Math-3
Lesson 6-7
The Sine and Cosine Functions.

What is a function?   f(x)

Function: a rule that matches each input to exactly one output.

What is the domain of a function?

Domain: the set of all allowable input values of a function (the ‘x’ values that have corresponding ‘y’ values).

What is the range of a function?

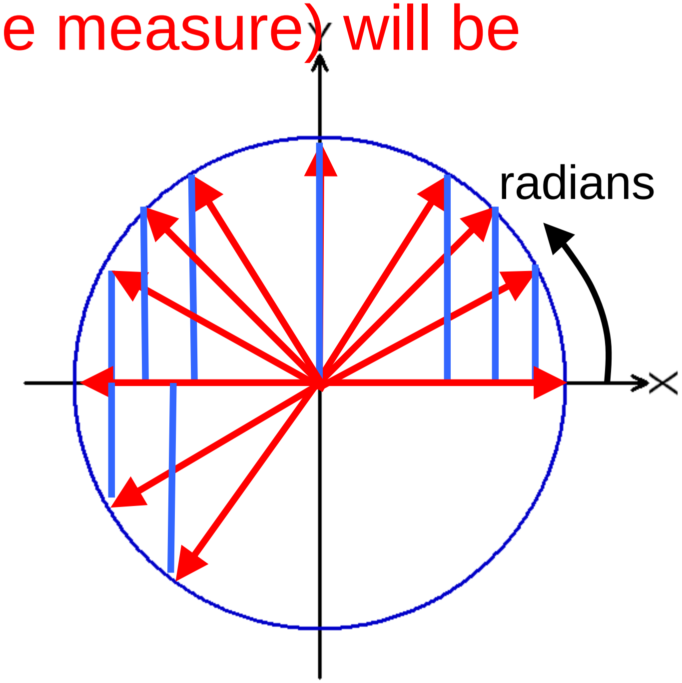
Range: the set of all output values of a function (the ‘y’ values).

Graph of the Sine Function $f(\theta) = \sin\theta$

Think of a point moving around the unit circle.

Input value: the angle

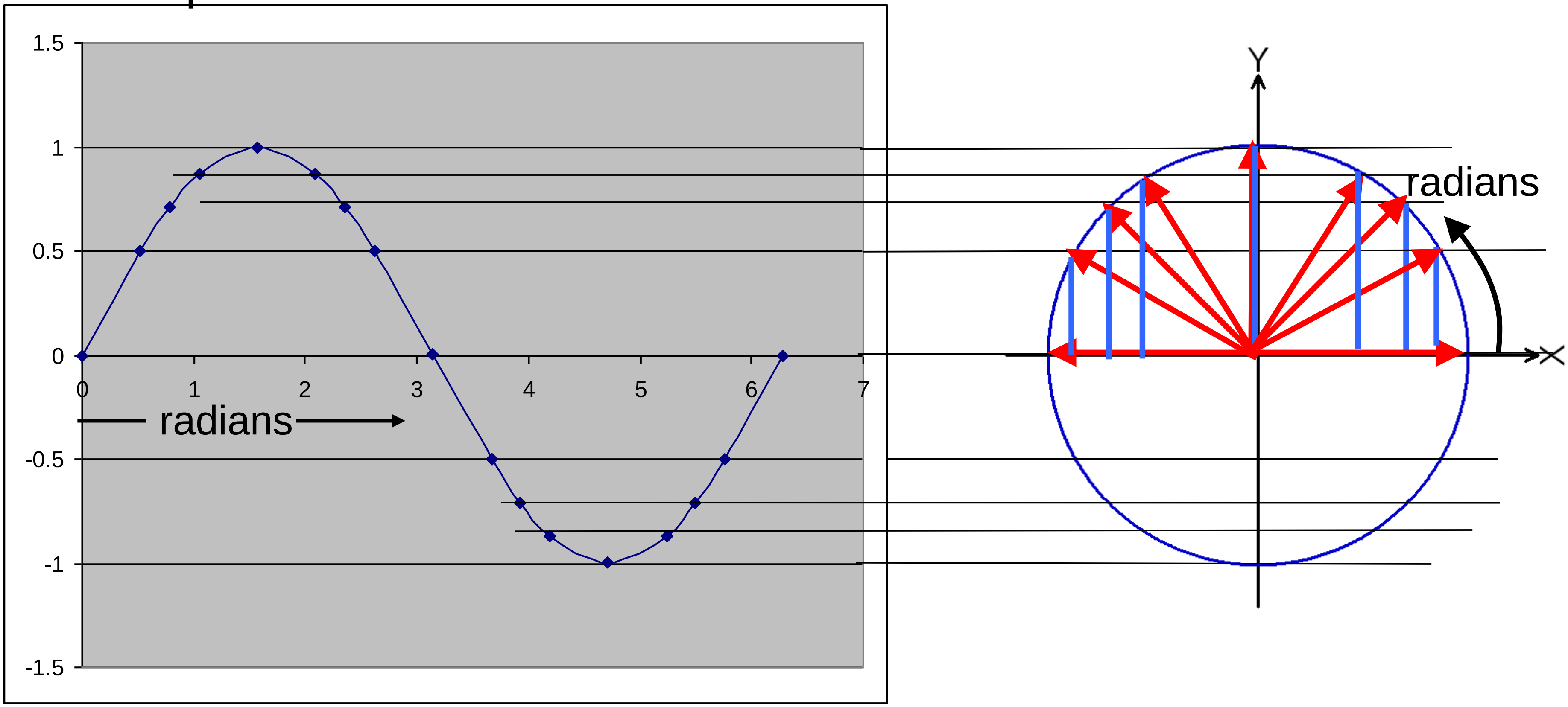
The input value (standard position angle measure) will be graphed on the ***x-axis of another plot***



Output value: the y-value of the point as it goes around the circle.

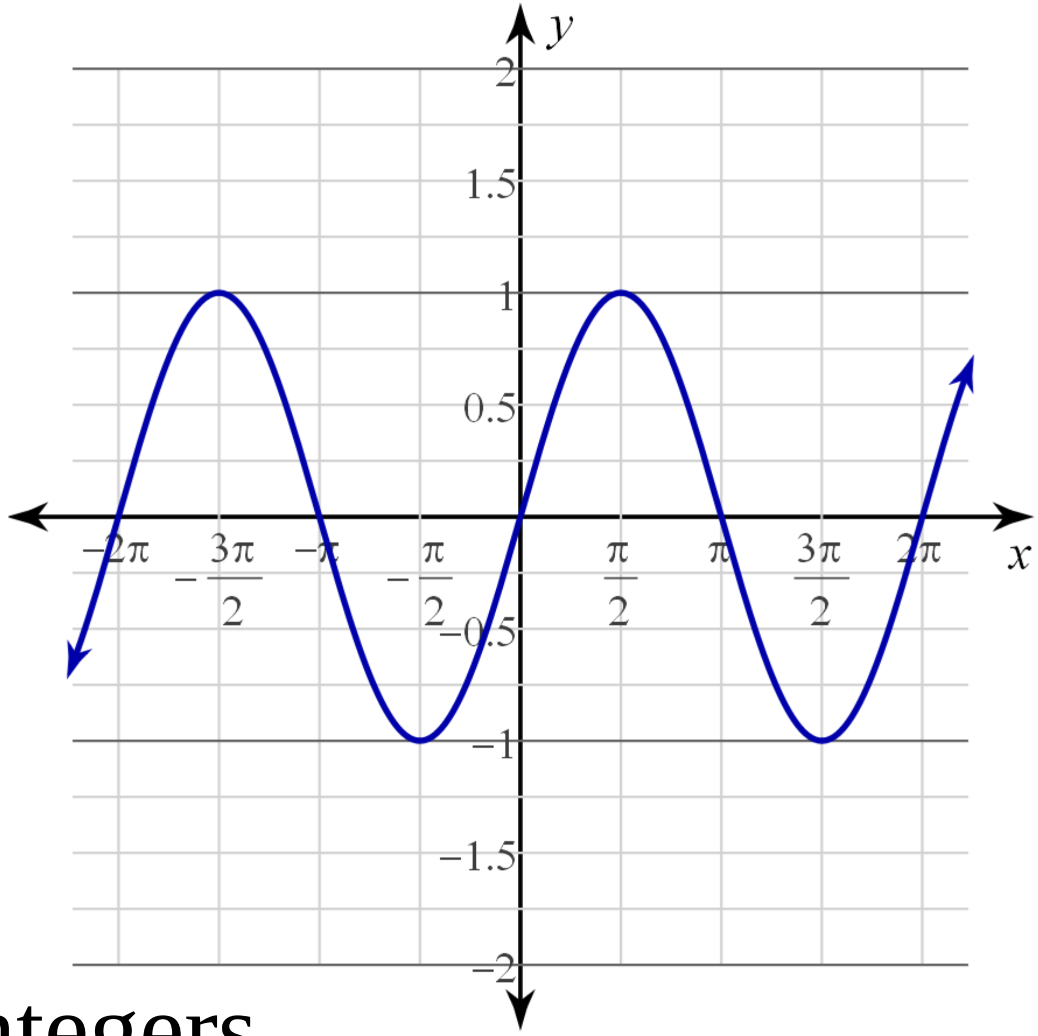
The output value (the y-value of the point) will be graph on the *y-axis of the other plot*

