

# Math-3

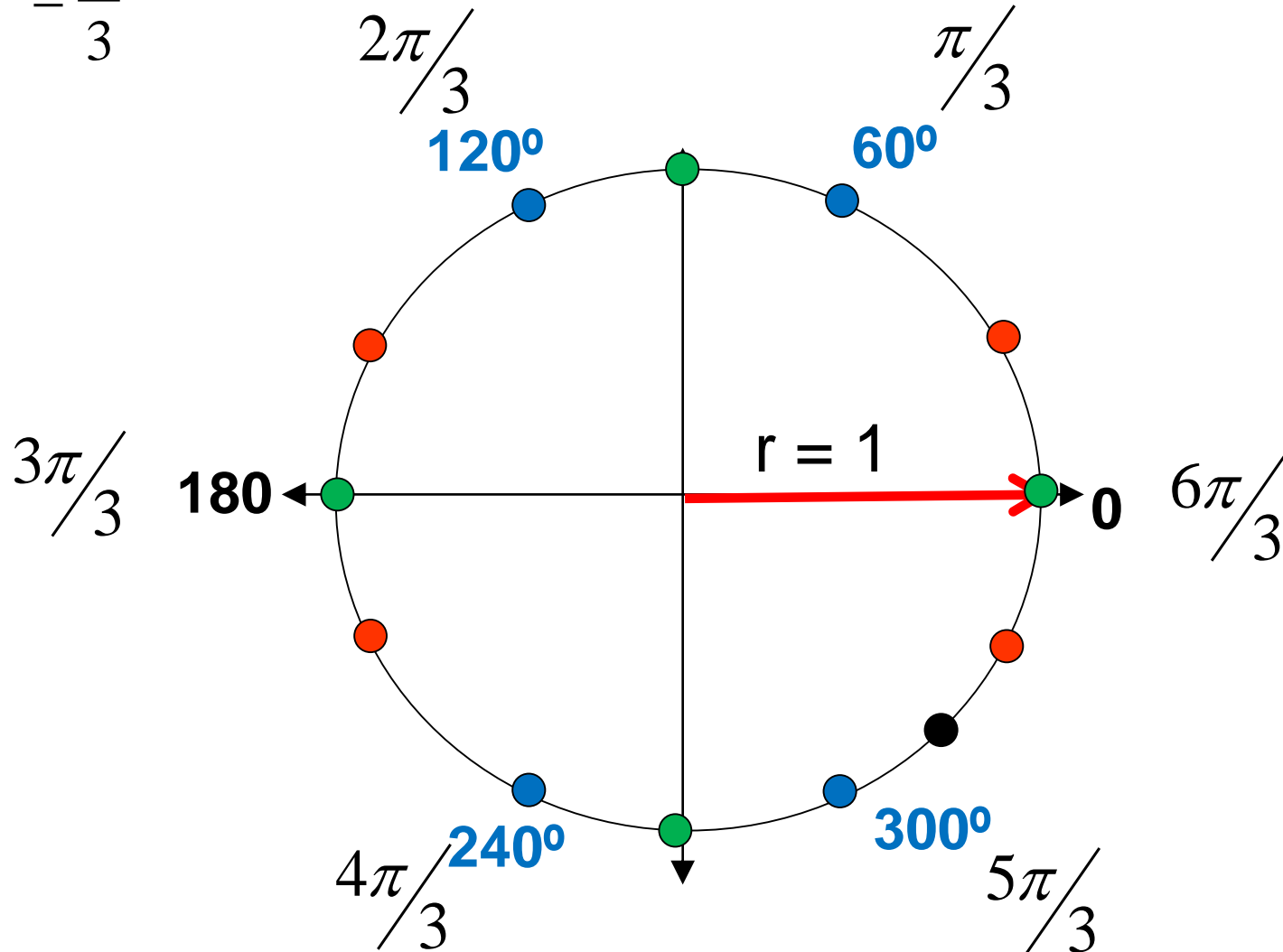
## Lesson 6-4

### The Law of Sines

$$60^\circ * \left( \frac{\pi}{180^\circ} \right) = \left( \frac{60^\circ \pi}{180^\circ} \right)$$

