are we using?

Inverse Property of Multiplication.

Can you do it this way?

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{\cancel{x}+\cancel{2}}{\cancel{2}*\cancel{x}*x}+\frac{\cancel{x}-(\cancel{2}*2)}{\cancel{2}*\cancel{x}*x} = \frac{1}{x}+\frac{-2}{x}$$

Why not?

You CANNOT use the Inverse Property of Multiplication on addends.

Your turn: add/subtract

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$= \frac{2(x-1)}{2 * x^2}$$

I <u>will not</u> allow you to simplify using the Inverse Property of Multiplication until you have factored it into two fractions.

$$= \frac{\cancel{2}}{\cancel{2}} * \frac{(x-1)}{x^2} = \frac{(x-1)}{x^2}$$

Only then will you be able to see how the Inverse Property of Multiplication changes the rational expression into <u>multiplication by one.</u>

Add/Subtract Rational Expression with Equal Denominators

Which one is correct?

$$\frac{2x-7}{x^2+2} - \frac{x-4}{x^2+2}$$

$$\rightarrow \frac{2x-7-x-4}{x^2+2}$$

$$\rightarrow \frac{2x-7-x+4}{x^2+2}$$

Group expressions to avoid distributive property errors.

$$\frac{(2x-7)}{(x^2+2)} - \frac{(x-4)}{(x^2+2)} \rightarrow \frac{(2x-7)-(x-4)}{x^2+2} \rightarrow \frac{2x-7-x-(-4)}{x^2+2}$$

$$\boxed{\rightarrow \frac{x-3}{x^2+2}}$$

Add/Subtract Rational Expressions whose Denominators are Additive Inverses of each other.

$$\frac{1}{x-2}+\frac{3}{2-x}$$

Rewrite the denominator in "standard form"

$$\frac{1}{x-2}+\frac{3}{-x+2}$$

Use the *Identity Property of Multiplication* (multiply by "one" in the form of -1/-1)

$$\frac{1}{x-2}+\frac{3}{(-x+2)}*\frac{(-1)}{(-1)}$$

Simplify

$$\frac{1}{x-2}+\frac{-3}{x-2}$$

Add (or subtract)

$$\frac{-2}{x-2}$$

No common denominator.

$$\frac{(2x-1)}{2x}+\frac{(x-2)}{3x}$$

Multiply the left side fraction by one in the form of 3/3

Multiply the right side fraction by one in the form of 2/2

$$\frac{3}{3}*\frac{(2x-1)}{2x}+\frac{(x-2)}{3x}*\frac{2}{2}=\frac{3(2x-1)}{2*3*x}+\frac{2(x-2)}{2*3*x}$$

$$=\frac{3(2x-1)+2(x-2)}{6x}=\frac{6x-3+2x-4}{6x}=\frac{8x-7}{6x}$$

Can you factor this into two fractions multiplied together?

$$\frac{x}{3} - \frac{x+1}{6} \qquad \frac{2}{2} * \frac{x}{3} - \frac{(x+1)}{6} \qquad = \frac{2x}{6} - \frac{(x+1)}{6}$$

$$= \frac{2x-(x+1)}{6} \qquad = \frac{2x-x-1}{6} \qquad = \frac{x-1}{6}$$

$$\frac{x-1}{2x}+\frac{2x+3}{x} = \frac{(x-1)}{2x}+\left(\frac{2}{2}\right)*\frac{(2x+3)}{x}$$

$$=\frac{(x-1)}{2x}+\frac{2(2x+3)}{2x} = \frac{(x-1)}{2x}+\frac{(4x+6)}{2x}$$

$$=\frac{(x-1)+(4x+6)}{2x} = \frac{5x+5}{2x}$$

Can you factor this into two fractions multiplied together?

$$\frac{3x+1}{2x}-\frac{1}{5} \qquad =\frac{5}{5}*\frac{(3x+1)}{2x}-\frac{1}{5}*\frac{2x}{2x}$$

$$=\frac{5(3x+1)}{10x}-\frac{2x}{10x} \qquad =\frac{5(3x+1)-2x}{10x}$$

$$=\frac{15x+5-2x}{10x} \qquad =\frac{13x+5}{10x}$$

Can you factor this into two fractions multiplied together?

$$\frac{12}{x^2+5x-24}+\frac{3}{x-3}$$ (step by step)

What is the factored version of the left denominator?

