Math-3
Lesson 7-4

Review

Compositions of Functions

f(x) = 2x + 3 and $g(x) = x^2$

f(g(x)) = ?

1. The input value to f(x) is g(x).

$$f(..) = 2(..) + 3$$

2. Replace the 'x' in f(x) with a set of parentheses.

$$f(x^2) = 2(x^2) + 3$$

3. Put the input value (g(x)) into the parentheses.

$$f(g(x)) = 2x^2 + 3$$

4. Find the output value.

Composition of Functions

f(x) = 2x + 1 g(x) = 3x + 2

$$f(g(x)) = ? \quad = 2(\qquad\qquad) + 1 \quad = 2(3x + 2) + 1$$

$$f(f(x)) = ? \quad = 2(\qquad\qquad) + 1 \quad = 2(2x + 1) + 1$$

$$f\left(g(4)\right) = ?$$

Relation: A "mapping" or pairing of input values to output values.

Function: A relation where each input has exactly one output.

There are at least 6 ways to show a relation between input and output values.

Data table:

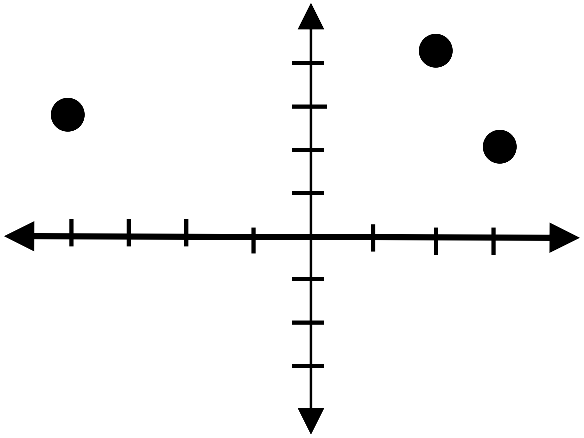
x	2	3	-4
y	4	2	3

Ordered Pairs:
(2, 4), (3, 2), (-4, 3)

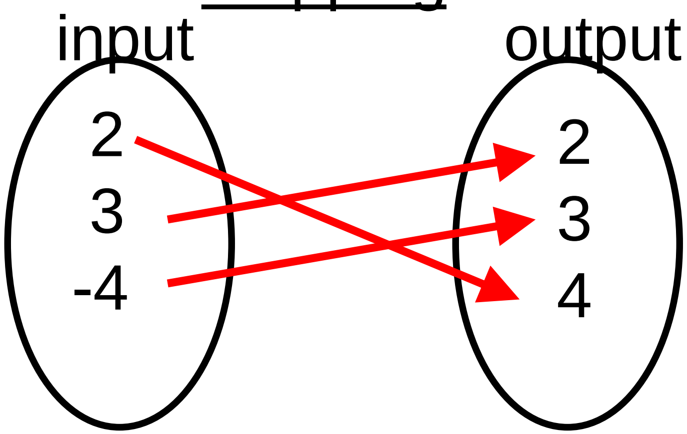
Equation: $y = 2x + 1$

Function notation: $f(2) = 4$

Graph:



Mapping



What is an "Inverse Function"?

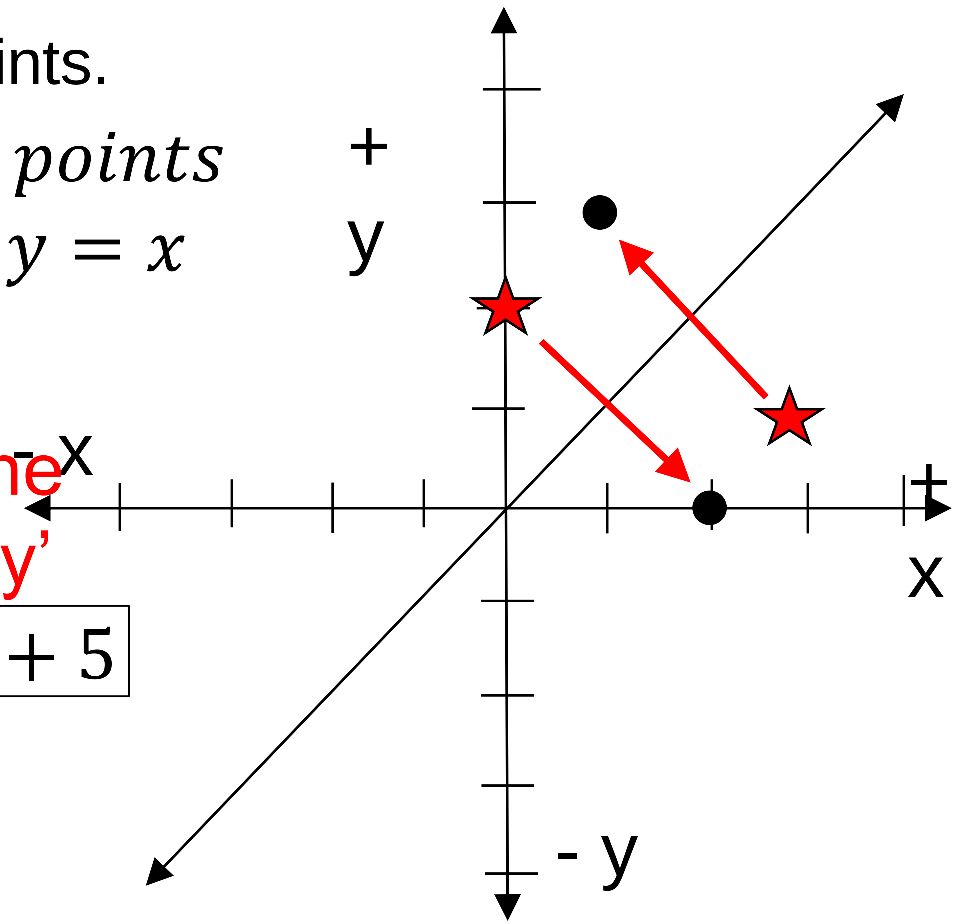
It depends upon which form of the function you're talking about.