Graph of the Sine Function:



Think of <u>sin θ</u> as the distance that the point is <u>above</u> (or below) the x-axis, determined by ‘<u>θ</u>’ (the standard position angle passing through the point).

Sinusoid $f(\theta) = \sin\theta$



Domain = ?

$$-\infty < \theta < \infty$$

Range = ?

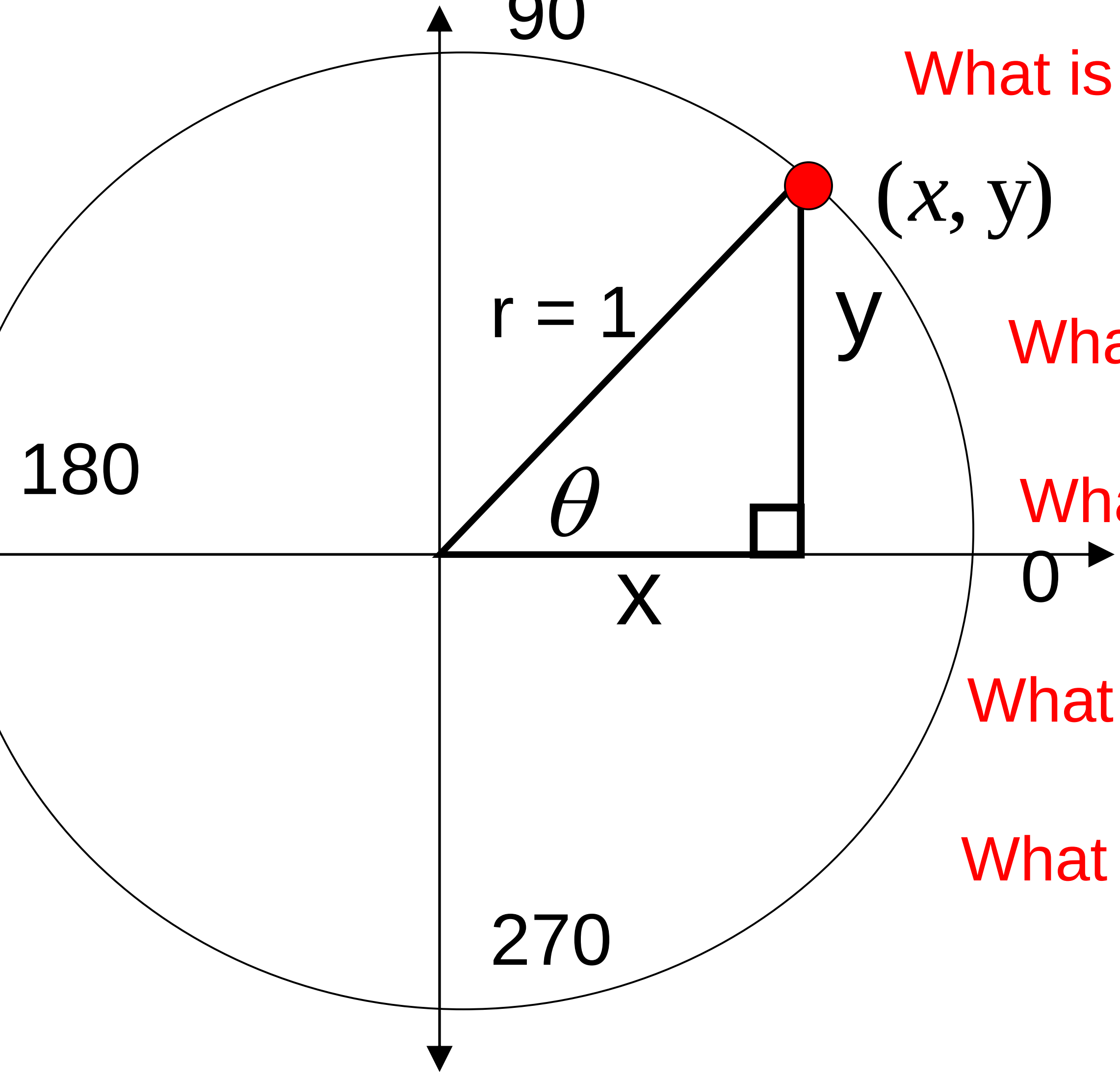
$$-1 \le y \le 1$$

x-intercepts?

$$x = n * \pi \quad \text{n} \in \text{integers}$$

The cosine function
(as opposed to the cosine ratio)

$$f(\theta) = \cos\theta$$



What is the input to the function?

angle

What is the output of the function?

x-value of the point

What is the maximum Ө-value?

∞

What is the minimum Ө-value?

-∞

What is the maximum y-value?

