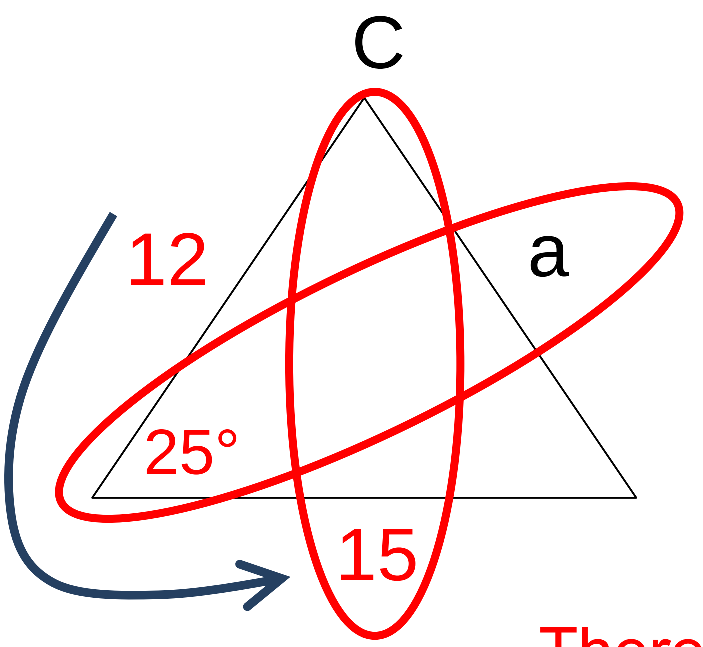
Math-3

Lesson 6-5

The Law of Sines

(Ambiguous Case)

If the following information is given "Walk around the block"



Start at the first side or angle that is known then list the order of the known items.

Side, Angle, Side → SAS

This means: "the measures of two sides and the included angle are known.

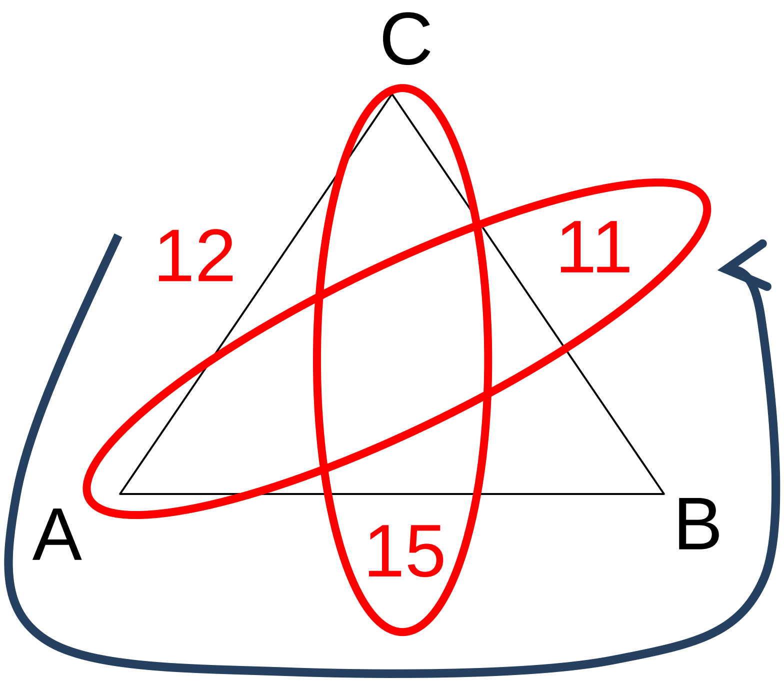
There are two unknown values in the "loops."

You cannot solve a single equation that has two unknown values!

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Law of Sines will NOT work for SAS

If the following information is given: "Walk around the block"



Start at the first side or angle that is known then list the order of the known items.

Side, Side, Side → SSS

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Law of Sines will NOT work for <u>SSS</u>.

