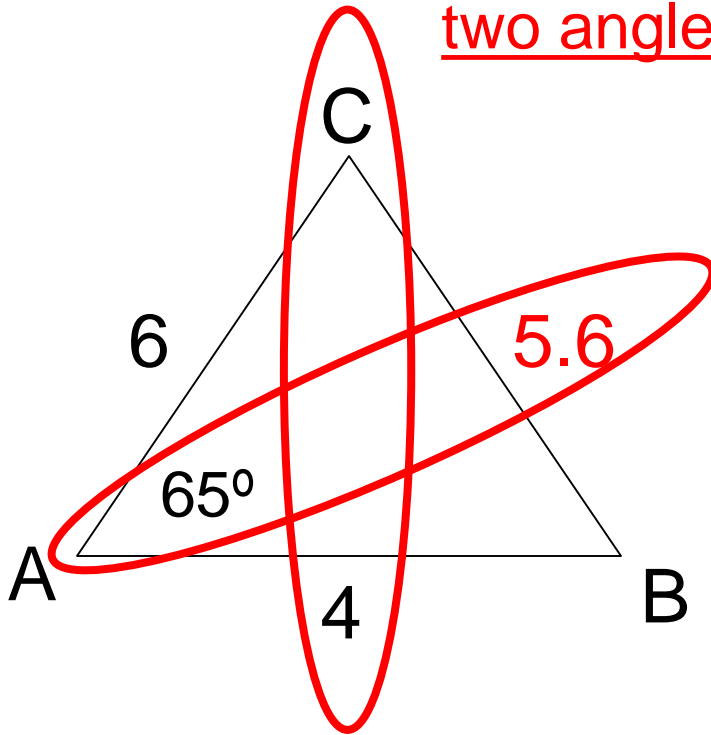# Math-3
# Lesson 6-9

Area of Triangles

Use Law of Sines to find the smaller of the two angles → Angle C.

C

6    5.6

65º

A    4    B

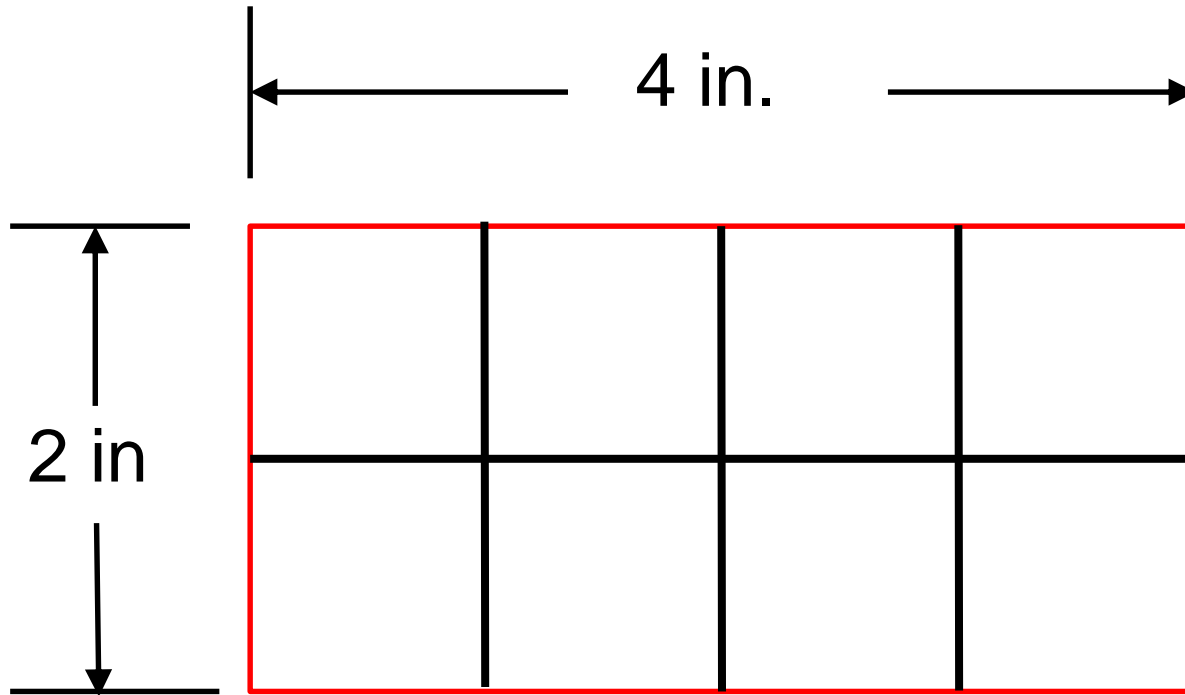$$\frac{\sin C}{4} = \frac{\sin 65}{5.6}$$

