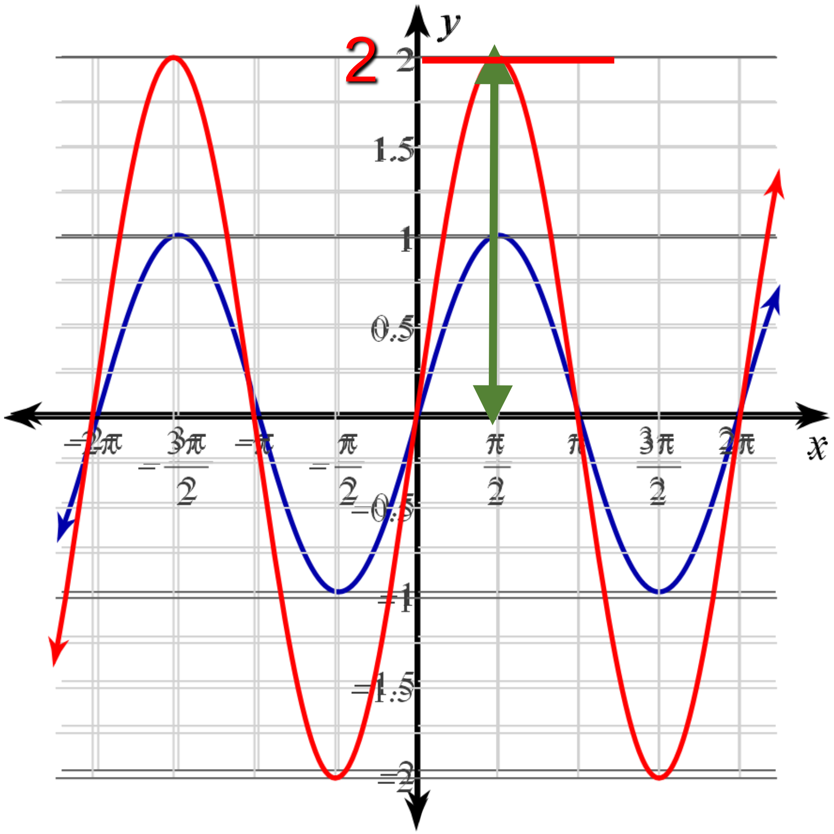
Math-3
Lesson 6-8
Periodic Behavior

Compare:

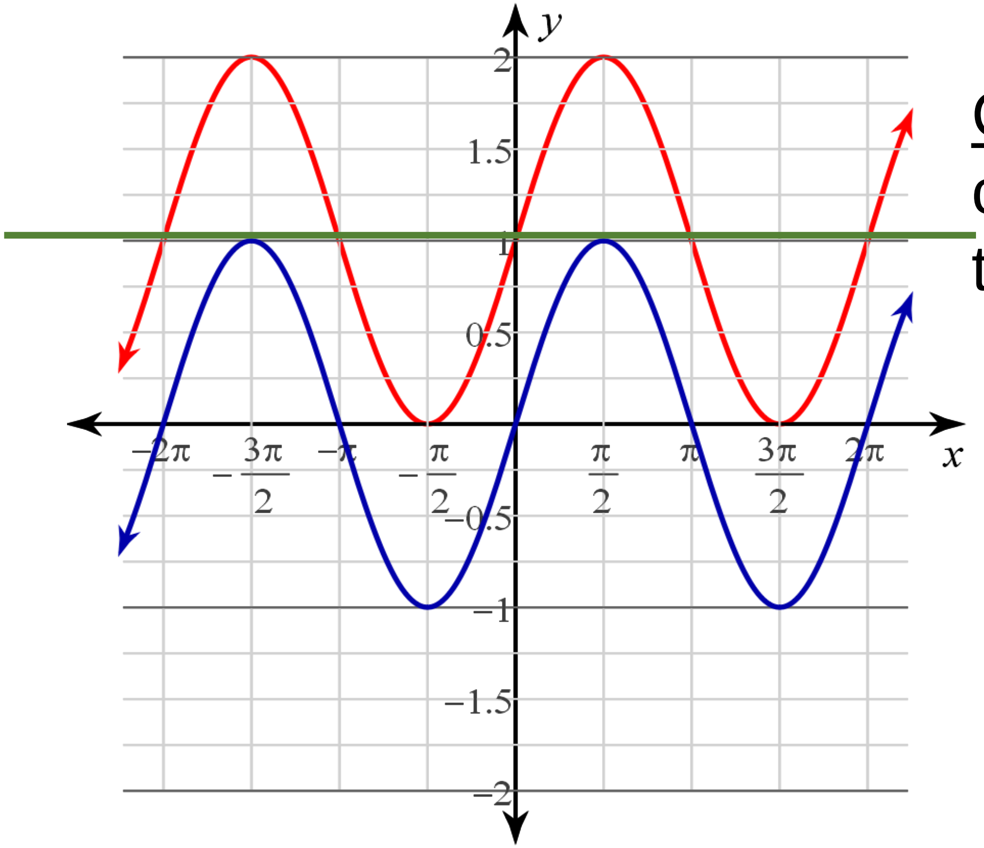
$$f(x) = \sin x \qquad g(x) = 2\sin x$$



$g(x) = \sin x$

$f(x) = 1 + \sin x$

shift



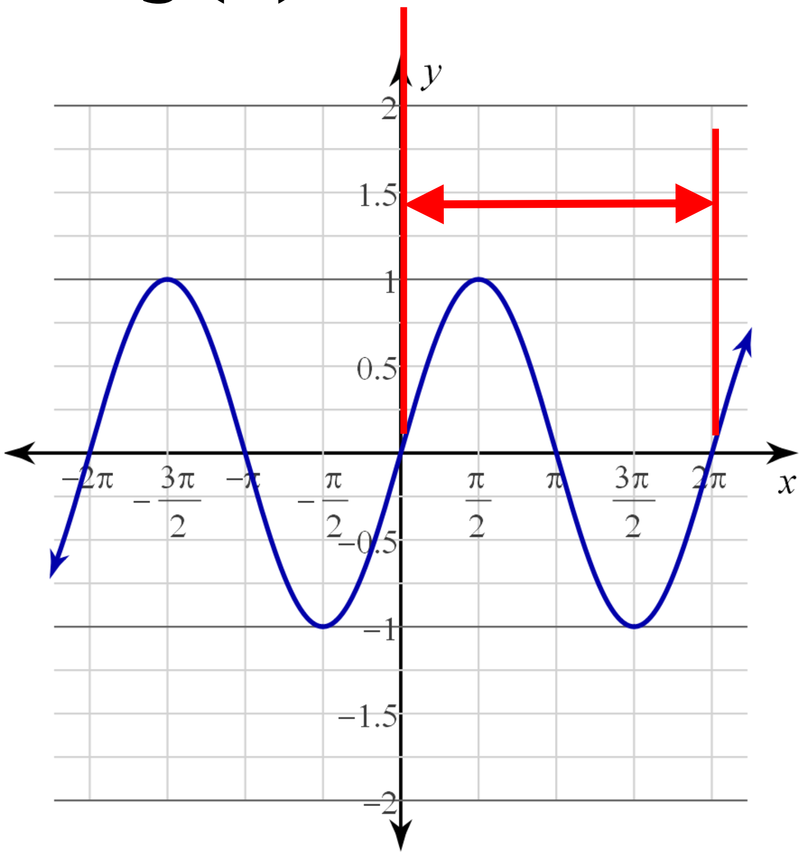
<u>Centerline of the Oscillation</u>: corresponds to the up/down translation.

Vocabulary

Period: the horizontal distance needed to complete one full cycle of the oscillation.

$$g(x) = \sin x$$

$$Period = 2\pi$$



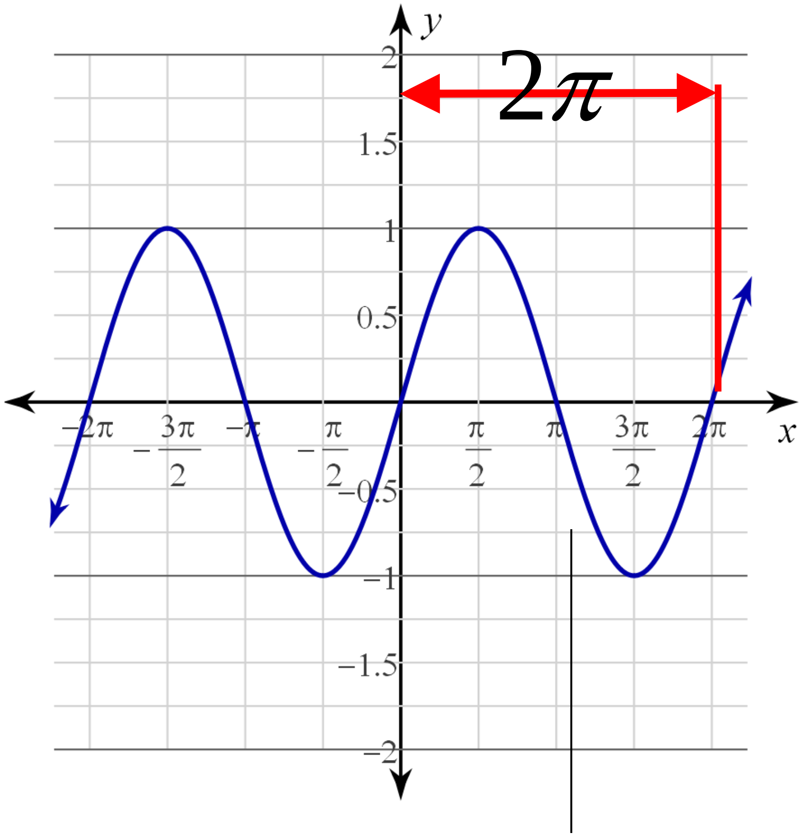
<u>Period:</u> the horizontal distance needed to complete one full cycle of the oscillation.