“count around” the circle by adding 60 ( $\pi/3$ ) to each angle

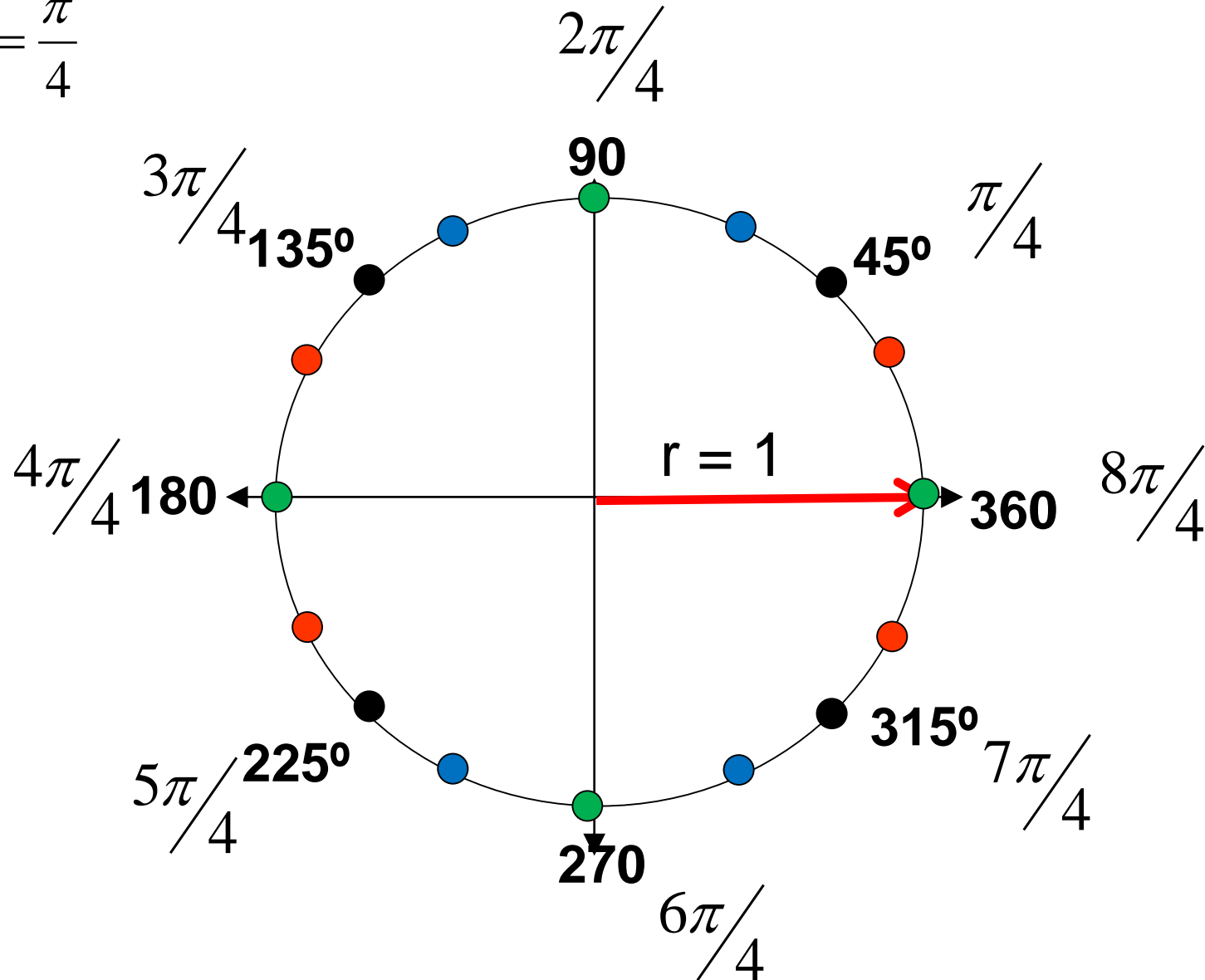
$$60^\circ = \frac{\pi}{3}$$



$$45^\circ * \left( \frac{\pi}{180^\circ} \right) = \left( \frac{45^\circ \pi}{180^\circ} \right)$$