$$\sin C = \frac{4\sin 65}{5.6}$$

$$C = \sin^{-1}\left(\frac{4\sin 65}{5.6}\right)$$

$$C = 40.3°$$

$$B = 180° - 65° - 40.3°$$
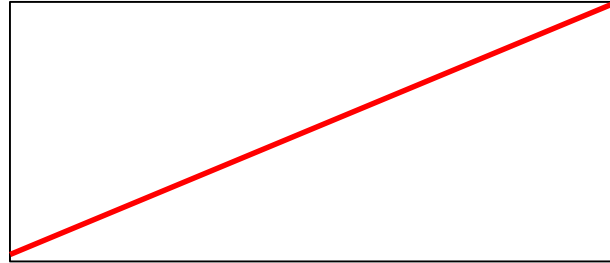
$$B = 74.7°$$

# The <u>area</u> of this rectangle is….?



$$\text{area}_{\text{rectangle}} = \text{length} * \text{width}$$

Rectangle area formula.

$$A_{rectangle} = L * W$$
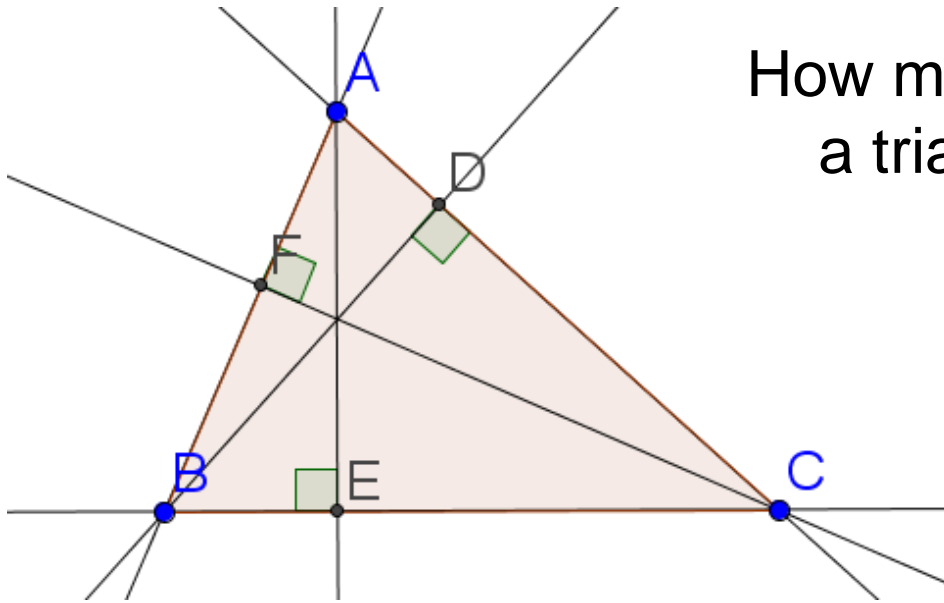
W = width

L = length

Triangle area formula.

$$A_{triangle} = \frac{1}{2} * A_{rectangle} = \frac{1}{2} * L * W$$

$$A_{triangle} = \frac{1}{2} * B * h$$

<u>Altitude of a triangle</u>: The perpendicular distance from
   any vertex to its opposite side.

<u>Altitude of a triangle</u>: means the same thing as the <u>height</u>
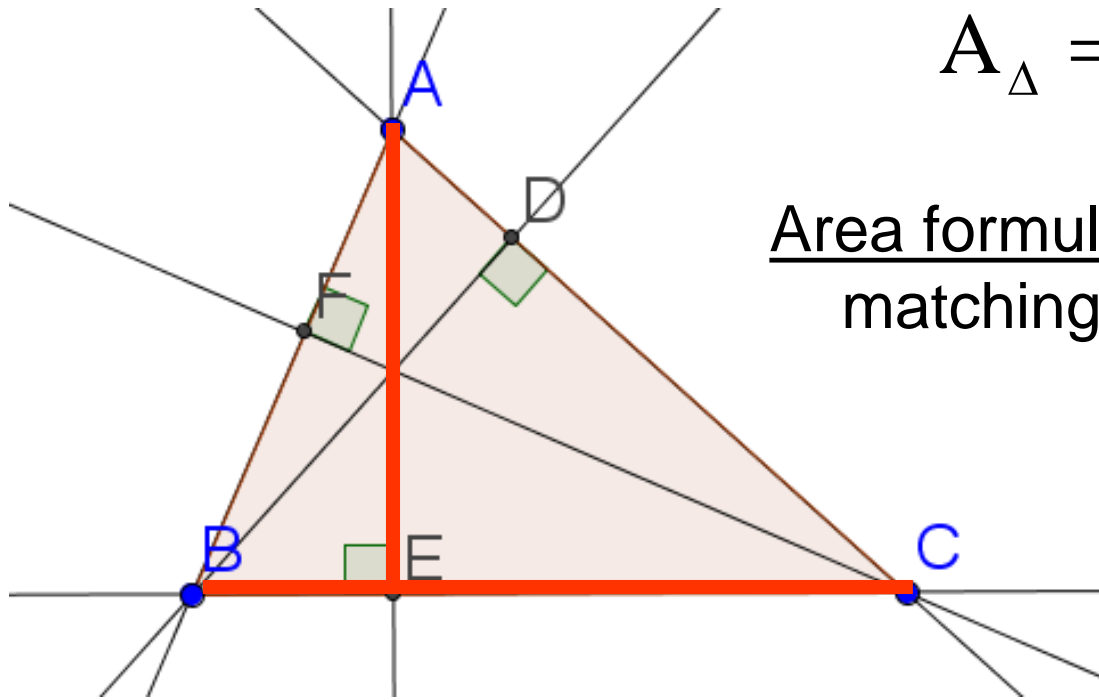   <u>of a triangle</u>.

