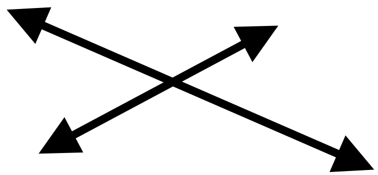
# Math-3
# Lesson 7-5

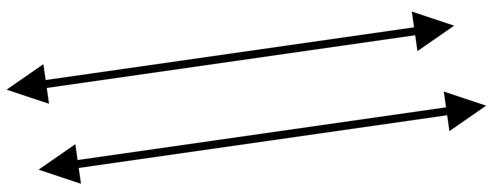
Solving Systems of Equations by Substitution and Elimination

# Categories of Solutions:

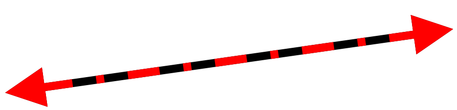
Ways 2 lines can be graphed:



Cross → one solution



Parallel →no solutions



Same line → infinitely many solutions

# How do you know how many solutions there are? (1, 0, or infinite #)

$$y = 3x + 1$$
$$y = 2x + 1$$

Not same line, not parallel → one solution.

$$y = -2x + 3$$
$$y = -2x - 4$$

parallel → no solutions

$$2x + 2y = 2$$
$$x + y = 1$$

1st equation is a multiple of the 2nd equation
→ same line

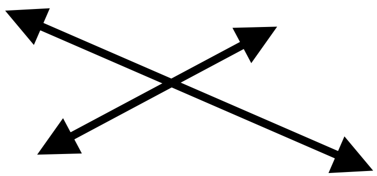
→ infinite # of solutions.

Which Category ?

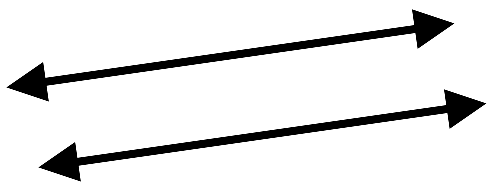
$$y = 2x + 6$$
$$y = 4x - 2$$

$$y = 2x + 4$$
$$y = 2x - 7$$

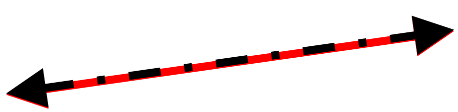
$$2x + 3y = 6$$
$$4x + 6y = 12$$



Cross → one solution



Parallel →no solutions



Same line → infinitely many solutions

# Algebraic Methods of Solving Systems of Equations

Substitution: Solve one equation for one of the variables. Substitute the equivalent expression for the variable into the other equation. This results in one equation with one variable.

Elimination: Add the equations (or multiples of the equations) to eliminate one of the variables. Then solve the single variable equation and "back substitute" the result.

# Substitution Method

1. Solve one equation for one of the variables (already done if in "y =" form).

$y = -2x + 8$

$y = 3x – 2$

2. Substitute the value of the variable into the other equation.

$(\ \ ) = -2x + 8$

$3x – 2 = -2x + 8$

$+2x \qquad +2x$

3. Solve for the single variable.

$5x – 2 = 8$

$+2 \quad +2$

$5x = 10$

$÷5 \quad ÷5$

$x = 2$

4. Substitute the value of the solved-for variable into either equation to find the other varable.

$y = 3x – 2$

$y = 3(\ \ ) – 2$

$y = 3(2) – 2$

$y = 6 – 2$

$y = 4$

5. Test your solution (2, 4) in the other equation.

$y = -2x + 8$

$(4) = -2(2) + 8$

Solve by subsitution

$$6x + 2y = 3$$
$$y = -3x + 1$$

→ No solutions

$$6x + 2y = 3$$
$$y = -3x + 1$$

→ $y = -3x + \frac{3}{2}$

The lines were parallel.

x – 3y = 5
-x + 5y = 3

Solution: (17, 4)

$$2x - y = 2$$
$$4x + 2y = 8$$

Solution: (3/2, 1)

# Equations in Standard Form

$2x + y = 8$

$-3x + 3y = -3$

1. Solve both equations for the same variable.

$y = -2x + 8$ $\qquad$ $y = x - 1$

2. Substitute the value of the variable into the other equation.

$-2x + 8 = x - 1$

$+2x \qquad +2x$

$8 = 3x - 1$

$+1 \qquad +1$

$9 = 3x$

$\div 3 \quad \div 3$

$x = 3$

3. Solve for the single variable.

4. Substitute the value of the solved-for variable into <u>either equation</u>.

$2x + y = 8$

$2(3) + y = 8$

$6 + y = 8$

$y = 2$

5. Test your solution (2, 4) in the <u>other equation.</u>

$-3(3) + 3(2) = -3$

$-9 + 6 = -3$

When you solve algebraically, how do you know how many solutions there are? (1, 0, or infinite #)

$$6x + 2y = 4 \qquad 6x + 2(-3x + 2) = 4$$

$$y = -3x + 2 \qquad 6x - 6x + 4 = 4 \qquad 4 = 4$$

All the variables "disappeared" and the equation is true:

→ Infinitely many solutions

How can that be?

$$6x + 2y = 4 \longrightarrow y = -3x + 2$$

$$y = -3x + 2$$

Different versions of the same equation!