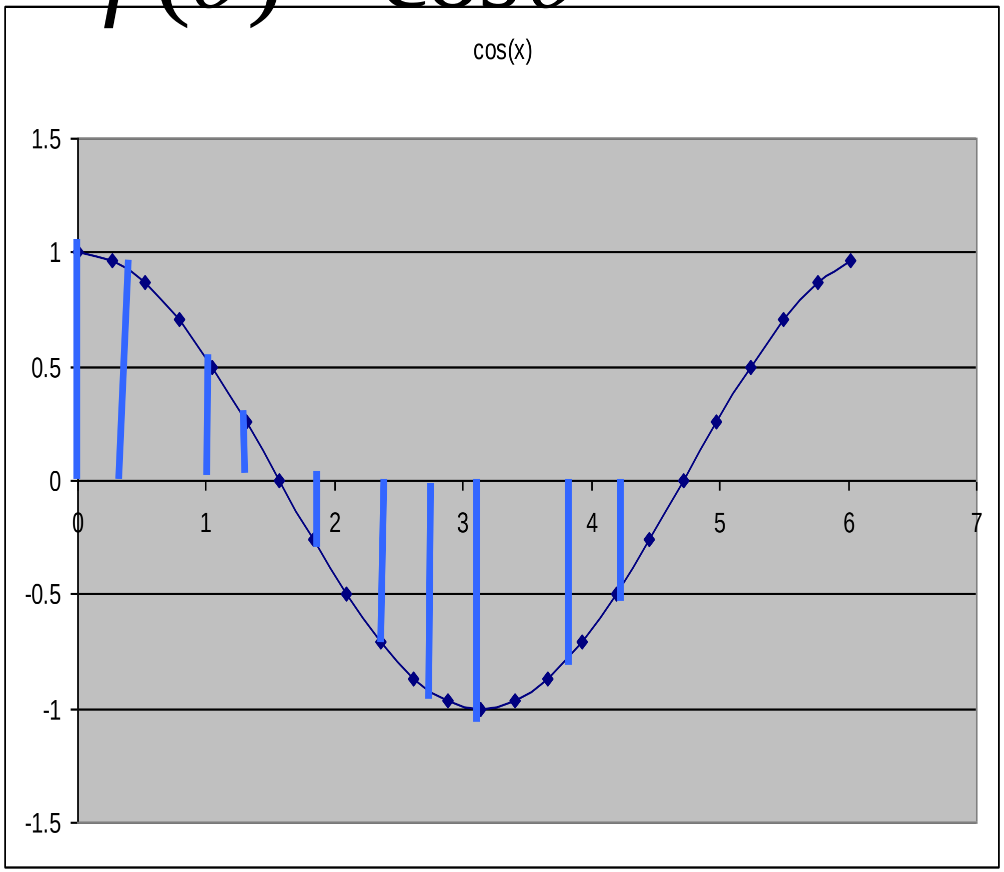
1

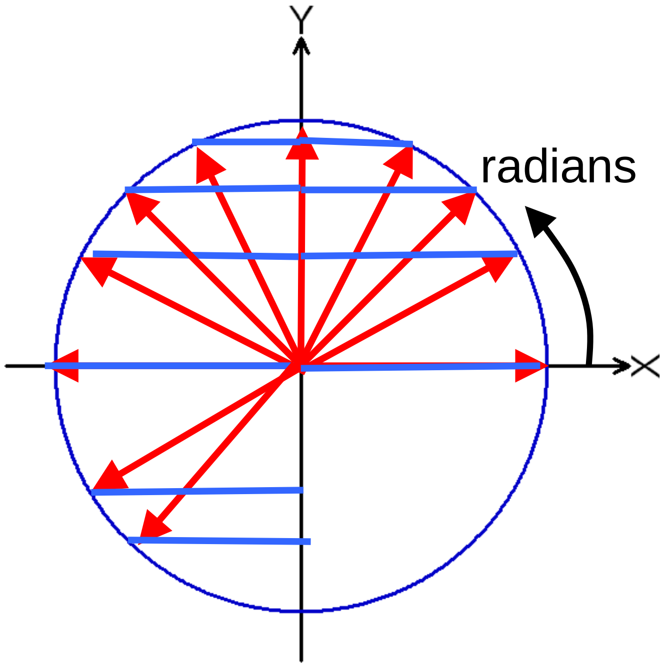
What is the minimum y-value?

-1

Graph of the Cosine Function

$$f(\theta) = \cos\theta$$





Think of a dot traveling around the circle to the right.

Think of <u>cos θ</u> as the distance <u>to the right</u> (or left) of the y-axis as determined by '<u>θ</u>' (the angle).

Degrees

$$f(\theta) = \cos\theta$$

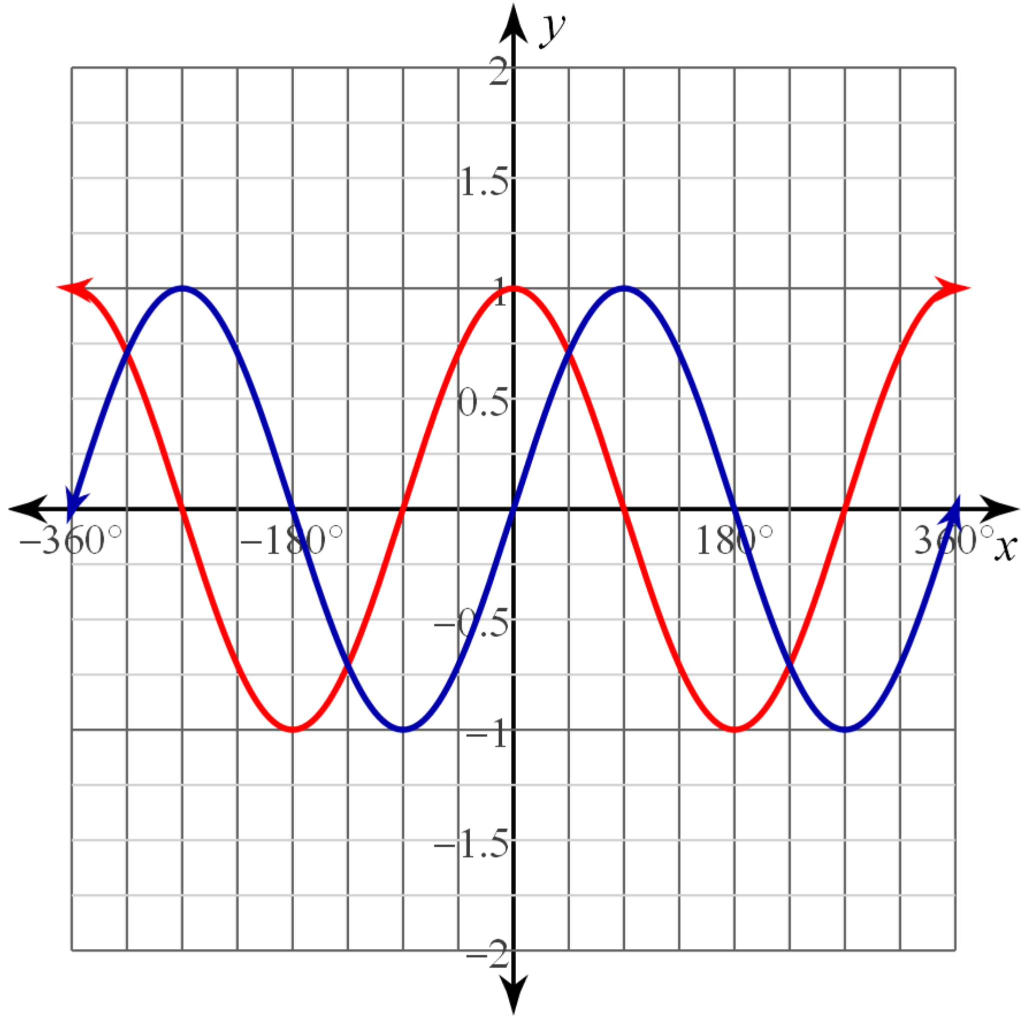
Domain = ?

$$-\infty < \theta < \infty$$

Range = ?

$$-1 \le y \le 1$$

x-intercepts?