“count around” the circle by adding 45 ( $\pi/4$ ) to each angle

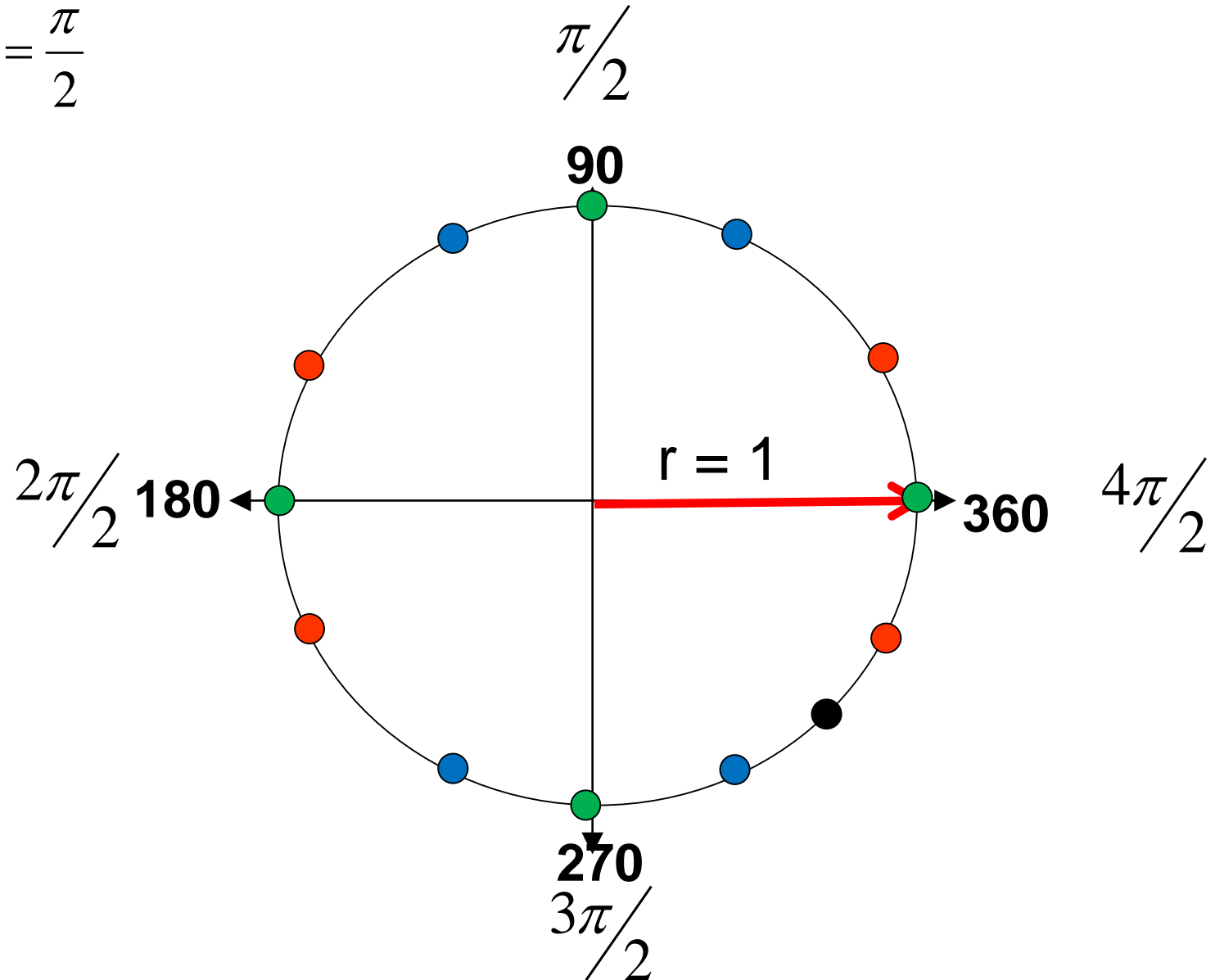
$$45^\circ = \frac{\pi}{4}$$



$$90^\circ * \left( \frac{\pi}{180^\circ} \right) = \left( \frac{90^\circ \pi}{180^\circ} \right)$$