$$\frac{12}{(x+8)(x-3)}+\frac{3}{(x-3)}$$

What is the least common denominator? $(x+8)(x-3)$

Multiply right-side term by "*one in the form of (x-8) over (x-8)*"

$$=\frac{12}{(x+8)(x-3)}+\frac{3}{(x-3)}*\frac{(x+8)}{(x+8)}$$

$$= \frac{12}{(x+8)(x-3)} + \frac{3}{(x-3)} * \frac{(x+8)}{(x+8)}$$

$$= \frac{12+3(x+8)}{(x+8)(x-3)} \quad = \frac{12+3x+24}{(x+8)(x-3)} \quad = \frac{3x+36}{(x+8)(x-3)}$$

$$= \frac{3(x+12)}{(x+8)(x-3)}$$

Can you factor this into two fractions multiplied together?

Complex Fraction is a fraction with a fraction in the numerator and a fraction in the denominator.

$$\frac{2/3}{4/5} = \frac{2}{3} \div \frac{4}{5} = \frac{2}{3} * \frac{5}{4} = \frac{2}{2} * \frac{5}{3*2} = \frac{5}{6}$$

How do you divide fractions?

Multiply by reciprocal.

Simplify the complex fraction.

$$\frac{\frac{5}{x+4}}{\frac{3}{x+4}} = \frac{5}{x+4} \div \frac{3}{x+4}$$

$$= \frac{5}{(x+4)} * \frac{(x+4)}{3} = \frac{\cancel{(x+4)}}{\cancel{(x+4)}} * \frac{5}{3}$$

Put binomials into parentheses!

$$= \frac{5}{3}$$

Simplify the complex fraction.

$$\frac{\frac{x}{x+2}}{\frac{3}{x+3}} = \frac{x}{x+2} \div \frac{3}{x+3} = \frac{x}{x+2} * \frac{x+3}{3}$$

$$= \frac{x}{x+2} * \frac{x+3}{3} = \frac{x(x+3)}{3(x+2)}$$

Combine the numerator fractions into <u>one</u> fraction.

$$\frac{\frac{x}{3}-6}{2+\frac{3}{x}} = \frac{\frac{x}{3}-\frac{6}{1}}{2+\frac{3}{x}} = \frac{\frac{x-18}{3}}{2+\frac{3}{x}}$$

2. Combine the <u>denominator</u> fractions into <u>one</u> fraction.

$$\frac{\frac{x-18}{3}}{\frac{2x+3}{x}} = \frac{x-18}{3} * \frac{x}{2x+3} = \frac{x(x-18)}{3(2x+3)}$$

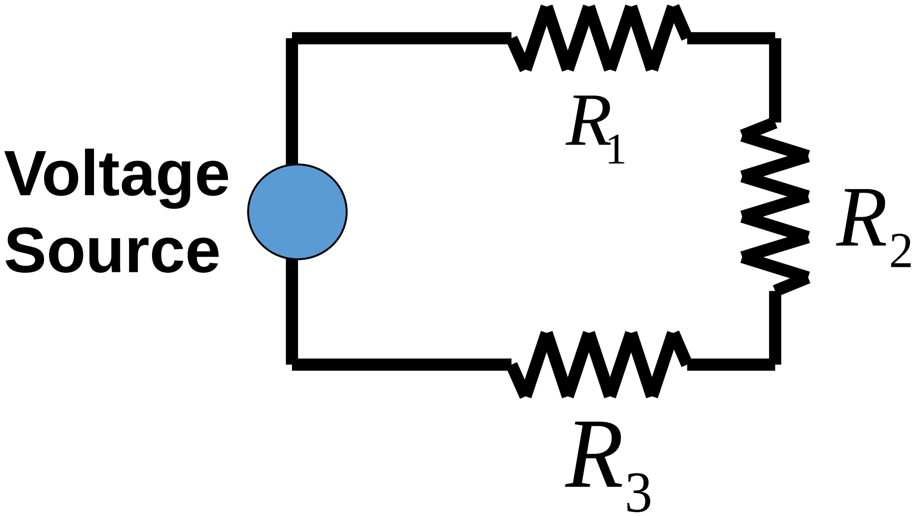
Simplify the complex fraction.