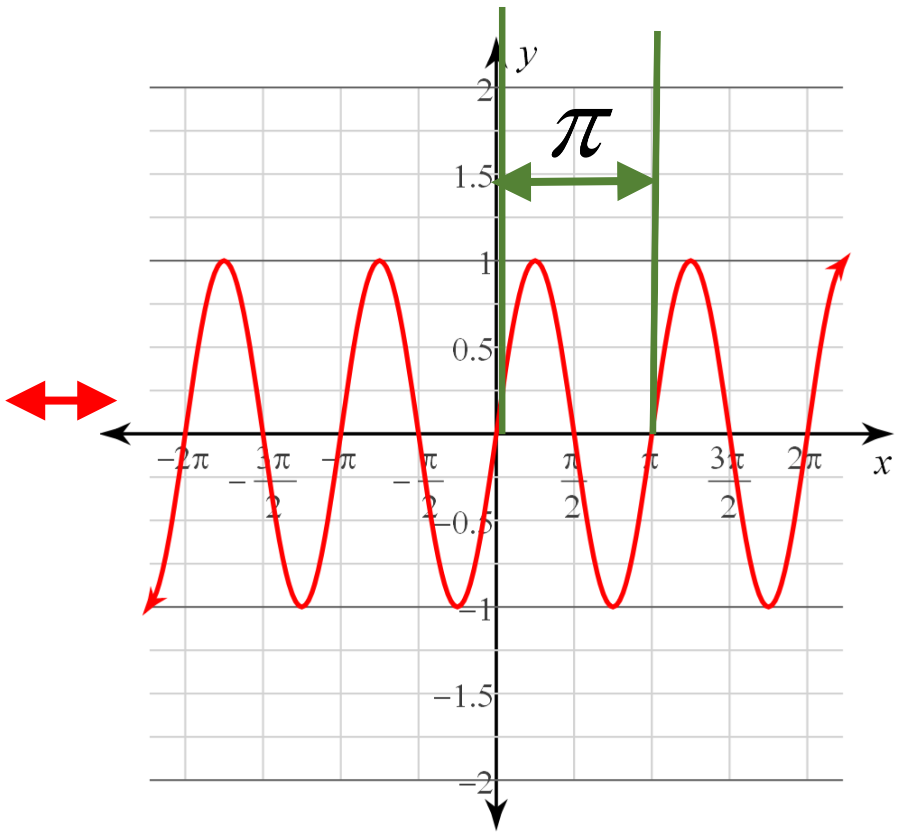
$$g(x) = \sin x$$

$$Period = 2\pi$$

$$f(x) = \sin 2x$$

$$Period = \frac{1}{2} * 2\pi \ = \pi$$





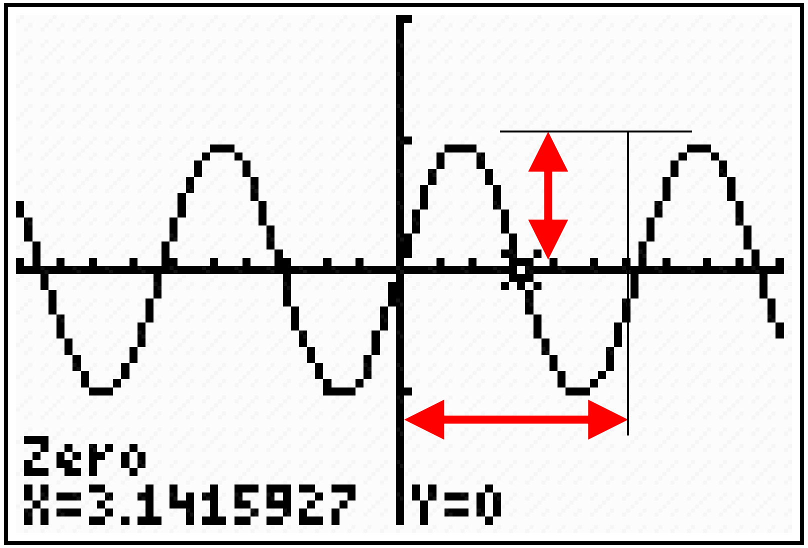
Sinusoid

$$f(x) = a \sin bx$$

If a is negative: reflection across x-axis

1. Amplitude:
   (½ of peak to peak distance) = $|a|$

2. Period: length of horizontal axis encompassing one complete cycle = $\dfrac{2\pi}{|b|}$



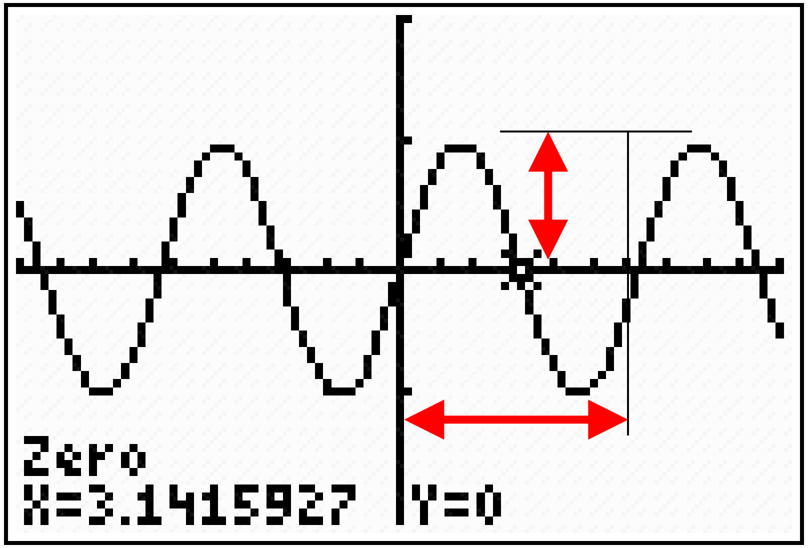
3. Frequency = $\dfrac{|b|}{2\pi}$

Frequency = 1/period

Sinusoid $f(x) = 3\sin 2x$

1. Amplitude:
(½ of peak to peak distance) = $|a| = 3$

2. Period: length of horizontal axis encompassing one complete cycle = $\frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$