Ordered Pairs: f(x) = (2, 4), (3, 2), (-4, 3)

$$f^{-1}(x) = (4, 2), (2, 3), (3, -4)$$

Graph: f(x) are the starred points.

$$f^{-1}(x)\ reflection\ of\ all\ points$$
$$in\ f(x)\ across\ the\ line\ y = x$$



Equation: $\boxed{f(x) = x - 5}$

Exchange 'x' and 'y' in the equation then solve for 'y'

$$y = x - 5$$
$$x = y - 5$$
$$x + 5 = y$$

$$\boxed{f^{-1}(x) = x + 5}$$

$$f(x) = (x+1)^{2/3} \qquad f^{-1}(x) = ?$$

$$x = (y+1)^{2/3}$$

$$x^{3/2} = \left((y+1)^{2/3} \right)^{3/2}$$

$$x^{3/2} = y+1$$

$$y = x^{3/2} - 1$$

Find the Inverse

1. $$y = x^4 - 3$$

2. $$y = (x-5)^{2/3}$$

3. $$y = (x+2)^5 - 3$$

4. $$f(x) = 1 + \sqrt{5x}$$

$$f(x) = \frac{2}{x-3} + 4$$

$$f^{-1}(x) = ?$$

$$x = \frac{2}{y-3} + 4$$

$$y - 3 = \frac{2}{(x-4)}$$

$$x - 4 = \frac{2}{y-3}$$

$$\boxed{y = \frac{2}{(x-4)} + 3}$$

$$(y-3)(x-4) = 2$$

$$f(x) = \frac{3x}{x+1} + 6$$

$$f^{-1}(x) = ?$$

$$x = \frac{3y}{y+1} + 6$$

$$x - 6 = \frac{3y}{y+1}$$

$$(y+1)(x-6) = 3y$$

multiply this out!

$$xy - 6y + x - 6 = 3y$$

$$xy - 6y - 3y = -x + 6$$

$$xy - 9y = -x + 6$$

$$y(x-9) = -x + 6$$

$$y = \frac{-x+6}{(x-9)}$$

Convert to logarithm form $\text{Log}_{\square}\square = \square$

$5^x = 25$

$4^x = 64$

$b^x = y$

$9^x = 81$

$10^x = 1000$

Convert to <u>exponential form</u>

$$\square^{\square} = \square$$

$$\log_{10} 100 = x$$

$$\log_3 27 = x$$

$$\log_9 1 = x$$

$$\log_4 x = 2$$

$$\log_2 x = 5$$

Solving an Exponential Equation: The easiest problem

$$2^x = 2^{4-x}$$

Exponents have to be equal to each other!

$$x = 4 - x$$

+x +x

$$2x = 4$$

÷2 ÷2

$$\boxed{x = 2}$$

Check your answer!

$$2^2 = 2^{4-2}$$

$$7^{2x+1} = 7^{13-4x}$$

$$2x + 1 = 13 - 4x$$

+4x +4x

$$6x + 1 = 13$$

-1 -1

$$6x = 12$$

$$\boxed{x = 2}$$

Solving using "convert to same base"

$$2^{4x-1} = 8^{x-1}$$

"convert to same base"

$$2^{4x-1} = (2^3)^{x-1}$$

Exponent of a power
Exponent Property

$$2^{4x-1} = 2^{3x-3}$$

$$\boxed{x = -2}$$

$$4x - 1 = 3x - 3$$

-3x -3x

$$x - 1 = -3$$

+1 +1

Check your answer!

$$2^{4(-2)-1} = 8^{-2-1}$$

$$2^{-9} = 8^{-3}$$

$$(\ 2^{-9} = 8^{-3}\ )^{-1}$$

$$2^9 = 8^3$$

$$512 = 512$$

Solving using "convert to same base"

$$9^{2x} = 27^{x-1}$$

"convert to same base"

$$\left(3^2\right)^{2x} = \left(3^3\right)^{x-1}$$

Power of a power
Exponent Property

$$3^{2*2x} = 3^{3(x-1)}$$

Check your answer!