$$\frac{\frac{1}{x}+\frac{2}{3x}}{\frac{3}{x+4}}$$

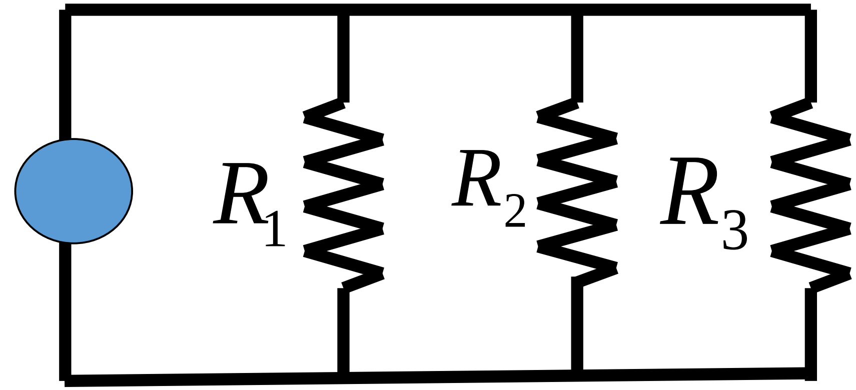
$$\frac{4}{x-3}-\frac{2x}{x^2-9}$$

$$\frac{2}{x-3}-\frac{3}{x+2}$$

$$R_{total} = R_1 + R_2 + R_3$$



$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

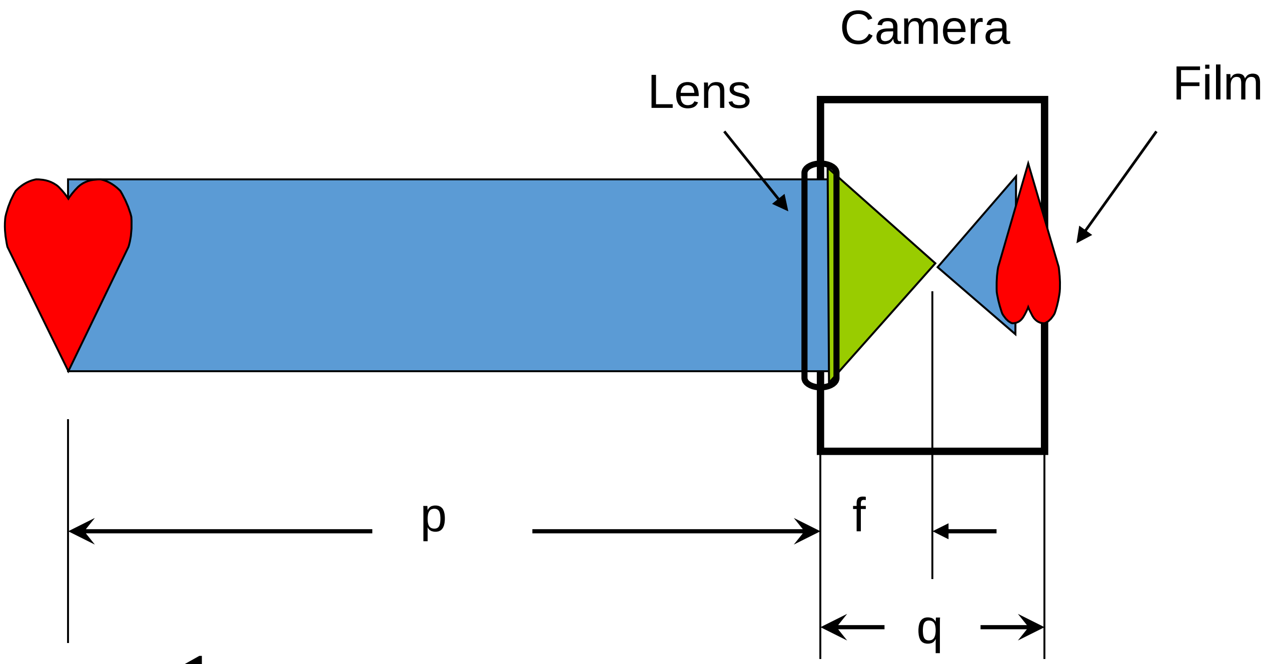


$$\frac{1}{R_T} = \frac{R_2 R_3}{R_1 R_2 R_3} + \frac{R_1 R_3}{R_1 R_2 R_3} + \frac{R_1 R_2}{R_1 R_2 R_3}$$

$$\frac{1}{R_T} = \frac{R_2 R_3 + R_1 R_3 + R_1 R_2}{R_1 R_2 R_3}$$

$$R_T = \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}$$

Focal length of a telescope/microscope



$$f = \frac{1}{\frac{1}{p} + \frac{1}{q}}$$

$$f = \frac{1}{\frac{q+p}{pq}}$$

$$f = \frac{pq}{q+p}$$