<u>Height = Altitude</u>

How many "heights" (altitudes) does
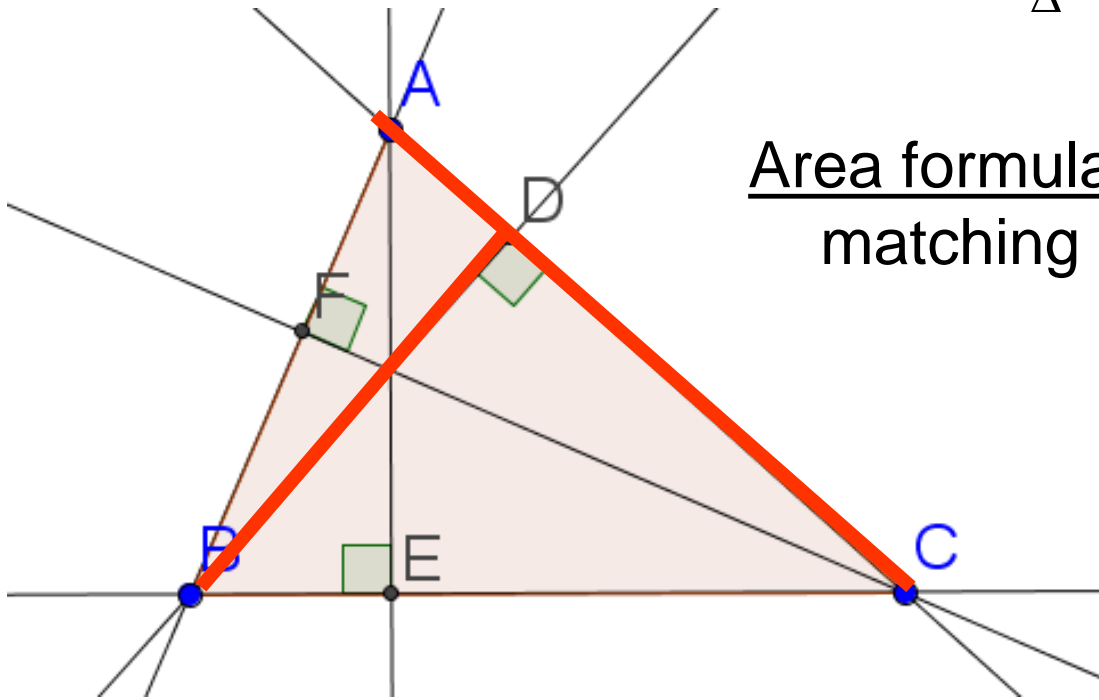   a triangle have?

<u>three</u>

# Triangle Area

$$A_{\Delta} = \frac{1}{2} * base * height$$

Area formula: requires the use of matching heights and sides.

Using segment BC as the base, requires the use of segment AE as the height.