Can the Law of sines be used for:

SAS ? no

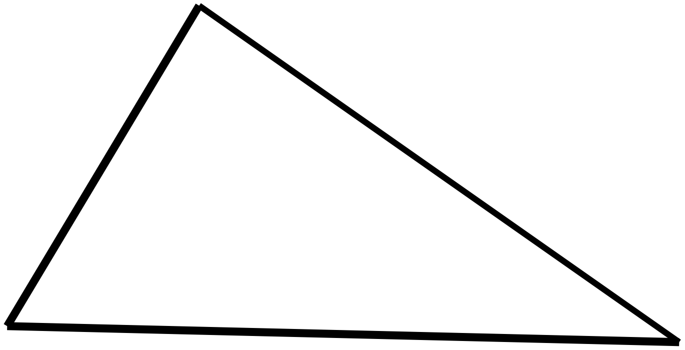
SSS ? no

SSA ? yes, BUT….

AAA? no, trangle can be scaled up or down in size (no unique triangle).

What is a triangle?

3 segments joined at their endpoints



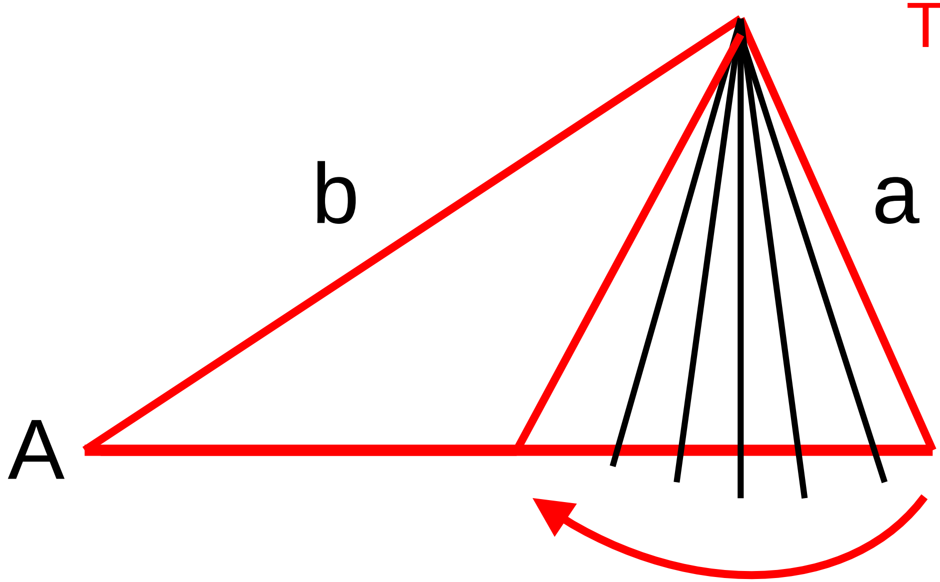
SSA: The "Ambiguous" Case

If an angle and its opposite side are known, and another side is known (Not SAS), we have a triangle.

We do not know the length of the bottom side.

We can "swing" side 'a' until it touches the bottom side at its end point. This makes another triangle.

Two triangles are possible!



SSA Case If the given information about a triangle is:

$A = 68^\circ$ $b = 88$ $a = 85$

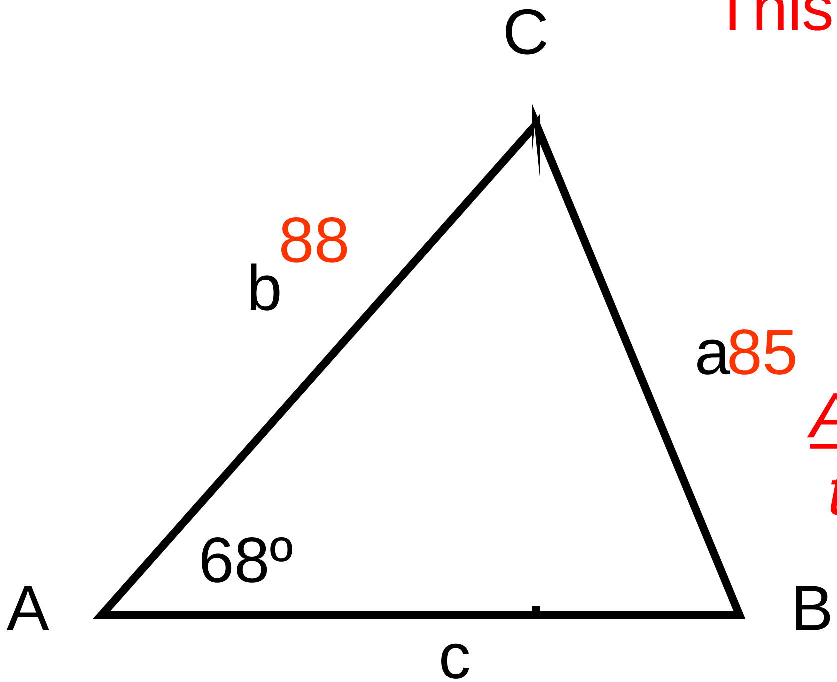
1. Draw the triangle.