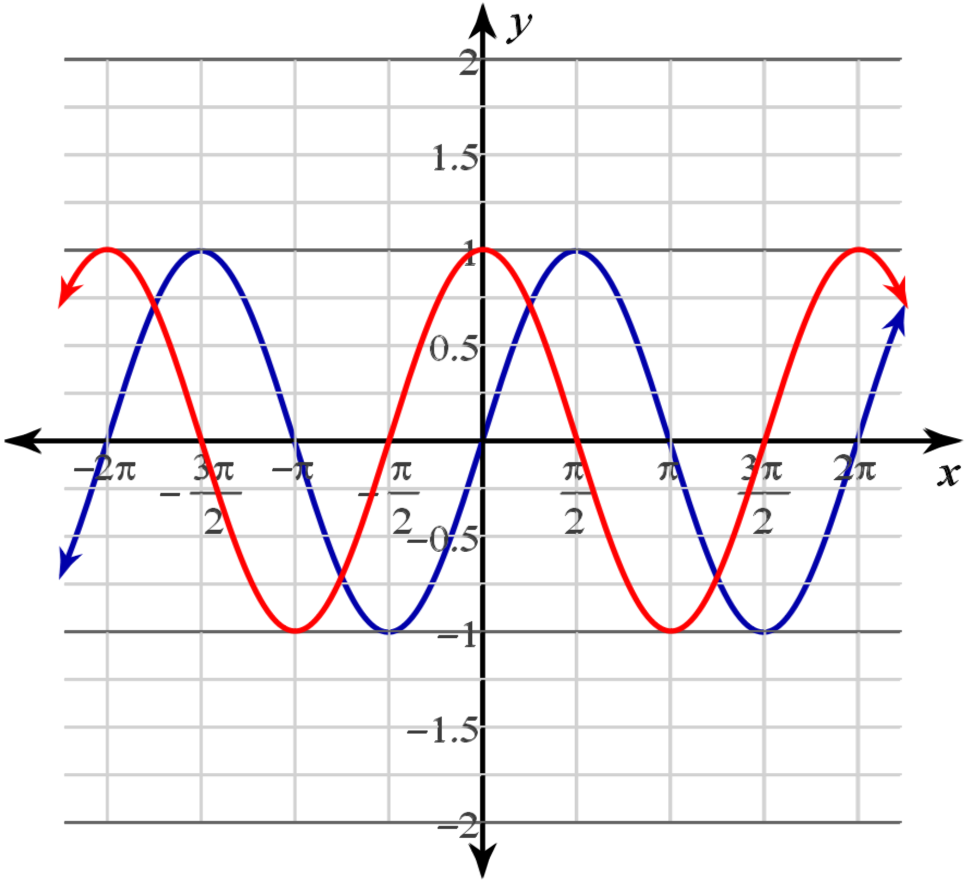


$$x = 90^0 + 180 * n \quad \text{n} \in \text{integers}$$

$$\cos(\,\theta - 90) = \sin(\,\theta)$$

Radians

$$f(\theta) = \cos\theta$$



x-intercepts?

$$x = \frac{\pi}{2} + n\pi \quad \text{n} \in \text{integers} \qquad \cos(\theta - \frac{\pi}{2}) = \sin(\theta)$$

The “Transformation Equation”

reflection Vertical stretch shift

$$y = (-1) * a * f(x - c) + d$$

shift

$$f(x) = -x^2$$ Reflection across x-axis

$$f(x) = a|x|$$ Vertical stretch

$$f(x) = |x - c|$$ Horizontal shift

$$f(x) = |x| + d$$ Vertical shift

Describe the transformations these function make on their parent functions.

$$f(x) = |x|$$

$$g(x) = 2|x-4|+5$$

Vertically stretched by a factor of 2, shifted right 4 and up 5

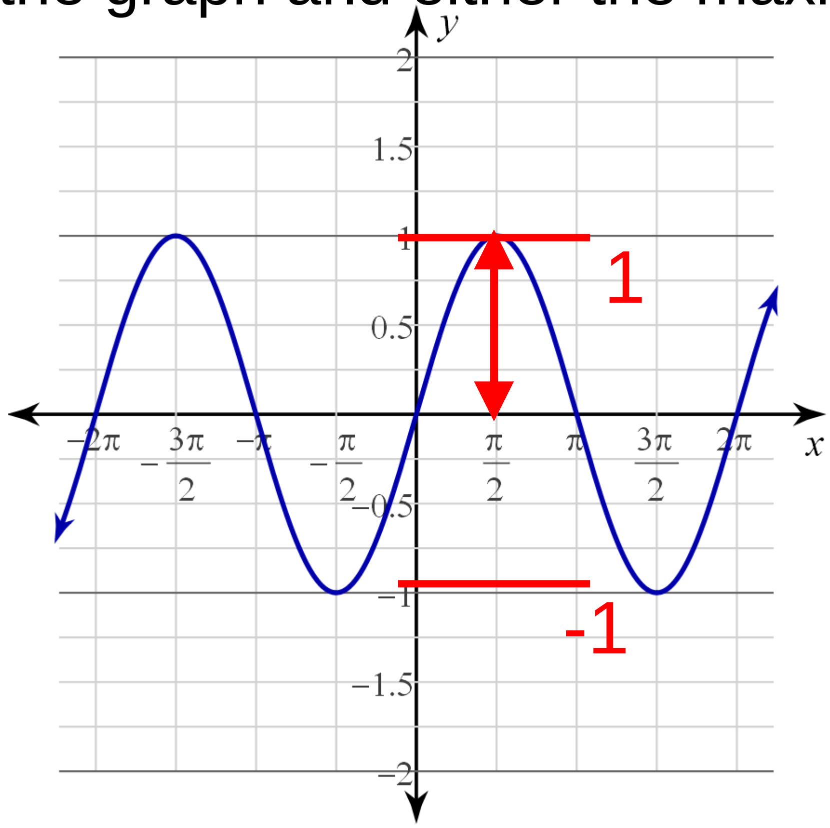
$$h(x) = x^2$$

$$k(x) = 0.25(x+1)^2 - 6$$

Vertically stretched by a factor of 0.25, shifted left 1 and down 6

$$f(x) = a \sin x$$

Amplitude: The vertical distance between the centerline of the graph and either the maximum or minimum output value.

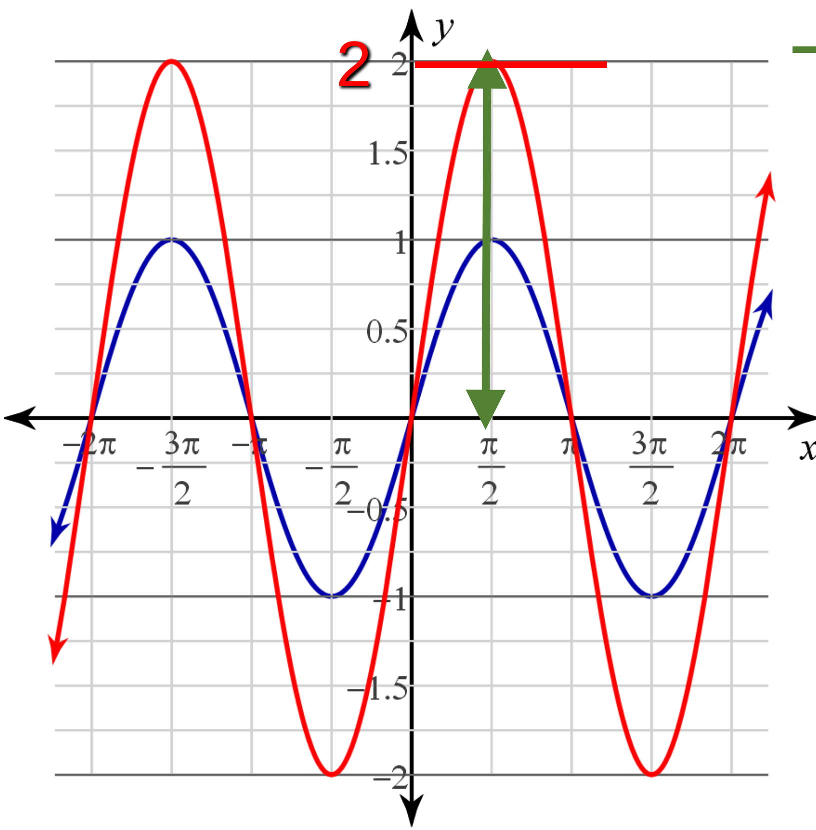


For the sine and cosine functions, we call the coefficient ‘a’ the amplitude of the function (which in general refers to the ***vertical stretch factor***).