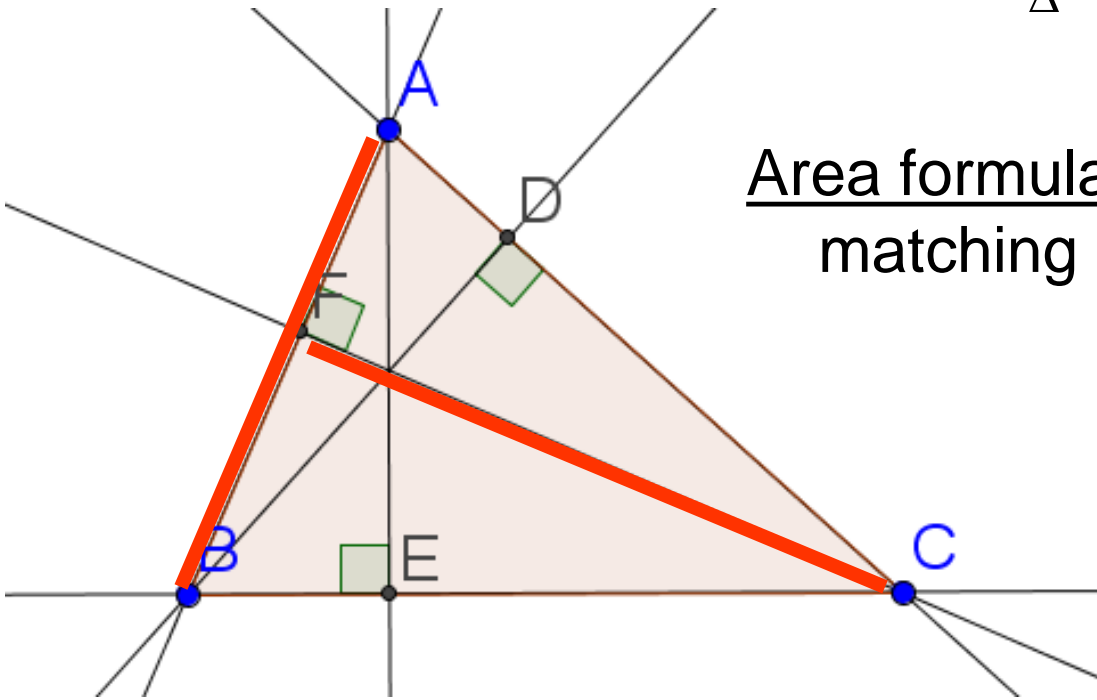$$A_\triangle = \frac{1}{2} * \text{base} * \text{height}$$

Area formula: requires the use of matching altitudes and sides.

Using <u>segment AC as the base</u>, requires the use of <u>segment BD as the height</u>.

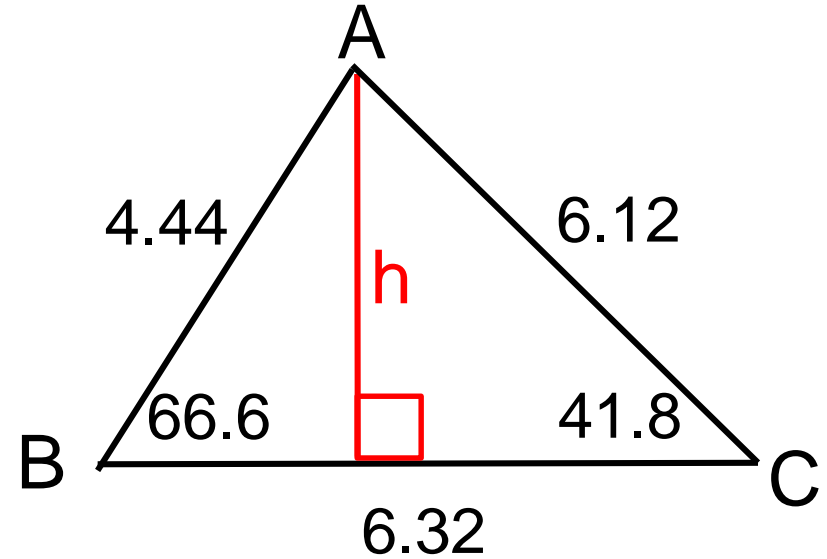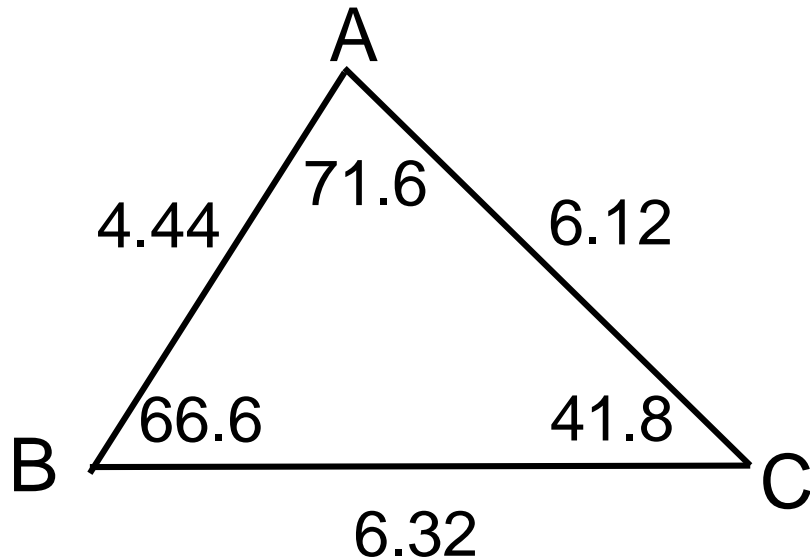$$A_\triangle = \frac{1}{2} * \text{base} * \text{height}$$

**Area formula**: requires the use of matching altitudes and sides.

Using <u>segment AB as the base</u>, requires the use of <u>segment FC as the height</u>.