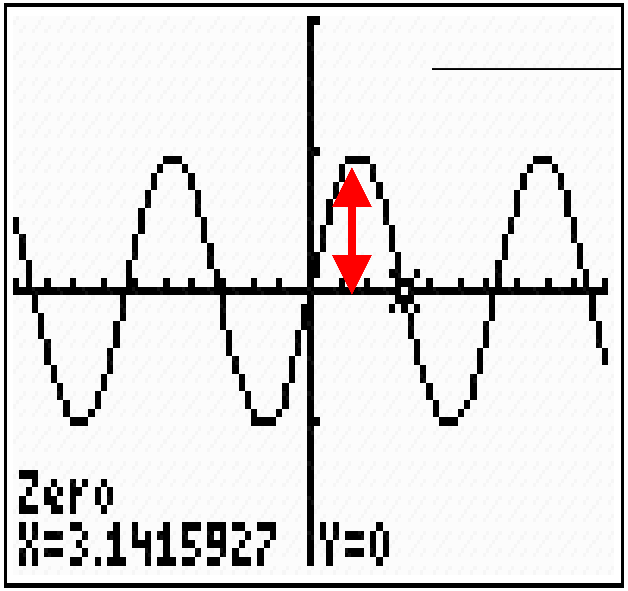
3. Frequency = $\frac{|b|}{2\pi} = \frac{2}{2\pi} = \frac{1}{\pi}$

Frequency = 1/period

Sinusoid: a function that models height vs. time.

$$h(t) = a\sin(bt) + k$$

What is the "centerline" of the sinusoid? $y = k$



Amplitude: The maximum distance above/below the centerline. $|a|$

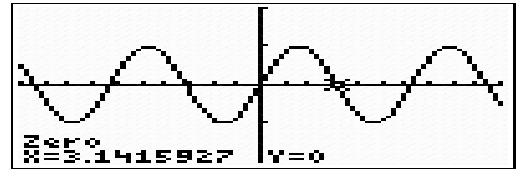
Horizontal stretch factor: $\frac{1}{|b|}$

Period = HSF * period of the parent (either 360 degrees, or 2π radians)

Period = $\frac{2\pi}{|b|}, \frac{360^\circ}{|b|}$ (time required to complete one cycle)

Frequency = $\frac{|b|}{2\pi}$ (# of oscillations per unit time: "Hz")

$$f(x) = a\sin\left(bx - h\right) + k$$



Horizontal stretch factor and horizontal shift <u>must be separated</u> (factor off the coefficient of ‘x’)

$$f(x) = a\sin b\left((x - h/b\right) + d$$

$$f(x) = 3\sin\left(2x + \pi/3\right) + 5 \;\; = 3\sin 2\left(x + \pi/6\right) + 5$$

1. <u>Amplitude</u>: (½ of peak to peak distance) = $|3| = 3$

2. <u>Left/right shift</u>: left $\pi/6$ radians

3. <u>Horizontal stretch factor</u> = reciprocal of the coefficient of the input variable. $1/b = 1/2$

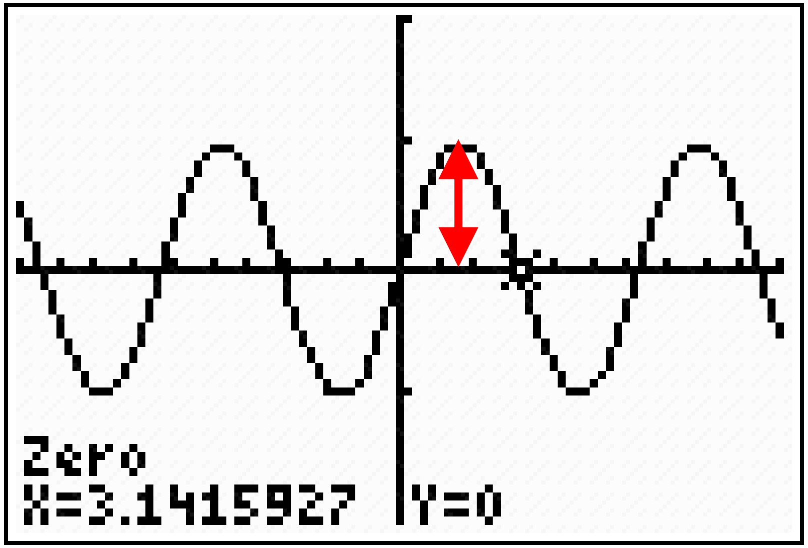
4. <u>Period of the function</u> = HSF * pi or HSF * 360 $\frac{2\pi}{2} = \pi$