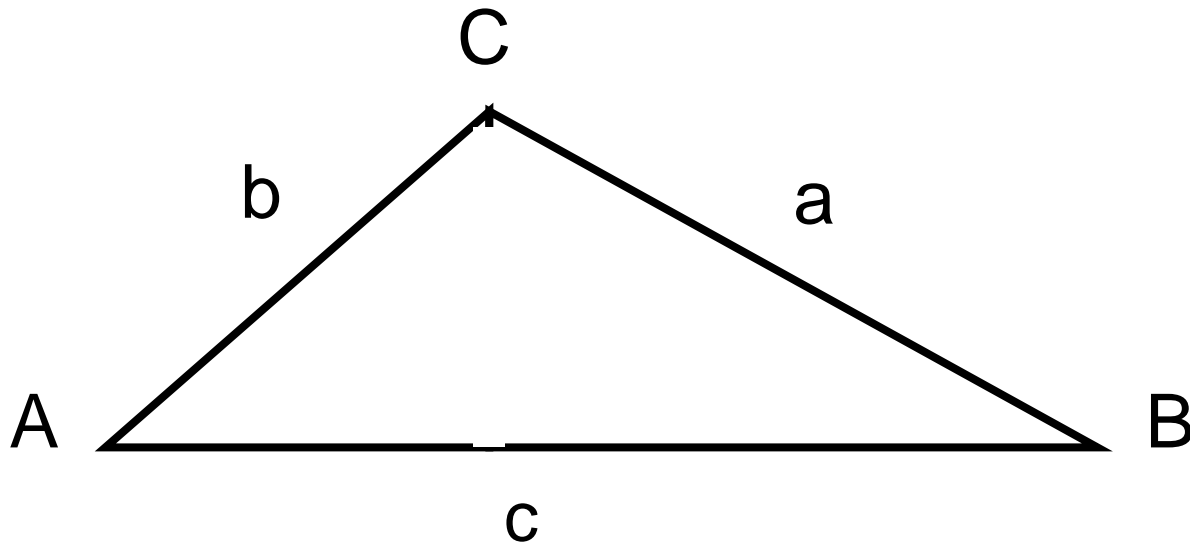
$$90^\circ = \frac{\pi}{2}$$

“count around” the circle by adding 90 ( $\pi/2$ ) to each angle



Sine, cosine, and tangent ratios are for solving Right triangles!

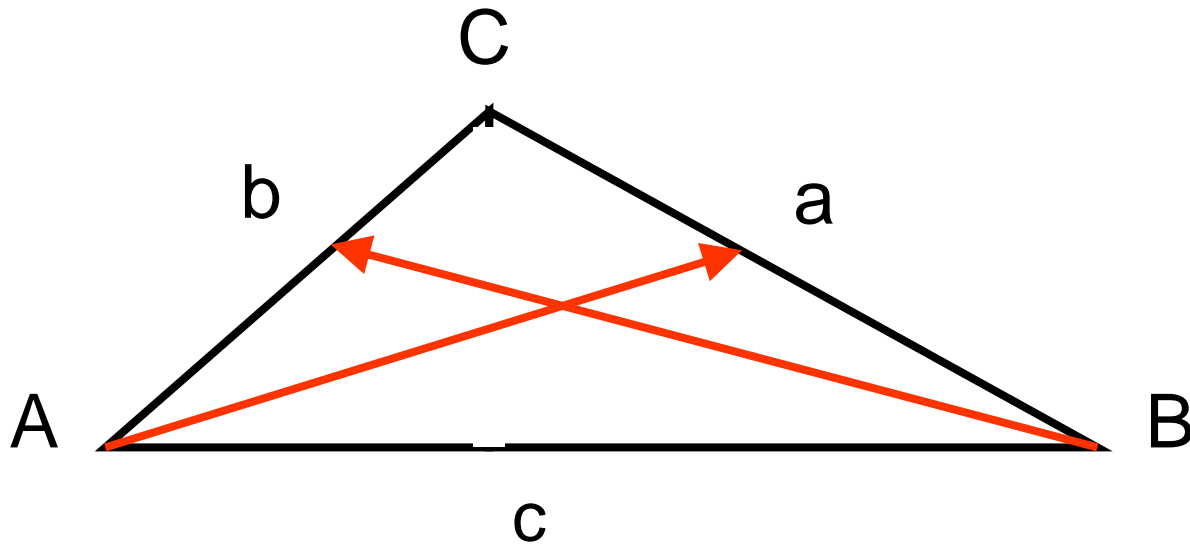
How do we solve for the unknown sides and angles of a triangle is not a right triangle?