Find the triangle area. (Use the altitude from point A as its height.)

A

71.6

4.44        6.12

66.6        41.8

B

6.32

C

A

4.44        6.12

h

66.6        41.8

B

6.32

C

$$A_\Delta = \frac{1}{2} * base * height$$
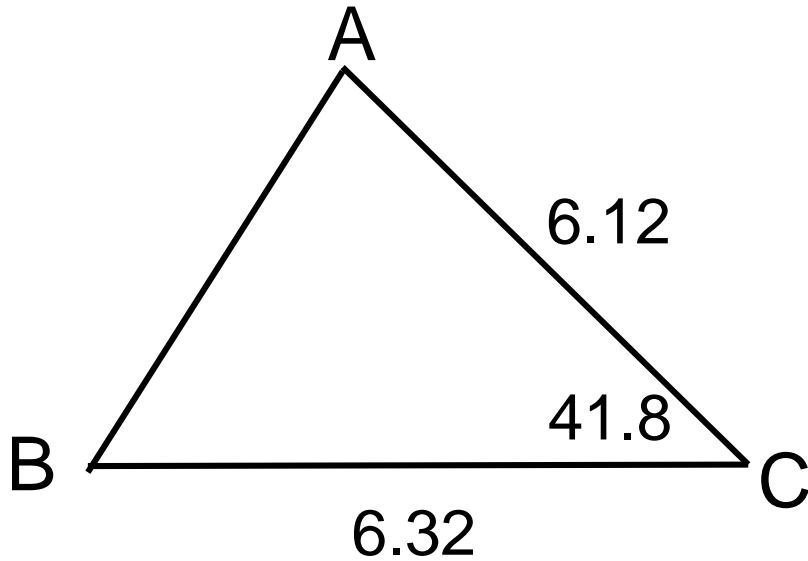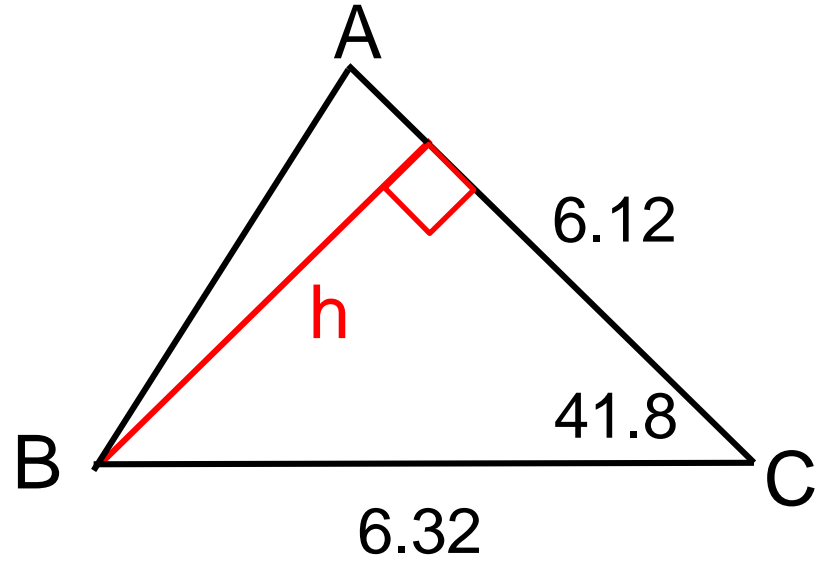
$$\frac{h}{6.12} = \sin 41.8°$$

$$h = 6.12 \sin 41.8°$$

$$A_\Delta = \frac{1}{2} * (6.32) * (6.12 sin(41.8))$$

$$A_\Delta = 12.89 \text{ units}^2$$

Find the triangle area. (Use the altitude from point B as its height.)