2. Label the triangle

3. Determine what case it is.

The angle is not between the two sides.

This may cause problems.

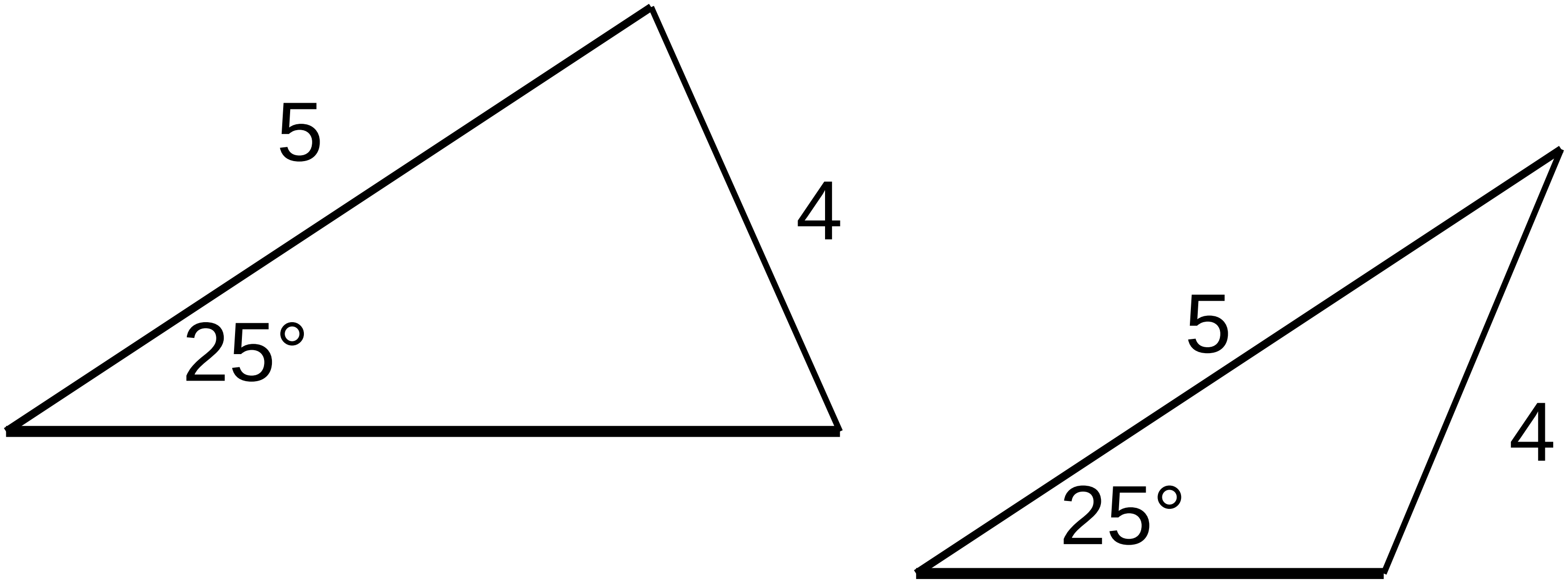


We call this the *ambiguous* case.

Ambiguous: Liable to more than one interpretation.

<u>SSA: The “Ambiguous” Case</u>

IF: (1) the side <u>opposite</u> the given angle is <u>shorter</u> than the <u>adjacent</u> side, and
(2) The angle is acute
→ you <u>can</u> have <u>two triangles.</u>



Which of the following cases might give you two possible triangles?

$A = 68^o$ $b = 68$ $a = 85$

$A = 25^o$ $b = 7$ $a = 5$

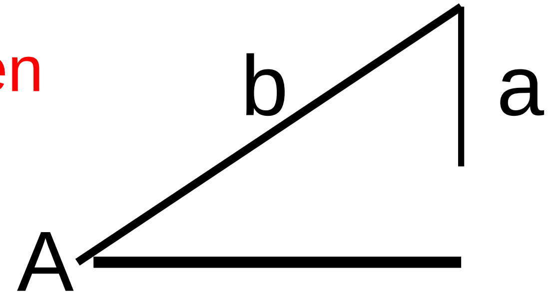
$A = 118^o$ $b = 8$ $a = 20$

Will this give you two triangles?