Compare:

$$f(x) = \sin x$$

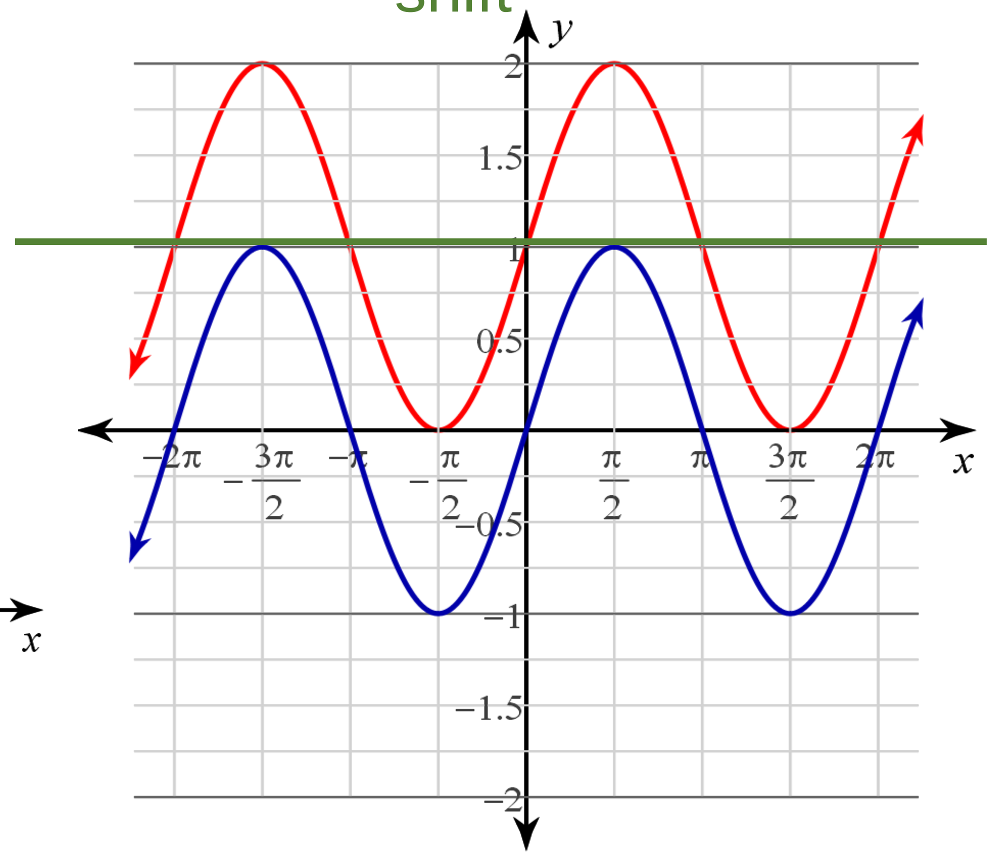
$$g(x) = 2\sin x$$



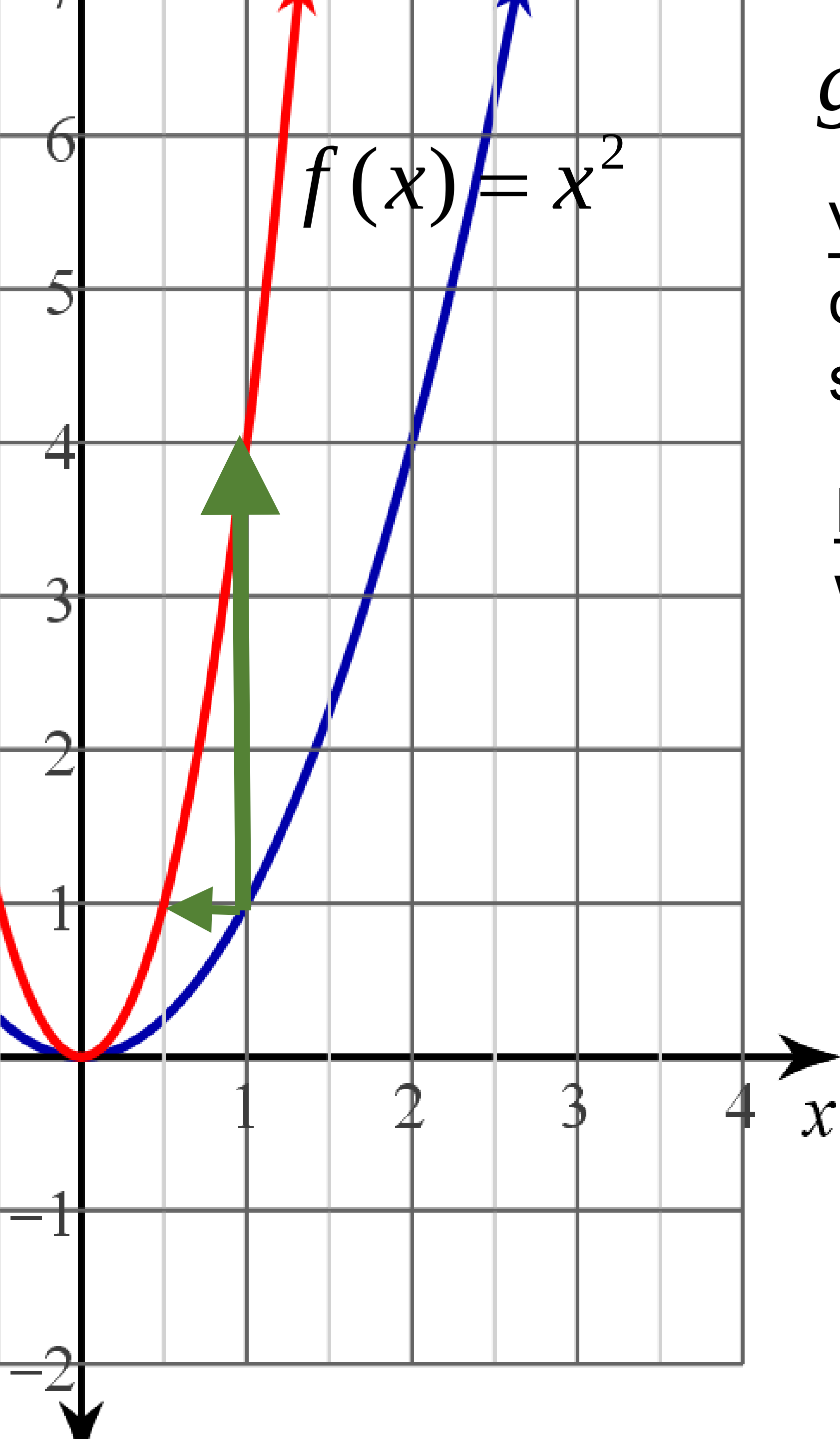
$$g(x) = \sin x$$

$$f(x) = 1 + \sin x$$

shift



<u>Centerline of the Oscillation</u>: corresponds to the up/down translation.



$$g(x) = 4f(x) \qquad g(x) = 4x^2$$

<u>Vertical Stretch</u>: multiplying the original function by 4, "vertically stretches" it by a factor of 4.