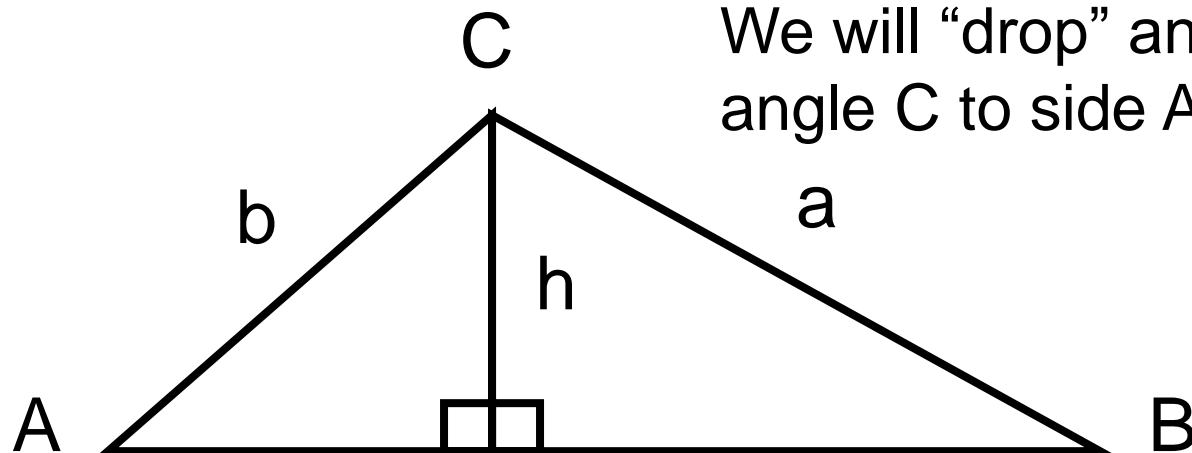


The Law of Sines can be used for ALL triangles.

The standard method of labeling triangles is:

The length of the side opposite Angle A is lower case a, etc.





We will “drop” an altitude from angle C to side AB.

$$\sin A = \frac{h}{b}$$

$$\sin B = \frac{h}{a}$$

We now have two right triangles that we can write sine ratios for.

What variable is common to both equations?

$$h = a \sin B$$

$$h = b \sin A$$

We solve each equation for ‘h’.

$$a \sin B = b \sin A$$

Since ‘h’ = ‘h’, we can set the equations equal to each other (by substitution) then rearrange.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

We could repeat this for any combination of sides and angle.

## Law of Sines

In  $\triangle ABC$  with angles  $A$ ,  $B$ , and  $C$  and sides  $a$ ,  $b$ , and  $c$ , respectively, the following equation is true :

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

By the Transitive Property this means each of the expressions are equal to each other.

We could also write it this way (using sequential property of equality steps):

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Which one we use depends upon whether we need to find the measure of an unknown angle or an unknown side.

Pick the version that puts the unknown variable in the numerator!

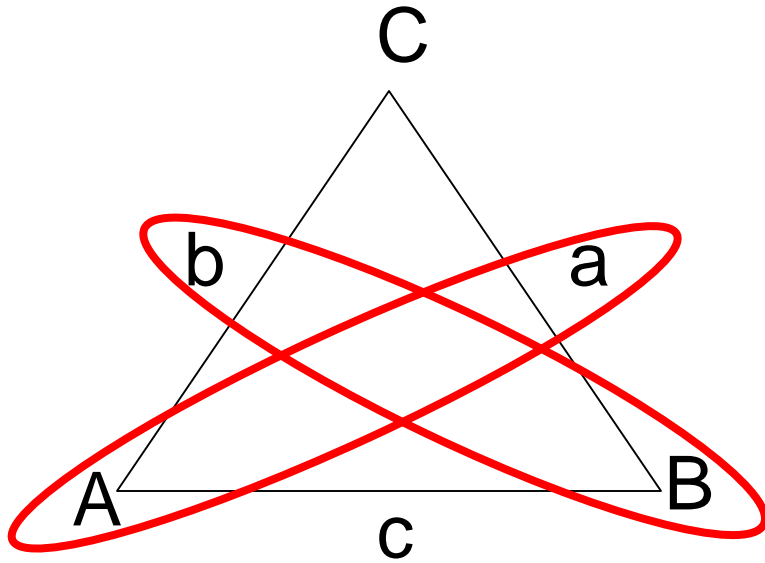


There is a pattern  
for Law of Sines

There are six possible unknowns in a  
triangle (3 sides, 3 angles).