$$9^{2(-3)} = 27^{-3-1}$$

$$3^{4x} = 3^{3x-3}$$

$$9^{-6} = 27^{-4}$$

$$4x = 3x - 3$$

$$(9^{-6} = 27^{-4})^{-1}$$

$$\boxed{x = -3}$$

$$9^{6} = 27^{4}$$

$$531441 = 531441$$

Solving using "log of a power" property

$$5^{2x} = 27^{x-1}$$

Take log base (your choices of base) of both side

$$log_5 5^{2x} = \log_5 27^{x-1}$$

"log of power property"

$$2x log_5 5 = (x-1)\log_5 27$$

$$2x log_5 5 = (x-1)(2.0478)$$

Distributive property

$$2x = 2.0478x - 2.0478$$

solve

+2.0478 +2.0478

$$2x + 2.0478 = 2.0478x$$

-2x -2x

$$2.0478 = 0.0478x$$

÷0.0478 ÷0.0478

$$x = 42.84$$

Solve for ‘x’ (how do you get the exponent ‘x’ all by itself?

$$8^x - 2 = 5$$ “Isolate the exponential”

+2 +2

$$8^x = 7$$ “convert to a log”

$$x = \log_8 7$$

$$\boxed{x = 0.9358}$$

Solve using "undo the exponential"

$$3^{2x-1}+5=7$$ "Isolate the exponential"

-5 -5

$$3^{2x-1}=2$$ "Undo the exponential"

$$2x-1=\log_3 2$$

$$2x-1=0.63093$$

+1 +1

$$2x=1.63093$$

÷2 ÷2

$$\boxed{x=0.815}$$

Check for extraneous solutions:

x = 10, ~~-5~~

$$\log 2x + \log(x - 5) = 2$$

$$\log(2 * 10) + log(10 - 5) = 2$$

$$\log(20) + log(5) = 2$$ All logarands are positive ☺

$$log 100 = 2$$ “Condense the product” $$10^2 = 100$$

Checks

$$\log(2)(-5) + log(-5 - 5) = 2$$

$$\log(-10) + log(-10) = 2$$ Negative logarands ☹

x = 5 is an extraneous solution.

More complicated Logarithmic Equations

$$2 + \log_2 5^{x-2} = 7$$

"Isolate the logarithm"

-2 -2

$$\log_2 5^{x-2} = 5$$

"undo the logarithm"

$$(x-2)\log_2 5 = 5$$

"push buttons"

$$(x-2)(2.3219) = 5$$

Divide 2.3219 left/right

$\div 2.3219$ $\div 2.3219$

$$x - 2 = 2.15338$$

Add '2' to both sides.

$+2$ $+2$

$$x = 4.1524$$

$$\log_4(5x-1)=3$$ Convert to exponential

$$5x-1=4^3$$ solve

$$5x-1=64$$ Add ‘1’ to both sides

$$5x=65$$ Divide both sides by ‘5’

$$x=13$$ Plug back in to check!

$$\log_4(5*13-1)=3$$

$$\log_4 64=3$$ Checks

$$\frac{x}{3} - \frac{x+1}{6}$$ Combine Ugly fractions with subtraction

$$\frac{2}{2} * \frac{x}{3} - \frac{(x+1)}{6}$$ Obtain common denominator

$$\rightarrow \frac{2x}{6} - \frac{(x+1)}{6}$$ Combine into one fraction

$$\rightarrow \frac{2x-(x+1)}{6} \qquad \rightarrow \frac{2x-x-1}{6} \qquad \boxed{\rightarrow \frac{x-1}{6}}$$

Solve Rational equations.

$$\frac{4}{x}+\frac{3x}{2}=-5$$ Obtain a common denominator

$$\frac{2}{2}*\frac{-4}{x}+\frac{3x}{2}*\frac{x}{x}=-5*\frac{2x}{2x}$$

$$\frac{-8}{2x}+\frac{3x^2}{2x}=\frac{-10x}{2x}$$ "undo" division by 2x

$$2x*\left(\frac{-8}{2x}+\frac{3x^2}{2x}\right)=\left(\frac{-10x}{2x}\right)*2x$$

$$3x^2-8=-10x$$ "non-standard" form

Solve Rational equations. $\frac{4}{x} + \frac{3x}{2} = 5$

$3x^2 - 8 = -10x$ Re-write in standard form

$3x^2 + 10x - 8 = 0$

Factor to solve

-8*3 = -24

-24 = -2 * 12

10 = -2 + 12

$3x^2 - 2x + 10x - 8 = 0$

Split 10x into -2x + 10x

Box factoring to solve

	x	4
3x	$3x^2$	12x
-2	-2x	-8

$(3x - 2)(x + 4) = 0$

x = 2/3 or x = -4

Find the zeroes of the following 3rd degree Polynomial

$$y = x^3 + 5x^2 + 4x$$

Set y = 0

$$0 = x^3 + 5x^2 + 4x$$

Factor out the common factor.

$$0 = x(x^2 + 5x + 4)$$

Factor the quadratic

$$0 = x(x+1)(x+4)$$

Identify the zeroes

0, -1, -4