<u>Horizontal Stretch</u>: replacing 'x' with a number multiplied by 'x',

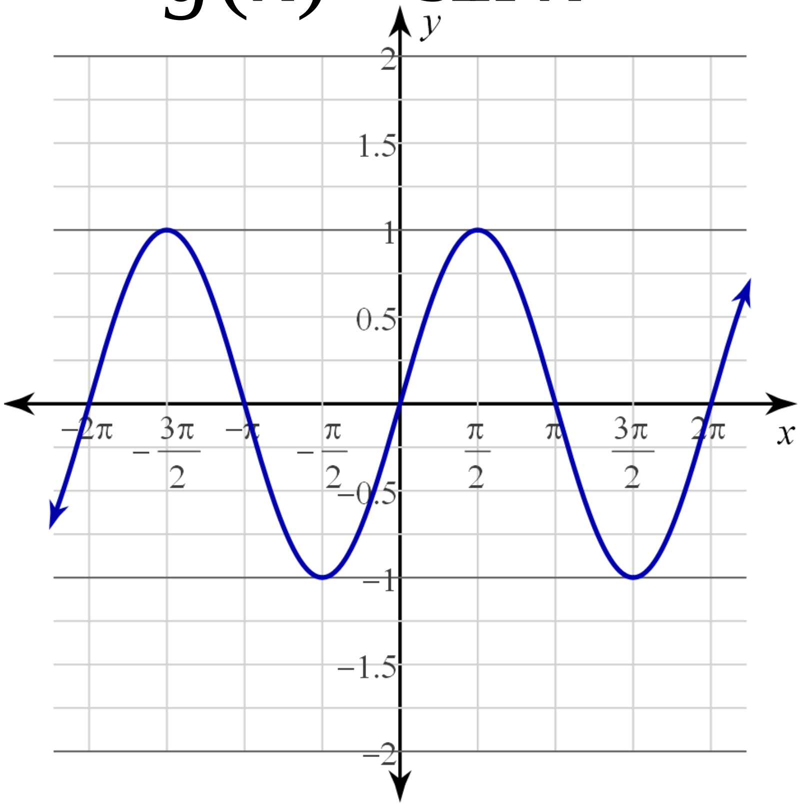
$$h(x) = (2x)^2 \quad = 4x^2$$

Multiplying the input value by '2' causes a horizontal stretch by a factor of ½.

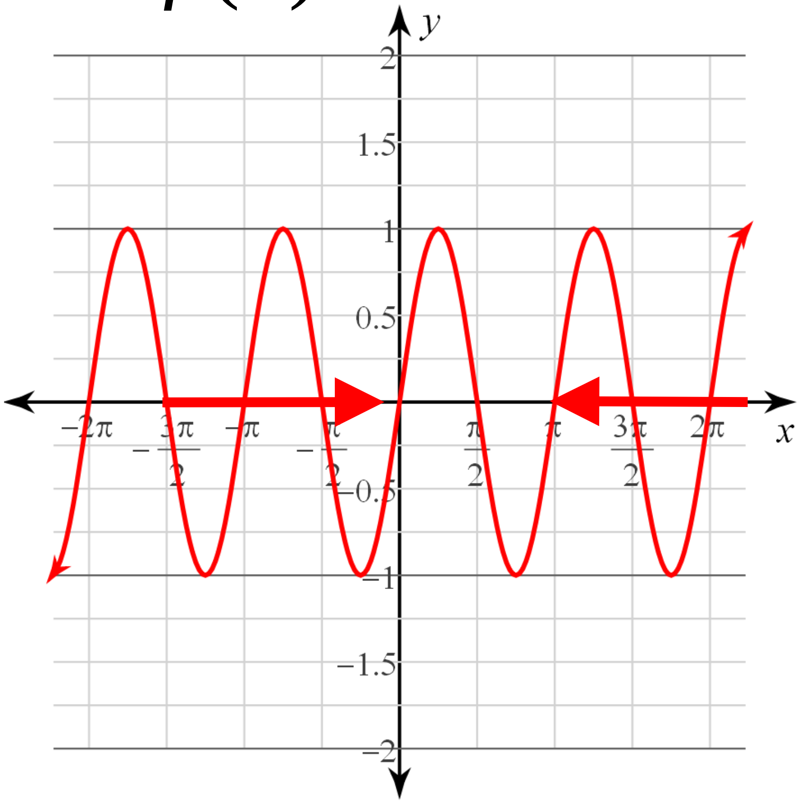
Predict what you think happens

Horizontal stretch or shrink?

$g(x) = \sin x$



$f(x) = \sin 2x$



horizontal shrink

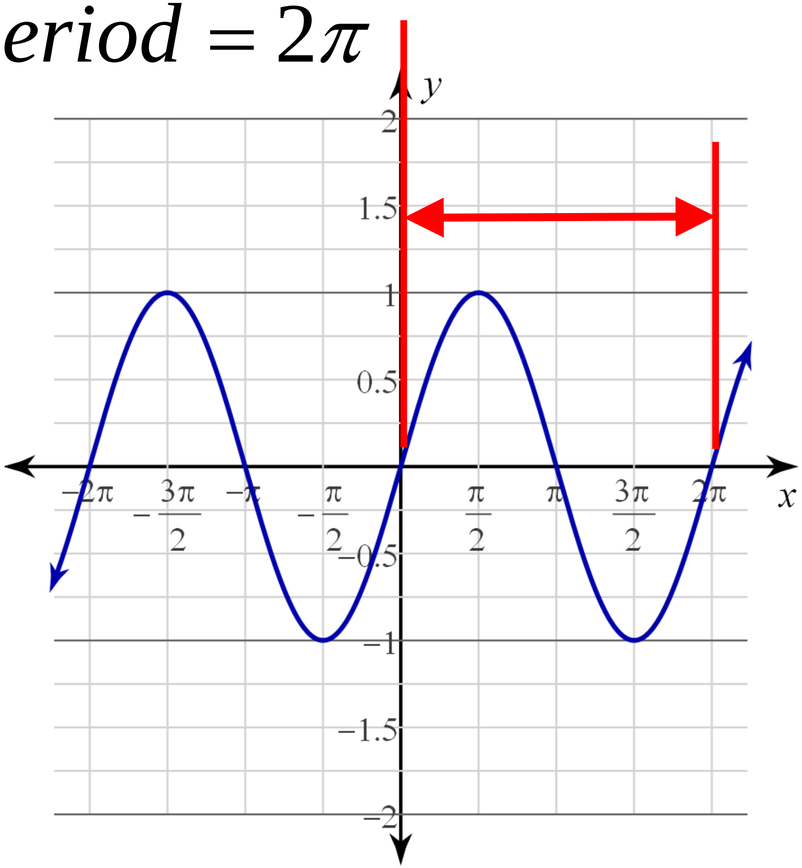
Stretched by a factor of ½ (we just use the word stretch)

$$f(x) = a \sin bx$$

Period: the horizontal distance along the x-axis needed to complete one full cycle of the oscillation.