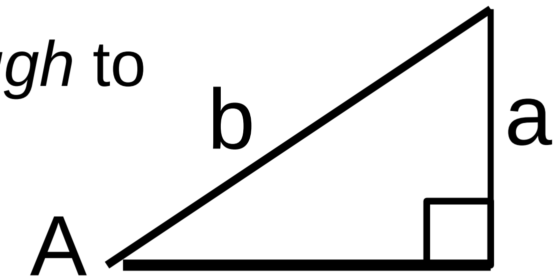
IF: The given angle is <u>acute (<90°)</u> the side <u>opposite</u> the given angle is <u>shorter</u> than the <u>adjacent</u> side, you may have <u>two triangles.</u>

IF: The given angle is acute (<90°) the side opposite the given angle is shorter than the adjacent side, ***there are three possibilities.***

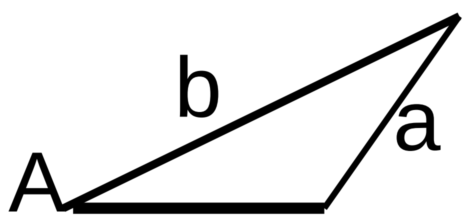
1. The opposite side is too short to even make a triangle.

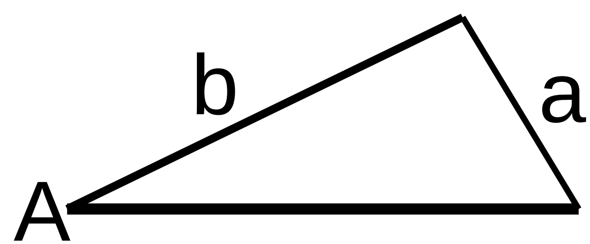


2. The opposite side is *just long enough* to touch once → right triangle.



3. The opposite side can touch in two places→2 triangles.

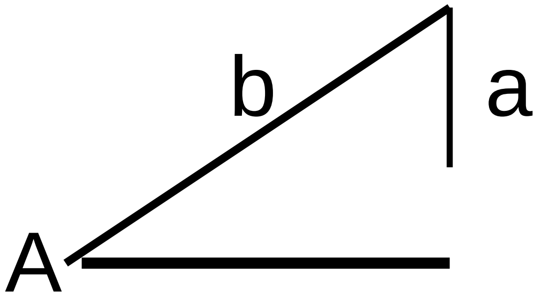




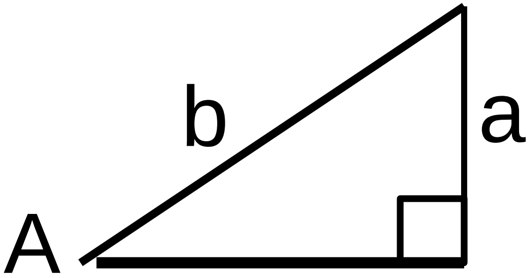
A = 25º   b = 7   a = 5

How can you tell which case it is?
(0, 1, or 2 triangles)

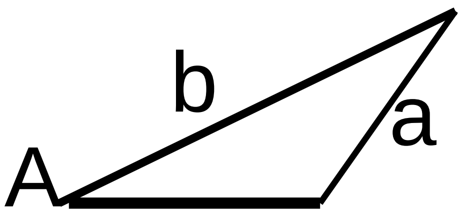
We calculate the "just right" opposite side length that will give us the right triangle.



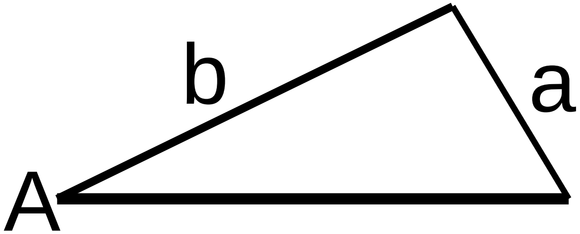
If the opposite side length is less than this "Goldilocks" length,
→ 0 triangles.