# How do you know how many solutions there are using the elimination method (1, 0, or infinite #) ?

When you perform the elimination step and <u>both</u> variables disappears and you get a number equal to another number:

a. and it's <u>true</u>:
( 3 = 3 or 0 = 0) → Infinite # of solutions (same line)

b. and it's <u>false</u>:
( -2 = 3 or 10 = 0) → No solutions (parallel lines)

Elimination Method: Eliminate one of the variables by adding the equations together.

$$x - 3y = 5$$
$$-x + 5y = 3$$

What property allows me to add equations together?

"Property of Equality"

Adding these equations will eliminate the 'x' variable.

$$2x - 3y = 5$$
$$-4x + 3y = 3$$

Adding these equations will eliminate the 'y' variable.

What variable will be eliminated if I add the following equations?

1. $$\begin{aligned} 2x + y &= -2 \\ -2x + 3y &= -8 \end{aligned}$$

2. $$\begin{aligned} 4x - 3y &= -2 \\ -2x + 3y &= -8 \end{aligned}$$

3. $$\begin{aligned} 3x + y &= -1 \\ 2x + 3y &= 18 \end{aligned}$$

Eliminate one of the variables by adding the equations together.

$$x - 3y = 5$$
$$-x + 5y = 3$$

$$x - x - 3y + 5y = 5 + 3$$

$$2y = 8$$

$$y = 4$$

$$x - 3(4) = 5$$

$$x = 17$$

Solution: (17, 4)

Replace ‘y’ with 4 in either of the original equations, then solve for ‘x’.

## Check the solution: (using substitution)

If your work indicated the solution to be (17, 4), replace 'x' with 17 and 'y' with 4 in both of the original equations, to see if the ordered pair (17, 4) is a solution to the system of equations.

$$x - 3y = 5$$
$$-x + 5y = 3$$

$$(17) - 3(4) = 5$$

$$-(17) + 5(4) = 3$$

Checks!

Checks!

Solution: (17, 4)

Solve

$$2x - 5y = 6$$
$$-x + 5y = 2$$

$$2x - x - 5y + 5y = 6 + 2$$

$$x = 8$$

$$-(8) + 5y = 2$$

$$5y = 10$$

$$y = 2$$

Replace ‘x’ with 8 in either of the original equations, then solve for ‘y’.

Solution: (8, 2)

Solve the equation using “elimination”

4x – 3 y = -2
-2x + 3y = -8

$$2x = -10$$

$$x = -5$$

$$-2(-5) + 3y = -8$$

$$10 + 3y = -8$$

$$3y = -18$$

$$y = -6$$

What if the coefficients are not the same?

$$5x - y = -2$$
$$-2x + 3y = -8$$

What is the LCM for the coefficients of ‘y’?

LCM = 3

You only have to fix one!

$$3*(5x - y) = -2 *3$$
$$-2x + 3y = -8$$

$$15x - 3y = -6$$
$$-2x + 3y = -8$$

$$3x - 4y = -10$$
$$6x + 3y = -42$$

(-2)3x – (-2)4y = -10(-2)

6x + 3y = -42

-6x + 8y = 20

6x + 3y = -42

$$11y = -22$$
$$y = -2$$

$$6x + 3(-2) = -42$$
$$6x - 6 = -42$$
$$6x = -36$$
$$x = -6$$

$$3x + 2y = 6$$
$$x - 4y = -12$$

$$3(0) + 2y = 6$$
$$(0) - 4y = -12$$

$$(2)3x + (2)2y = 6(2)$$
$$x - 4y = -12$$

$$2y = 6$$
$$-4y = -12$$

$$6x + 4y = 12$$
$$x - 4y = -12$$

$$y = 3$$

$$5x = 0$$

$$x = 0$$

Solution is (0, 3)

What if the coefficients are not the same?

$5x - 5y = -2$
$-2x + 3y = -8$

What is the LCM for the coefficients of ‘x’?

LCM = 10

You have to fix both!

$2*(5x - 5y) = -2*2$
$5*(-2x + 3y) = -8*5$

$10x - 10y = -4$
$-10x + 15y = -40$

## Linear Equation in 3 Variables:

$Ax + By + Cz = D$ $3x + 2y - z = 5$

System of Linear Equations: 3 equations, each with the same 3 variables

(3 equations in 3 unknowns)

$$Ax + By + Cz = D$$
$$Ex + Fy + Gz = H$$
$$Jx + Ky + Lz = M$$

# Solving by Elimination

Pick two equations and remove one of the variables.

$Ax + By + Cz = D$

$Ex + Fy + G z = H$

$Jx + K y + L z = M$

Pick two other equations and remove the same variable.

$x + y = G$

$x + y = G$

Solve the system of 2 equations in 2 variables.

Eq#1: $x + 2y - 2z = -15$

Eq#2: $2x + y - 5z = -21$

Eq#3: $x - 4y + z = 18$

Eq#1: $-2(x + 2y - 2z) = (-15)(-2)$

Eq#2 $2x + y - 5z = -21$

$-2x - 4y + 4z = 30$

Eq#1/#2 $-3y - z = 9$

Eq#1: $-1(x + 2y - 2z) = (-15)(-1)$

Eq#3: $x - 4y + z = 18$

$-x - 2y + 2z = 15$

Eq#1/#3 $-6y + 3z = 33$

Eq#1/#2 $-3y - z = 9$

$\div 3(-6y + 3z) = (33)(\div 3)$

Eq#1/#3 $-2y + z = 11$

$-3y - z = 9$

$-5y = 20$

$\boxed{y = -4}$

$-3(-4) - z = 9$

$12 - z = 9$

$\boxed{z = 3}$

$x - 4(-4) + (3) = 18$

$x + 16 + 3 = 18$

$\boxed{x = -1}$