$$g(x) = \sin x$$

$$Period = 2\pi$$



Period $= \dfrac{2\pi}{|b|}$

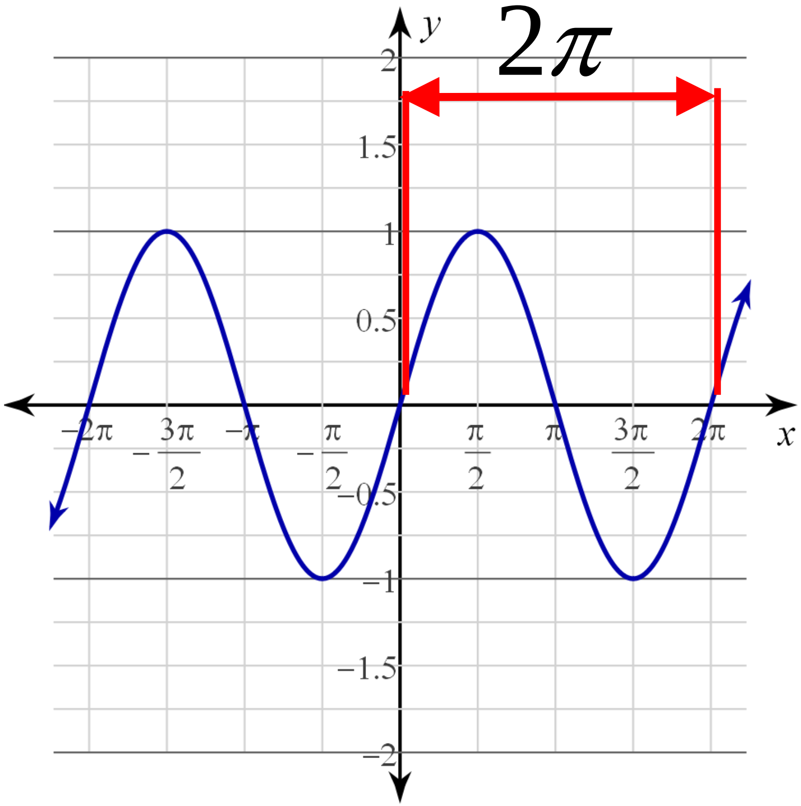
Frequency $= \dfrac{|b|}{2\pi}$

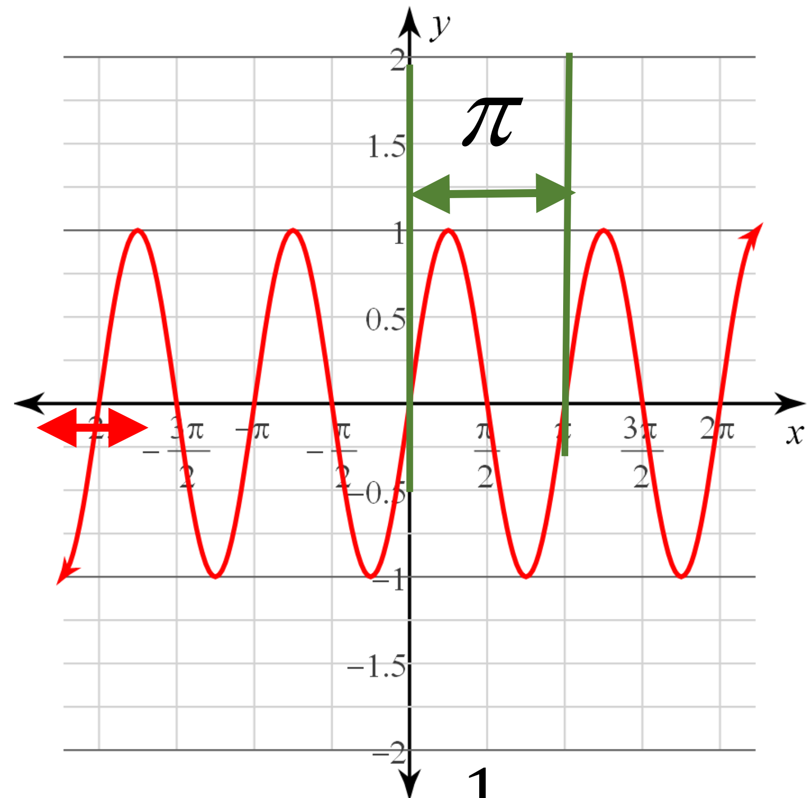
Frequency = 1/period

$$f(x) = \sin 2x$$

$$g(x) = \sin x$$

$$\text{horizontal stretch factor} = \frac{1}{2}$$





Frequency = 1 cycle every 2pi radians.

$$Period = \frac{1}{2} * 2\pi = \pi$$

Frequency = 1 cycle every pi radians.

Compare: $f(x) = a \sin bx$

$g(x) = \sin x$ $f(x) = \sin 3x$

horizontal stretch factor = $\frac{1}{b}$ $= \frac{1}{3}$

What is the period of g(x) ? $= 2\pi$

What is the period of f(x) ? $= \frac{1}{b} * 2\pi$ $= \frac{1}{3} * \frac{2\pi}{b}$ $= \frac{2\pi}{3}$

What is the frequency of f(x) ? Frequency = 3 cycles every 2pi radians.

Your turn: $f(x) = a\sin bx$

$g(x) = \cos x$ $f(x) = 4\cos 5x$

What is the horizontal stretch factor ?. $= \frac{1}{5}$

What is the <u>period</u> of g(x) ? $= 2\pi$

What is the <u>period</u> of f(x) ? $= \frac{2\pi}{b} = \frac{2\pi}{5}$

What is the <u>amplitude</u> of f(x) ? $= 4$

What is the <u>frequency</u> of f(x) ? $= \frac{5}{2\pi}$ 5 cycles every 2pi radians.

Vertical and now horizontal stretch factors

$$f(x) = a \sin bx$$

a: Vertical stretch factor $= a$

b: horizontal stretch factor. $= \frac{1}{b}$

$$f(x) = -2\sin\left(\frac{\pi}{2}x\right)$$

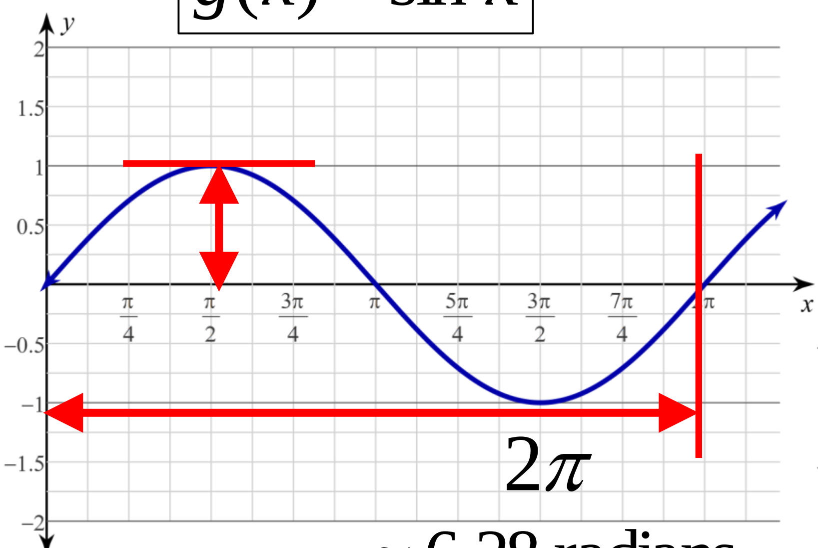
Reflected across x-axis.

Vertically stretched by a factor of 2.

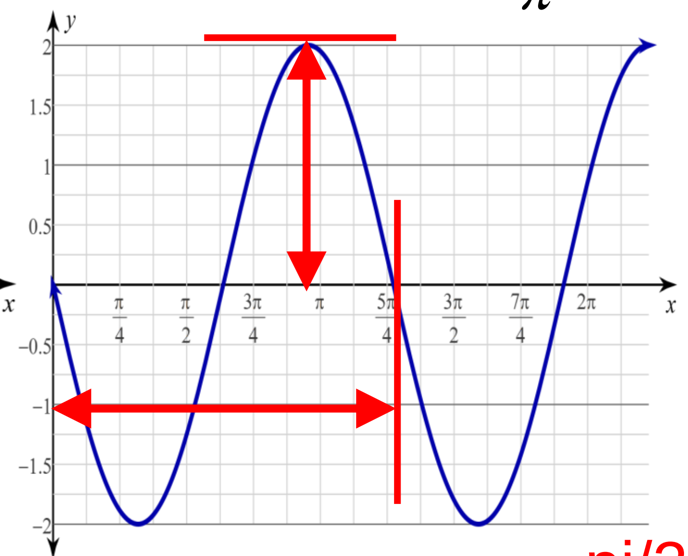
Horizontally stretched by a factor of $\frac{2}{\pi}$

Period = HSF*2π $\frac{2}{\pi} * 2\pi = 4 \text{ radians}$

$\boxed{g(x) = \sin x}$



≈ 6.28 radians



Frequency = 1/period

pi/2 cycles every 2pi radians.

$g(x) = \sin x$ $\quad\quad$ $f(x) = -5\sin 3x$

Describe how f(x) is a transformation of the parent function g(x).