A problem will give you three of  
the six unknowns.

After labeling the triangle with  
the given information, draw the  
following pattern (loop the  
2 sides and their opposite angles).



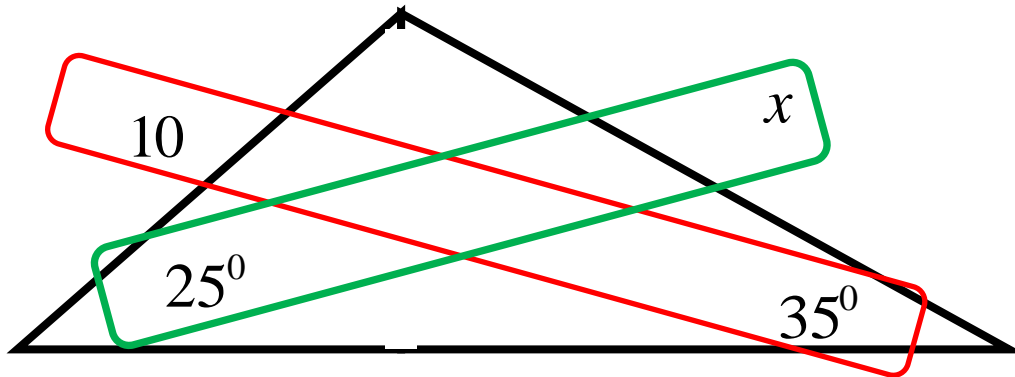
If three of the four items circled  
are known, use law of sines.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{or} \quad \frac{a}{\sin A} = \frac{b}{\sin B}$$

## Law of Sines

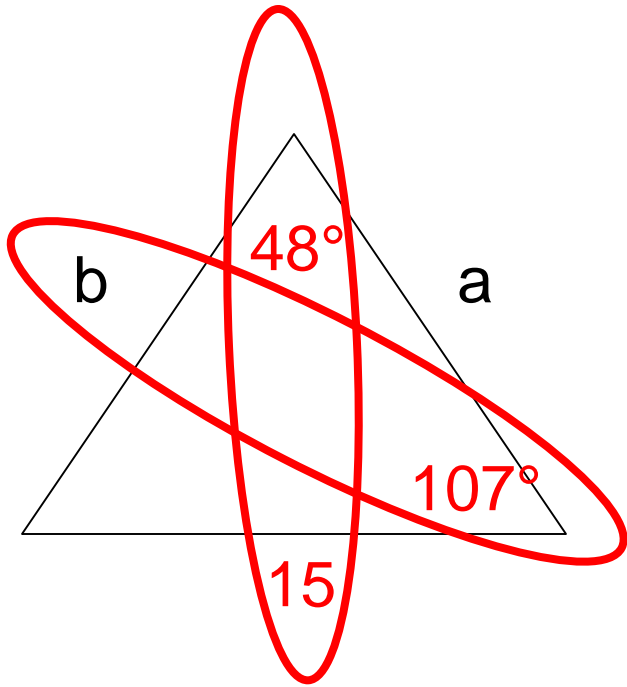
$$\frac{\textit{side}_1}{\sin \textit{Angle}_1} = \frac{\textit{side}_2}{\sin \textit{Angle}_2}$$

There is a pattern to the Law of Sines (sides and their opposite angles).



Three of the four numbers are known. You can use the proportion to solve for the unknown number.

## Draw the Law of Sines Loops



You can come up with one combination of loops that gives you only one unknown out of 4.

$$\frac{15}{\sin 48} = \frac{b}{\sin 107}$$

Can solve for 'b'