If the opposite side length is the "Goldilocks" length, it is a right triangle → 1 triangle.

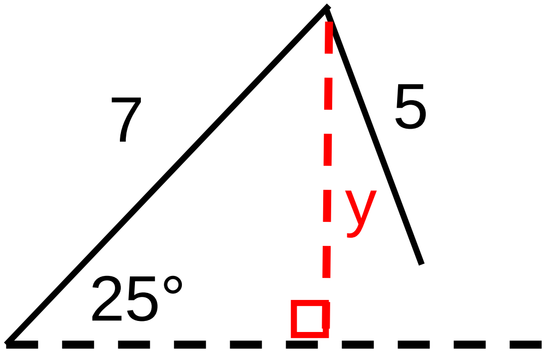


If the opposite side length is greater than this "Goldilocks" length AND is shorter than the adjacent side
→ 2 triangles.



SSA Case: Is it 0, 1, or 2 triangles?

You must calculate if the opposite side makes a right angle!



y = “Goldilocks” length

$A = 25^o$, b = 7, a = 5

To make a right angle: ‘a’ = 2.95.

$$\sin 25 = \frac{y}{7}$$

“just right” length < ‘a’ < adjacent side length

2.95 < (a = 5) < 7

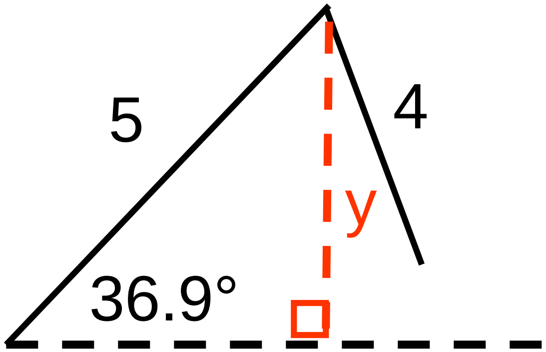
$y = 7 \sin 25^o$

→ 2 triangles.

y = 2.95

A = 36.9º, b = 5, a = 4

Is this the ambiguous case?



y = length for a single, right-triangle.

3 < 4 < 5

→ 2 triangles.

$$\sin 36.9 = \frac{y}{5}$$

y = 5 sin 36.9º

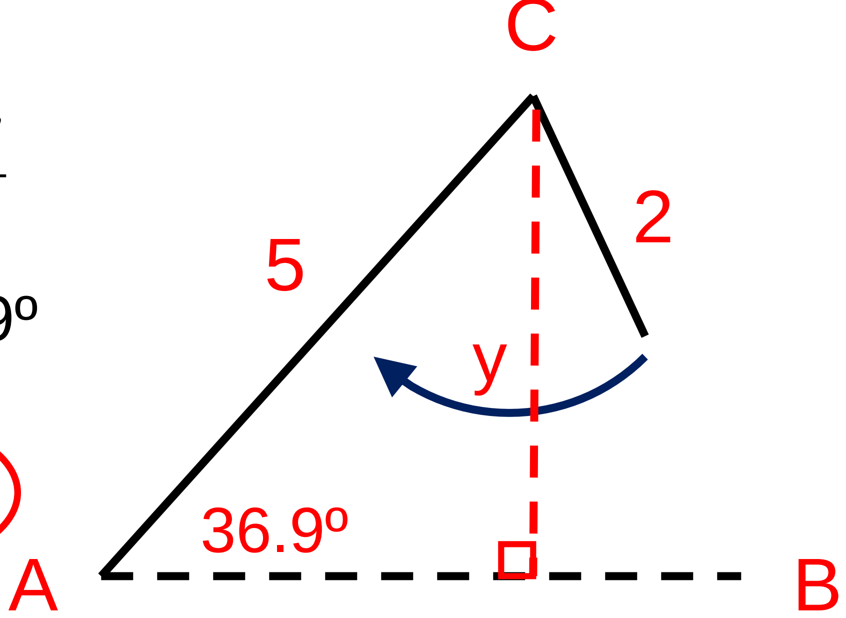
y = 3

<u>SSA Case</u>

$$\sin 36.9 = \frac{y}{5}$$

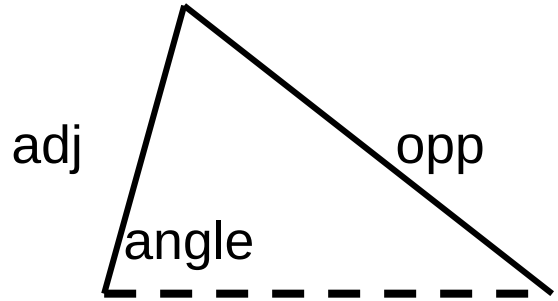
y = 5 sin 36.9º

y = 3



Opposite side is too short → 0 triangles.