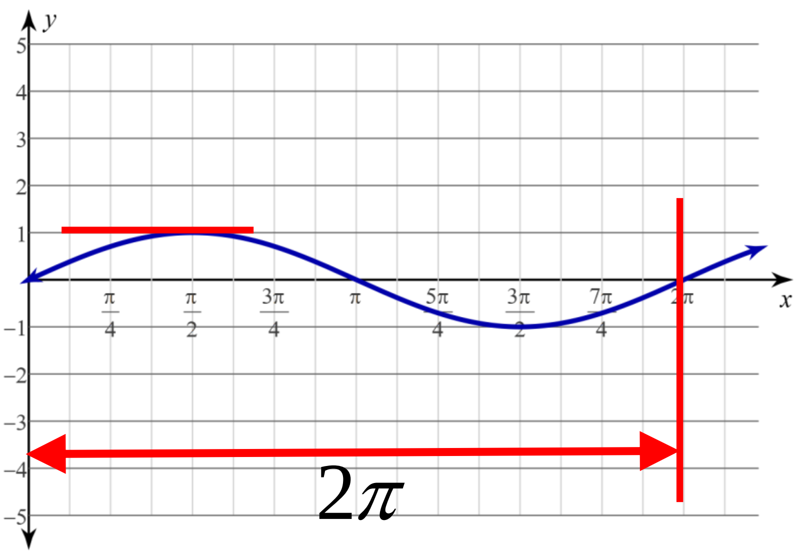
$f(x) = -5\sin 3x$ Reflected across x-axis.

Vertically stretched by a factor of 5 (amplitude).

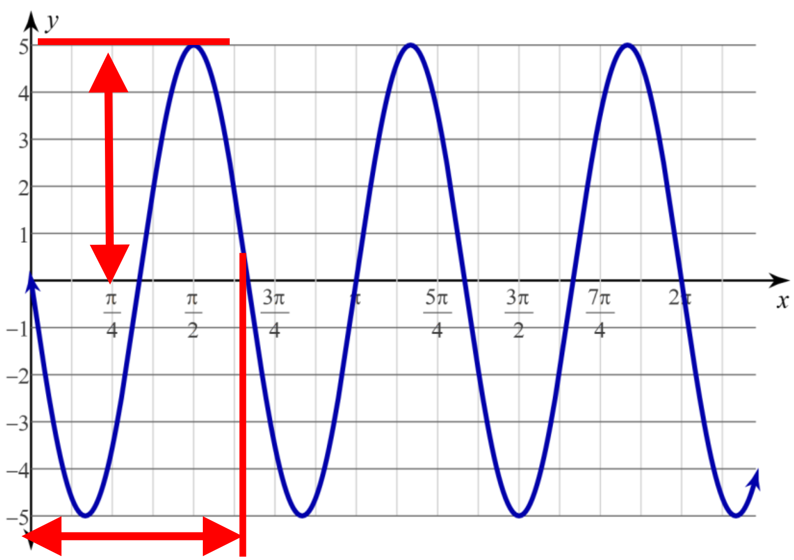
Horizontally shrunk by a factor of $\dfrac{1}{3}$

Period = HSF * 2π = $\dfrac{2\pi}{3}$ radians

Frequency = 3 cycles every 2pi radians.



$g(x) = \sin x$



$\dfrac{2\pi}{3}$ radians

Equivalent Equations
(input variable is ***theta***)

$$f(x) = \mathrm{a}\sin(b\theta - c) + k$$

In this version, the left/right shift is “mixed together” with the horizontal stretch factor.

$$f(x) = a\sin b\,(\theta - c/b) + k$$

By factoring out the coefficient of theta, we have separated the HSF from the phase shift.

$$f(x) = 4sin\,(3\theta - \pi) + 2$$

$$f(x) = 4\sin 3\,(\theta - \pi/3) + 2$$

$$f(x) = -5sin\,(\frac{\theta}{3} - \frac{\pi}{2}) + 2$$

$$f(x) = 4\sin\frac{1}{3}(\theta - \frac{3\pi}{2}) + 2$$

$$f(x) = 3sin\left(2\theta + \frac{\pi}{2}\right)$$

$$f(x) = a\sin(bx - c) + d$$

$$f(x) = a\sin b(x - c/b) + d$$

$$f(x) = +3\sin 2\left(\theta - \frac{\pi}{4}\right)$$

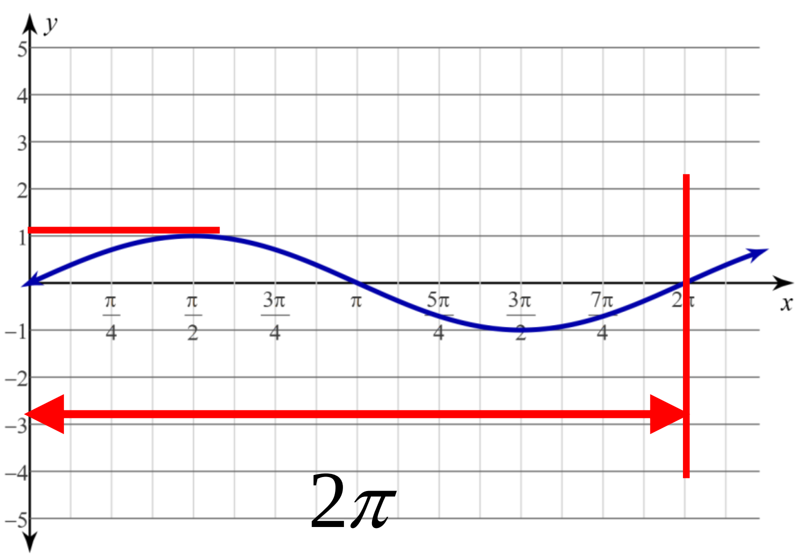
VSF = 3 HSF = 1/2

Not Reflected across x-axis.

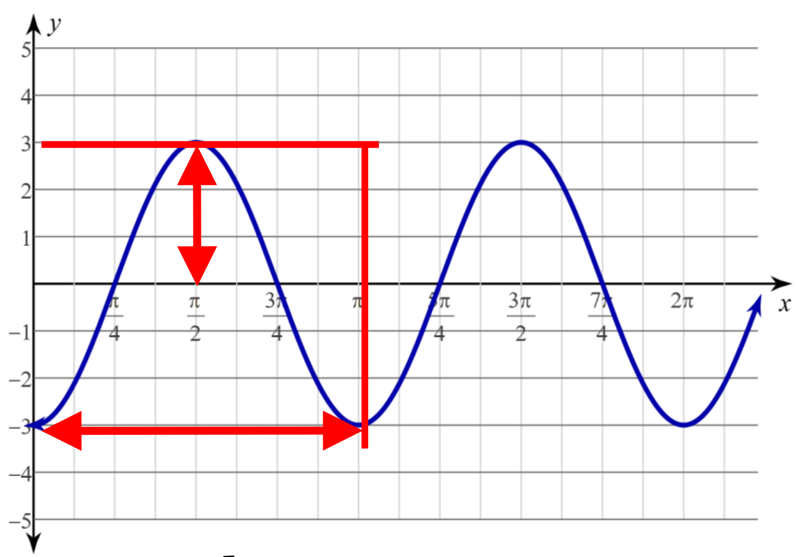
Frequency = 2 cycles every 2pi radians → 1/pi

Shifted right by π/4 radians

$$g(x) = \sin x$$



≈ 6.28 radians



π radians

$$f(x) = -0.5\sin\left(3x + \frac{\pi}{12}\right) - 2$$

Horizontal stretch & left/right shift mixed together → separate them

$$f(x) = -0.5\sin 3\left(x + \frac{\pi}{4}\right) - 2$$

Amplitude = ? 0.5 units

Phase shift = ? Left pi/4

Period = ? 2pi/3 radian per cycle.

Frequency = ? 3 cycles every 2pi radians.

Center line: y = -2