A

6.12

B

41.8

C

6.32

$$A_\Delta = \frac{1}{2} * \text{base} * \text{height}$$

A

6.12

h

41.8

B

C

6.32

$$\frac{h}{6.32} = \sin 41.8°$$

$$h = 6.32 \sin 41.8°$$

$$A_\Delta = \frac{1}{2} * (6.12) * (6.32 sin(41.8))$$

$$A_\Delta = 12.89 \text{ units}^2$$

There are <u>two</u> right triangles that can be used to solve for 'h'.

$$\sin A = \frac{opp}{hyp}$$

$$\sin(48) = \frac{h}{8}$$

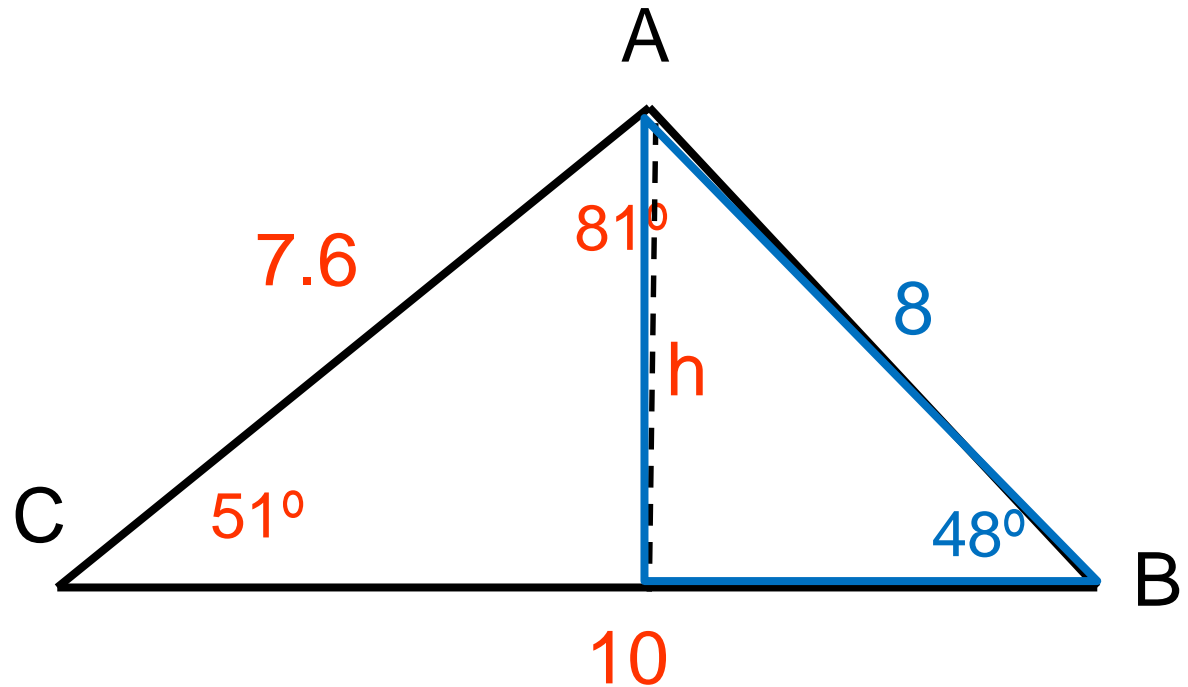h = 8*Sin(48⁰)

h = 5.9

$$Area = 0.5 * base * height$$

Area = ½(10)(5.9)