Summary (ambiguous case)



A’s & S’s	Given Angle and Sides	#of Triangles:
SSA	Adj > opp and opp > “just right”	2
	Adj > opp and opp = “just right”	1
	Adj > opp and opp < “just right”	0
	Adjacent side < opposite side	1

a. What is the situation? (SSA, ASA, AAS)
b. How many triangles? (one, two, or none)

$A = 52^\circ$, $a = 32$, $b = 42$

$A = 28^\circ$, $C = 75^\circ$, $c = 20$

$A = 40^\circ$, $a = 13$, $b = 16$

(Hint: draw the triangle!!!)

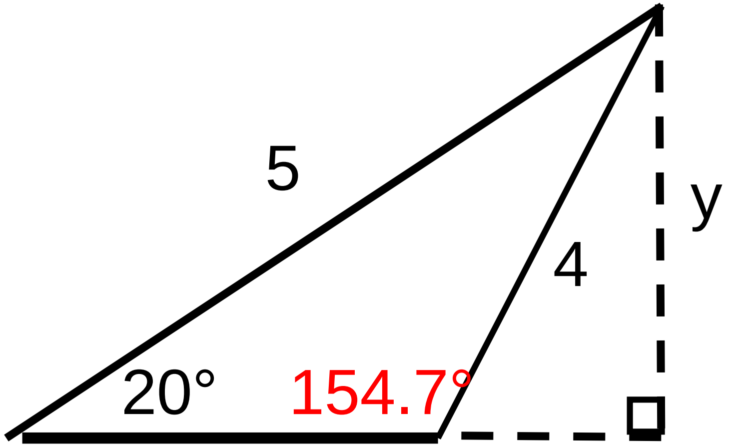
SSA: The "Ambiguous" Case A = 20º, a = 4, b = 5

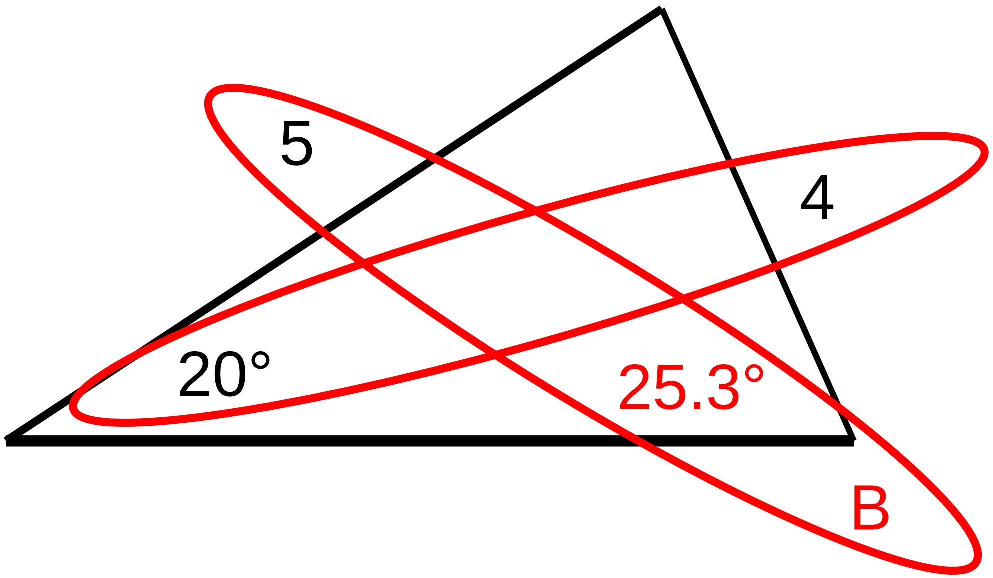
$$\sin 20 = \frac{y}{5} \qquad y = 5\sin 20 \qquad = 1.71$$