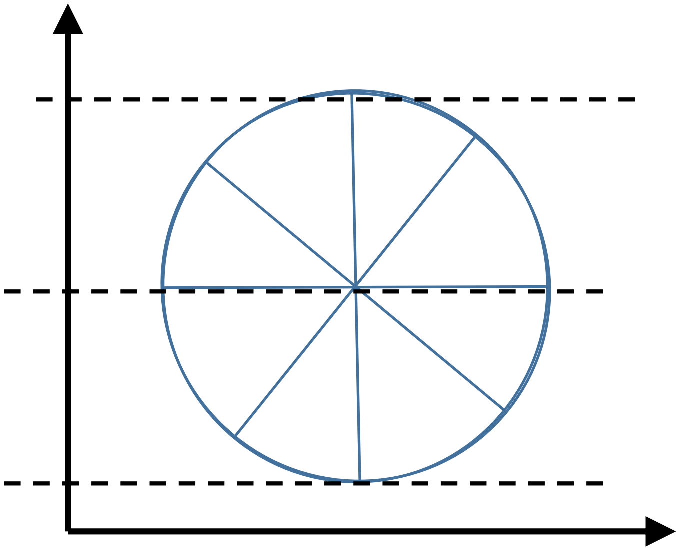
4. <u>Centerline of oscillation</u> = vertical shift number “k”: y = 5

Sinusoid: $h(t) = a\sin(bt) + k$

The Radius of the Lagoon Ferris Wheel is 21.8 m. The bottom of the Ferris Wheel is 4 feet above ground level (you have to go up onto a platform to get on the Ferris Wheel). Once the Ferris Wheel is spinning it takes 40 seconds to complete one revolution.

Write an equation that models the height of a point on the Ferris wheel as a function of time.

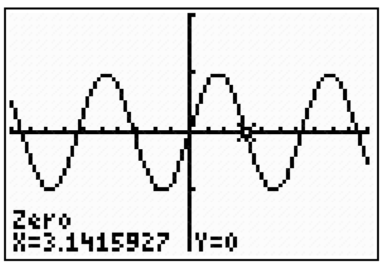




Sinusoid: $h(t) = a\sin(bt) + k$

The Radius of the Lagoon Ferris Wheel is 21.8 m. The bottom of the Ferris Wheel is 1.2 m above ground level (you have to go up onto a platform to get on the Ferris Wheel). Once the Ferris Wheel is spinning it takes 40 seconds to complete one revolution.

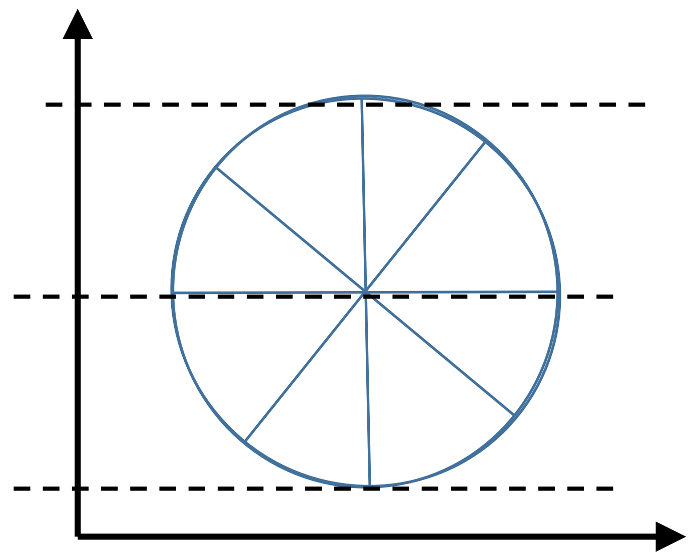
Period = $\dfrac{2\pi}{|b|}, \dfrac{360^\circ}{|b|}$ (time required to complete one cycle)



$$40 = \frac{360}{|b|}$$

$$b = \frac{360}{40} = 9$$

$$h(t) = a\sin(9t) + k$$



Sinusoid: $h(t) = a\sin(bt) + k$ $h(t) = a\sin(9t) + k$

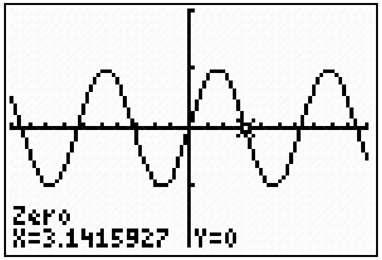
The Radius of the Lagoon Ferris Wheel is 21.8 m. The bottom of the Ferris Wheel is 1.2 m above ground level (you have to go up onto a platform to get on the Ferris Wheel). Once the Ferris Wheel is spinning it takes 40 seconds to complete one revolution.