Area = 29.5 square units

A

7.6

81⁰

8

h

C

51⁰

48⁰

B

10

What is the area of $\triangle ABC$ given that $c = 25, b = 23,$ and $a = 14.$

**2.** Using angle C, find the appropriate height..
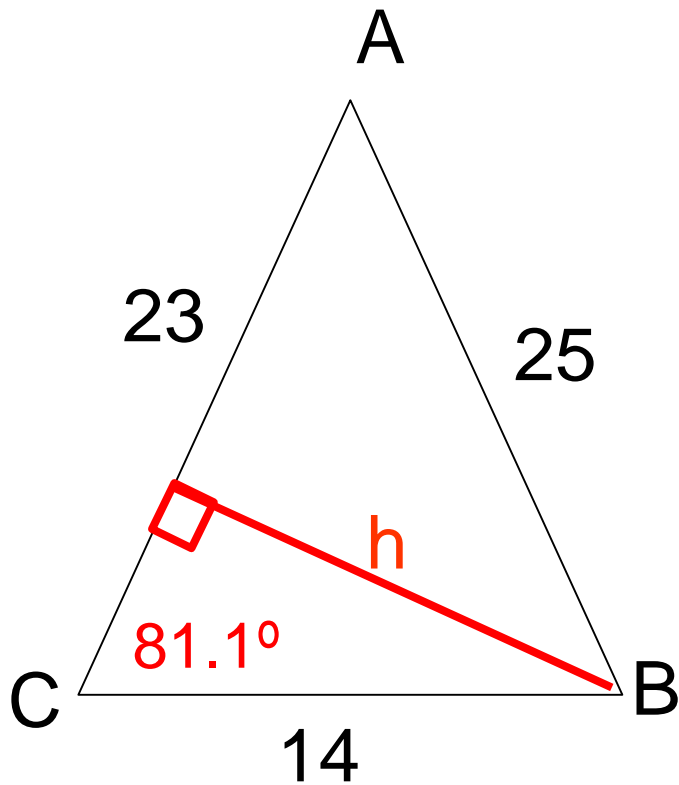


$$\sin 81.1 = \frac{h}{14}$$

$$h = 14\sin 81.1$$

$$h = 13.8$$

$$Area = 0.5 * \text{base} * \text{height}$$

$$Area = 0.5(23)(13.8)$$

$$Area = 158.7 \text{ units}^2$$

**Let's look at it again.**

$$Area = 0.5 * base * height$$
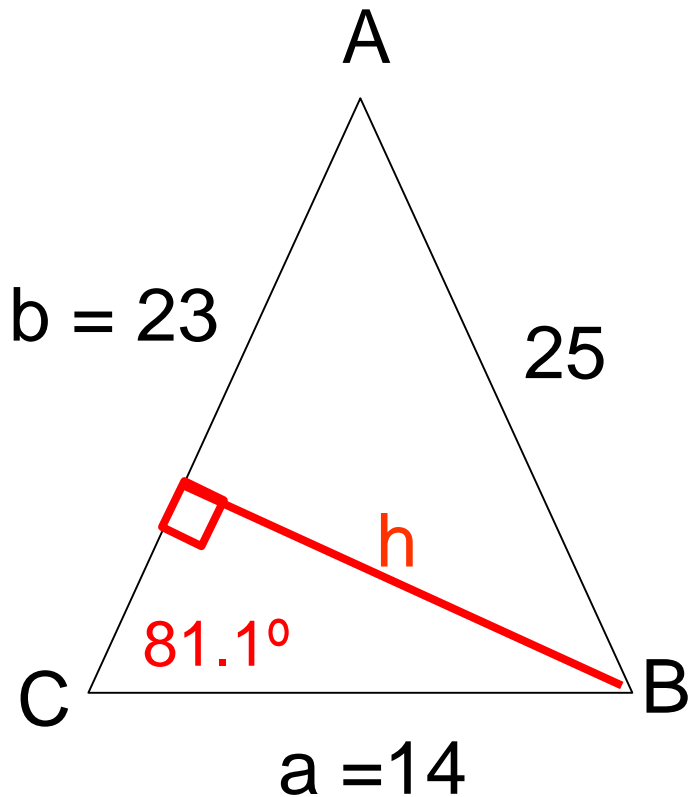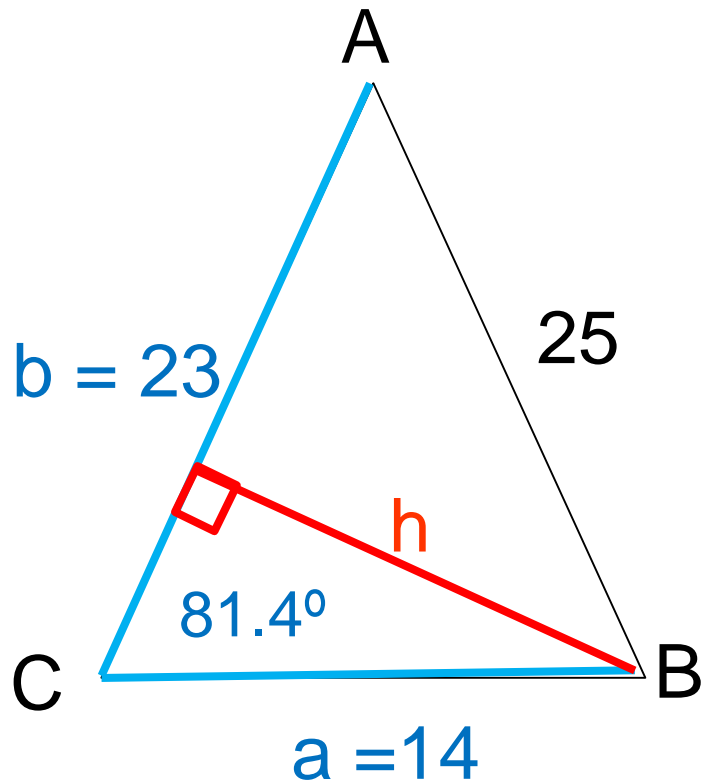
$$Area = 0.5(23)(13.8)$$