Two triangles, the law of sines will give the acute angle:

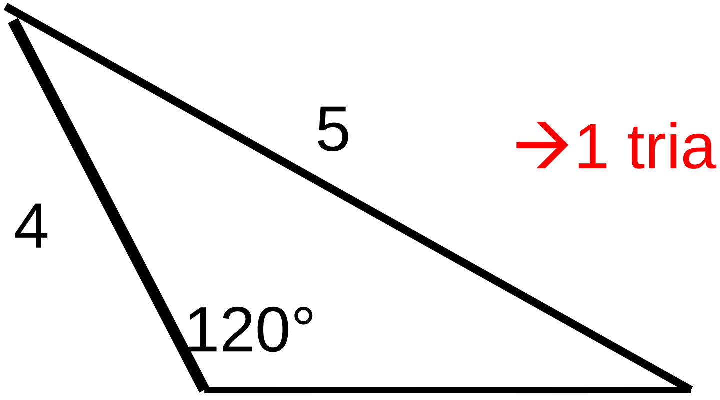
$$\frac{\sin B}{5} = \frac{\sin 20}{4} \qquad \sin B = \frac{5\sin 20}{4} \qquad \boxed{B = 25.3}$$

The obtuse angle is (180° - 25.3°) = 154.7°

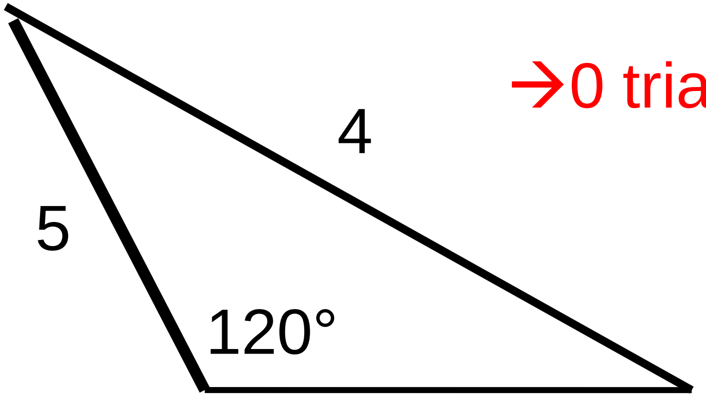




<u>SSA</u>: The angle is obtuse → 0 triangles or 1 triangle.



→1 triangle

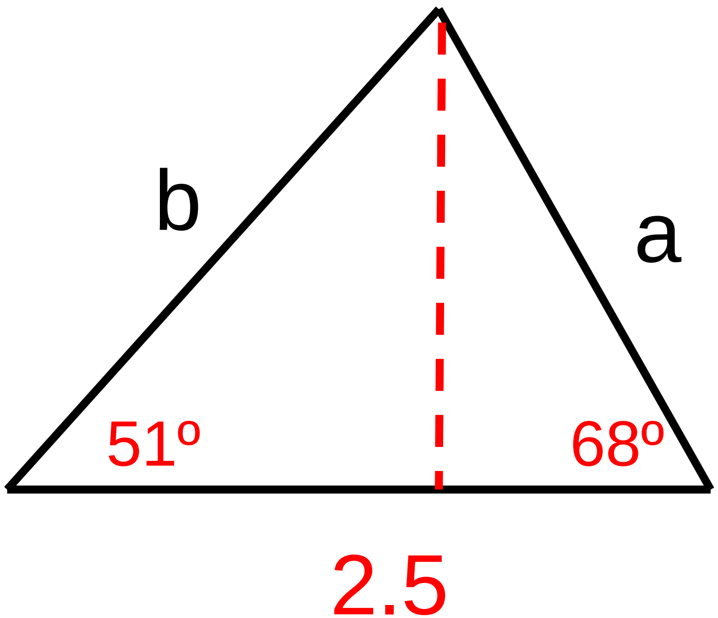


→0 triangles, why?

The longest side of a triangle *<u>must be opposite</u>* the largest angle.

Finding the Height of a Pole

Two people are 2.5 meters apart on opposite sides of a pole. The angles of elevation from the observers to the top of the pole are 51° and 68°. Find the height of the pole.



1. Find either ‘a’ or ‘b’ using Law of Sines.

2. Solve the right triangle using right triangle rules where height is the side opposite the angle.