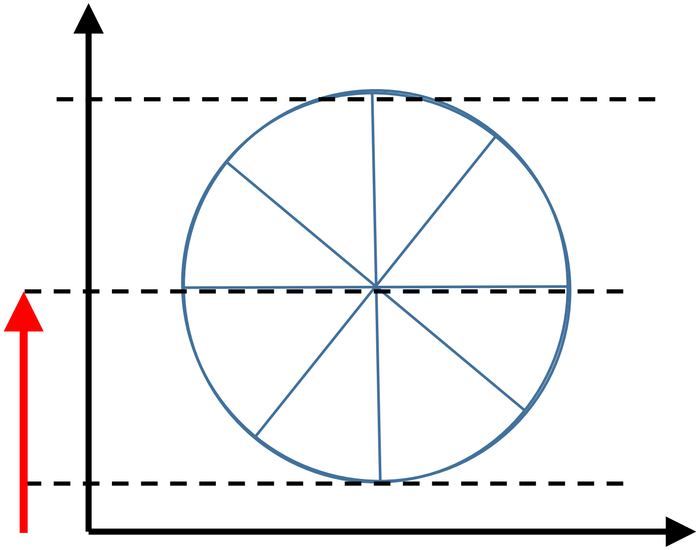
k = Vertical shift = centerline of the oscillation.

$$k = 1.2\text{ m} + 21.8\text{ m}$$

$$k = 23\text{ m}$$

$$h(t) = a\sin(9t) + 23$$





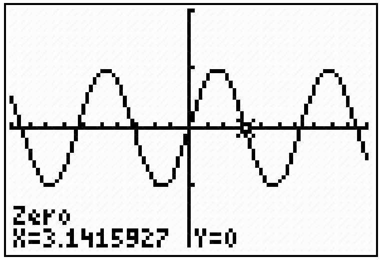
Sinusoid: $h(t) = a\sin(bt) + k$ $h(t) = a\sin(9t) + 23$

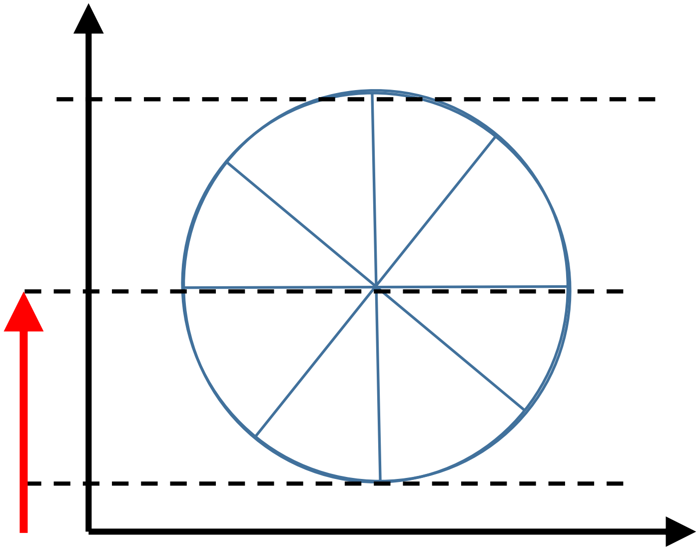
The Radius of the Lagoon Ferris Wheel is 21.8 m. The bottom of the Ferris Wheel is 1.2 m above ground level (you have to go up onto a platform to get on the Ferris Wheel). Once the Ferris Wheel is spinning it takes 40 seconds to complete one revolution.

amplitude = one half the "peak to peak" distance = circle radius

$a = 21.8 \text{ m}$

$h(t) = 21.8\sin(9t) + 23$





Harmonic Motion $d(t) = a\sin(bt) + k$

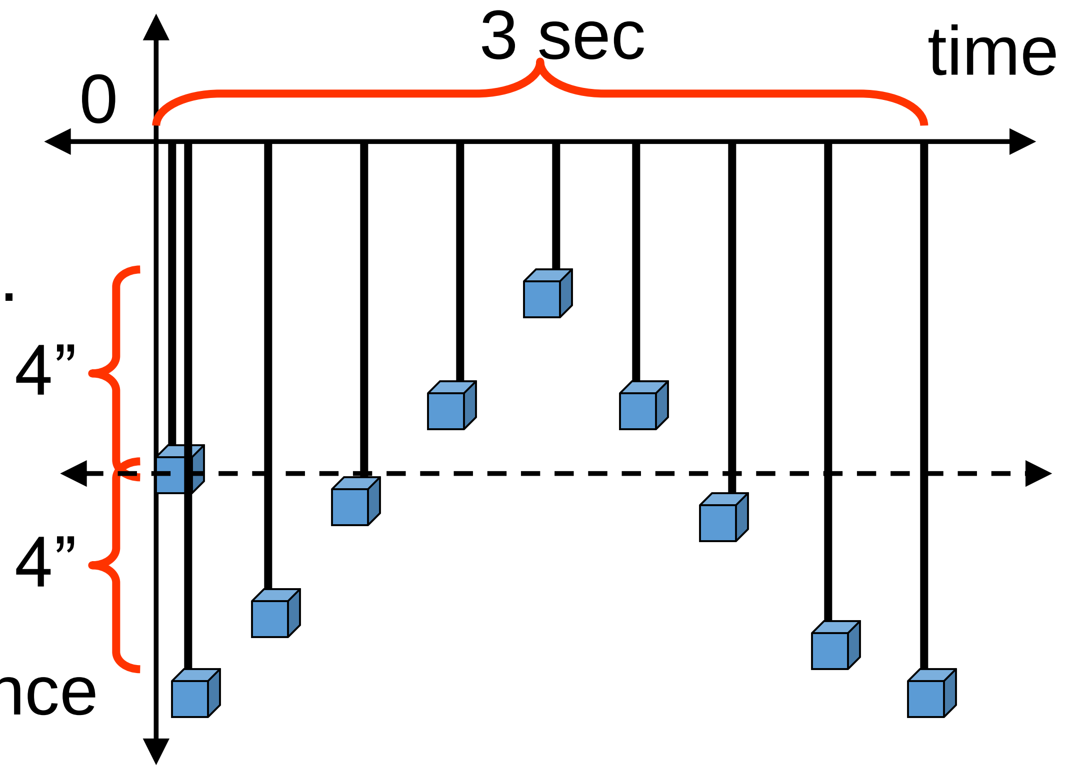
A mass oscillating up and down on the bottom of a spring can be modeled as harmonic motion (assuming perfect elasticity and no friction or air resistance).