$$h = 14 \sin 81.1$$

$$h = a \sin C$$

$$Area = 0.5 * a * b * \sin C$$

A

b = 23

25

h

81.1°

C

B

a = 14

**Let's look at it again.**

**SIDE ANGLE SIDE**

A
b = 23
25
h
81.4°
C
a = 14
B

$Area = 0.5 * base * height$

$Area = 0.5(23)(13.8)$

$h = 14 \sin 81.4$

$h = a \sin C$

$Area = 0.5 * a * b * \sin C$

**Notice: in the formula you see all three letters 'a', 'b', and 'C'**

Deriving "<u>Heron's</u>" Formula

$$Area = \frac{1}{2} * base * height$$

$$\sin A = \frac{opp}{hyp}$$

Substitute into area formula

$$\sin C = \frac{h}{b}$$

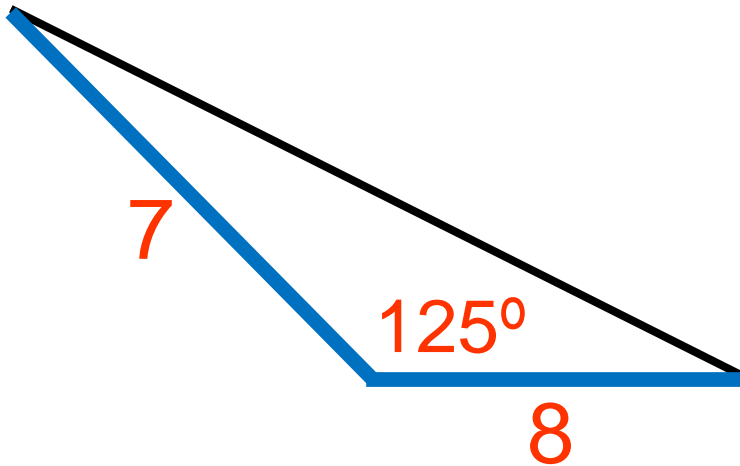$$Area = \frac{1}{2} * base * \boxed{height}$$

$$h = b \sin C$$

Area = ½*a*b*sin C

**SIDE ANGLE SIDE**



**Notice: in the formula you see all three letters 'a', 'b', and 'C'**
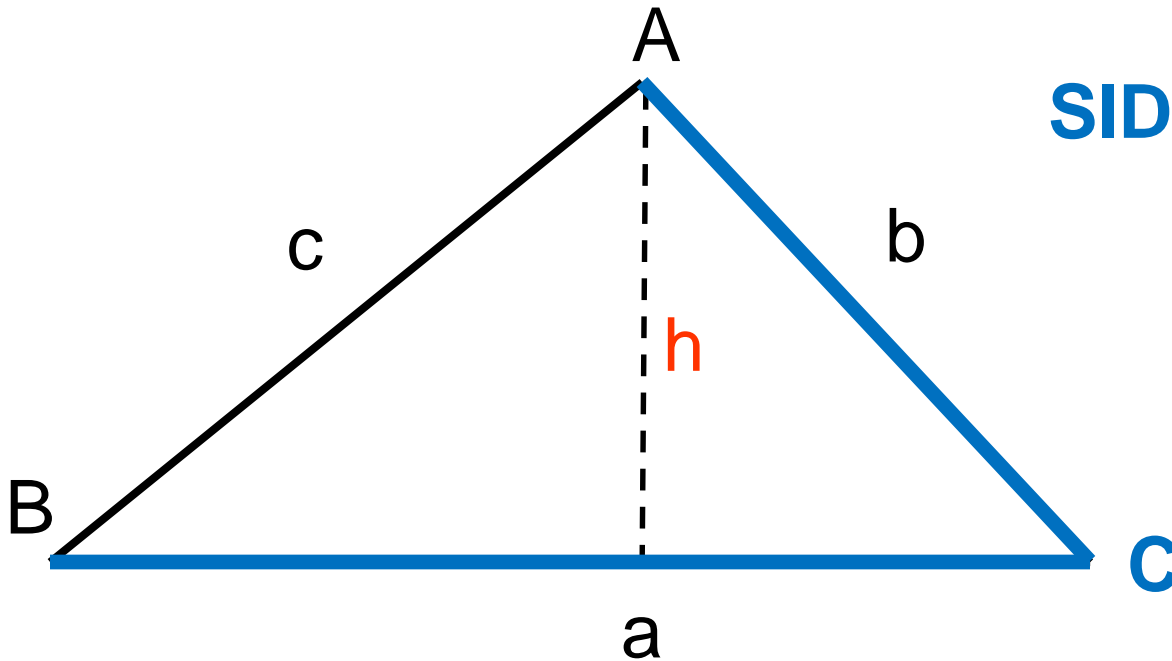
Find the area of triangle ABC

Area = ½*a*b*sin C

7

125⁰

8

Area = ½(7)(8)sin(25)

Area = 11.8

**SIDE ANGLE SIDE**