If the weight is displaced a maximum of 4 cm, find the modeling equation if it takes 3 seconds to complete one cycle.

1. Draw the picture.

x-axis: time

y-axis: Distance below spring attachment point.



2. Write the equation.

$$d = a\sin(\omega t)$$

Calculating Harmonic Motion

1. Draw the picture. x-axis: time y-axis: Distance below spring attachment point.

2. Write the equation.

$$d = a\sin(\omega t)$$

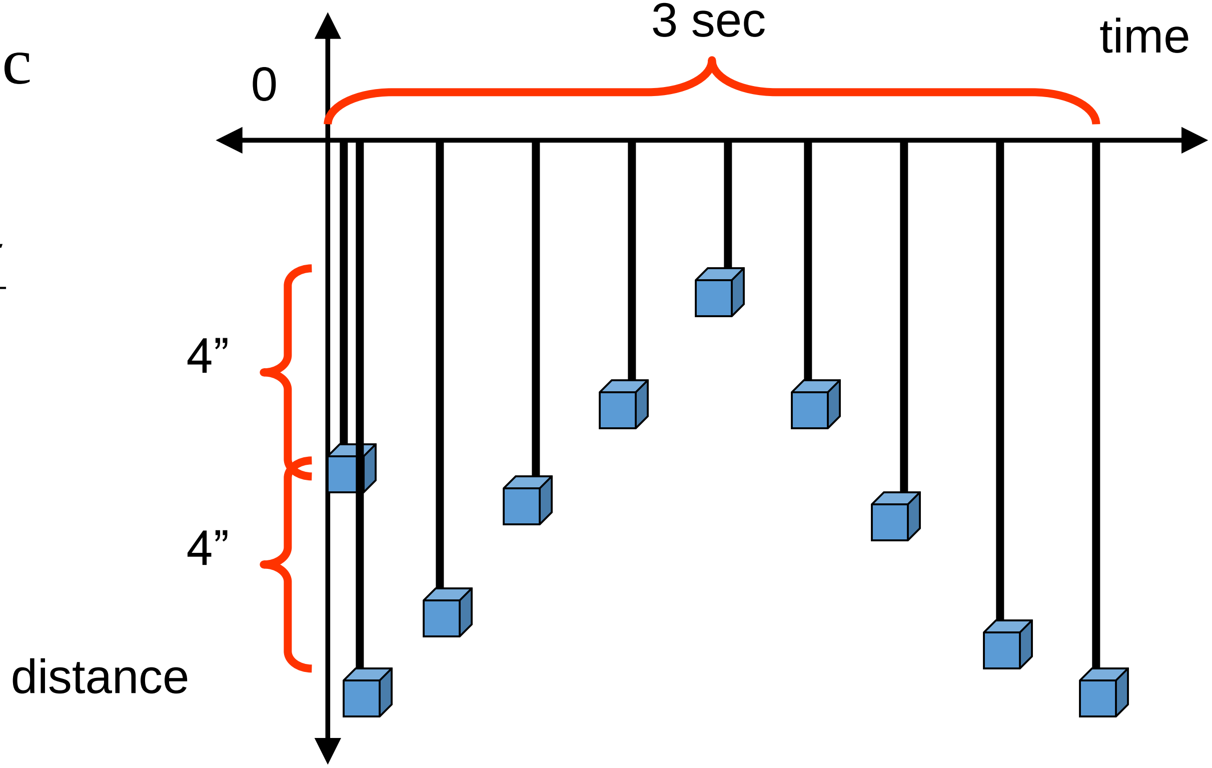
3. Solve the equation.

Amplitude = 4"

$$period = \frac{2\pi}{\omega} = 3\sec$$

$$\omega = 2\pi * \frac{1}{3} hz \qquad \omega = \frac{2\pi}{3}$$

$$\boxed{d = 4\sin\left(\frac{2\pi}{3}t\right)}$$



Your turn:

5. A mass oscillating up and down on the bottom of a spring can be modeled as harmonic motion (assuming perfect elasticity and no friction or air resistance).

If the weight is displaced a maximum of 2 cm, find the modeling equation if it takes 6 seconds to complete one cycle.

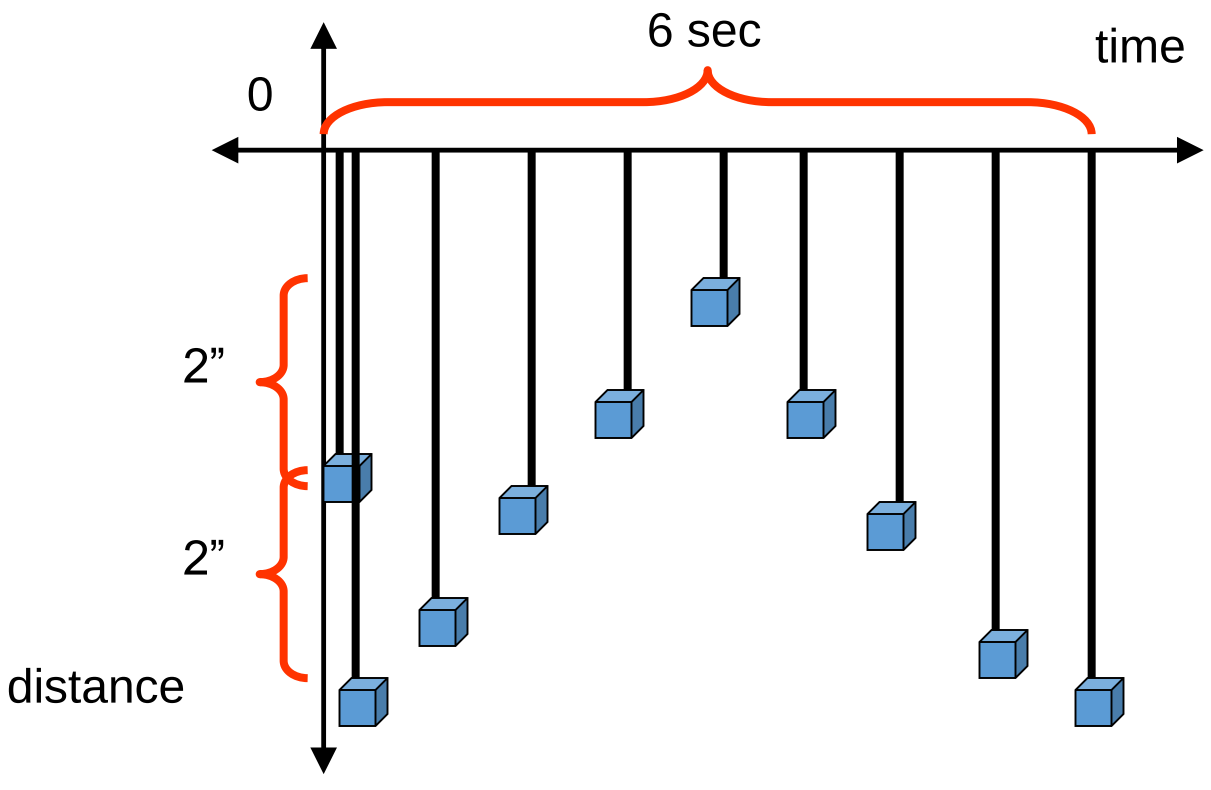
1. Draw the picture.

x-axis: time

y-axis: Distance below spring attachment point.

2. Write the equation.

$$d = a\sin(\omega t)$$



Alternation Current (AC)

Frequency: 60 Hz
Amplitude: 120 volts

60 Hz = 60 cycles per second.

Neglecting any phase shift, what is the equation that models the voltage as a function of time?

$V(t) = a\sin bt$ Amplitude = 120 $V(t) = 120\sin bt$

We take sines of *angles* so we must have a unit conversion factor (from time (in seconds) to degrees) in the coefficient of 't'.

$$Period = \frac{2\pi}{b} \qquad frequency = \frac{b}{2\pi} \qquad \frac{60}{sec} = \frac{b}{2\pi}$$

$$b = 120\pi\frac{degree}{sec} \qquad V(t) = 120\sin 120\pi t$$

Alternation Current (AC)

Frequency: 60 Hz

Amplitude: 120 volts

60 Hz = 60 cycles per second.

Neglecting any phase shift, what is the equation that models the voltage as a function of time?

$V(t) = a\sin bt$ Amplitude = 120 $V(t) = 120\sin bt$

We take sines of *angles* so we must have a unit conversion factor (from time (in seconds) to degrees) in the coefficient of 't'.

$$Period = \frac{360}{b} \qquad frequency = \frac{b}{360} \qquad \frac{60}{sec} = \frac{b}{360^0}$$

$$b = 21600 \frac{degree}{sec} \qquad V(t) = 120\sin 21600t$$

A tuning fork vibrates at a frequency of 6000 Hz (6000 cycles per second) The amplitude of motion of the tuning fork is 0.05 cm. Find the equation for harmonic motion for this situation.

$$d = a\sin(\omega t)$$

$$\text{frequency} = \frac{\omega}{2\pi}$$

$$6000 = \frac{\omega}{2\pi}$$

$$12000\pi = \omega$$

$$d = 0.05\sin(12000\pi * t)$$