Can solve for the 3<sup>rd</sup> angle

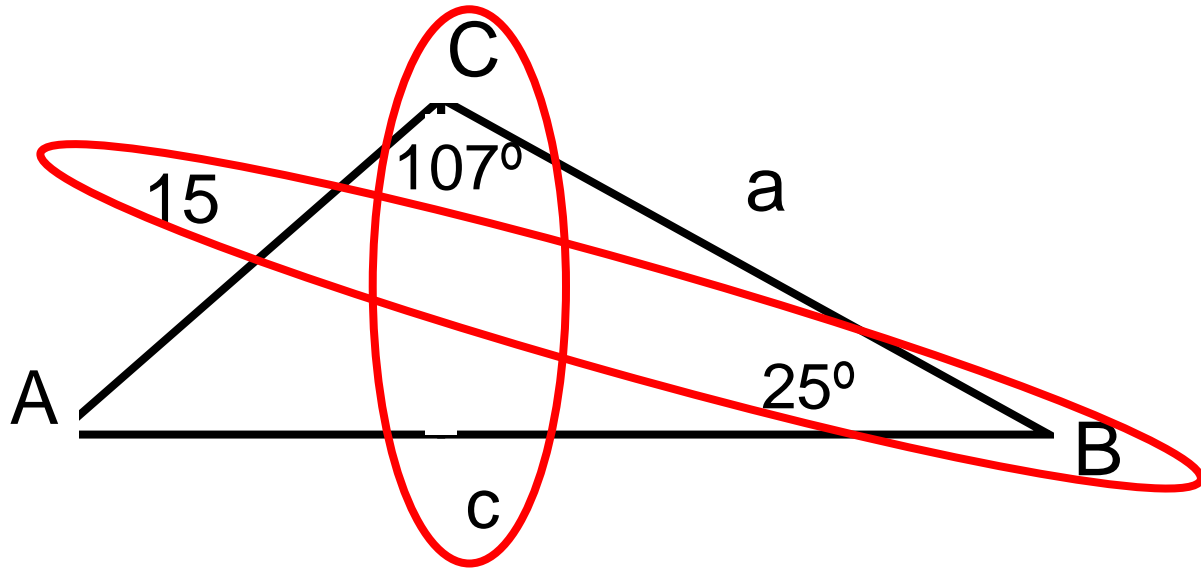
Can solve for the 'a'

## Solve for Side 'c'

1. Draw the Law of Sines Loops

2. Three of four numbers are known →

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$



$$\frac{c}{\sin 107^\circ} = \frac{15}{\sin 25^\circ}$$

3. Plug numbers into the Law of Sines equation and solve the unknown variable.

$$c = \frac{15 \sin 107^\circ}{\sin 25^\circ} = 33.9$$

Solve for Angle A 4. All three angles of a triangle add up to  $180^\circ$

$$m\angle A + m\angle B + m\angle C = 180$$

$$m\angle A + 25 + 107 = 180$$

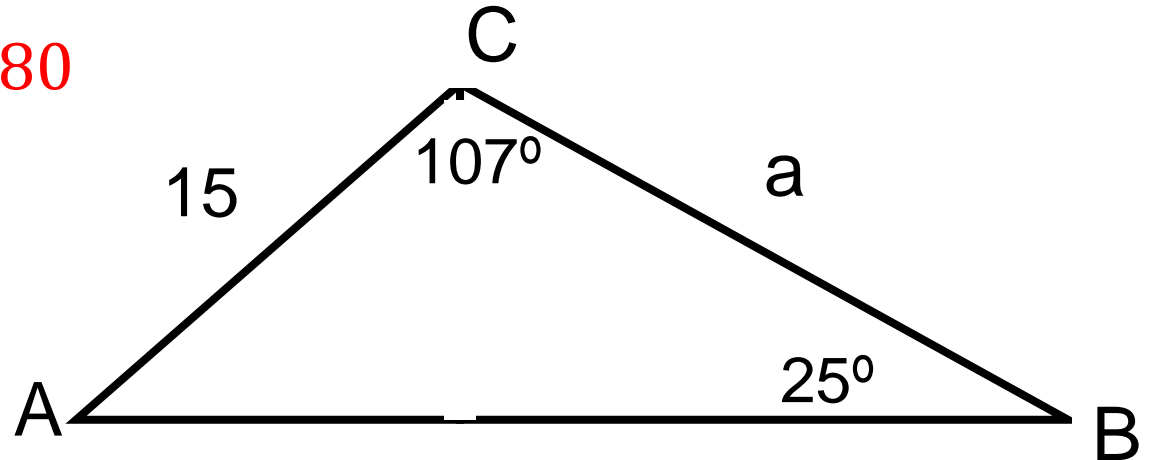
$$m\angle A = 48$$

4. Solve for 'a'  
(Use Law of Sines)

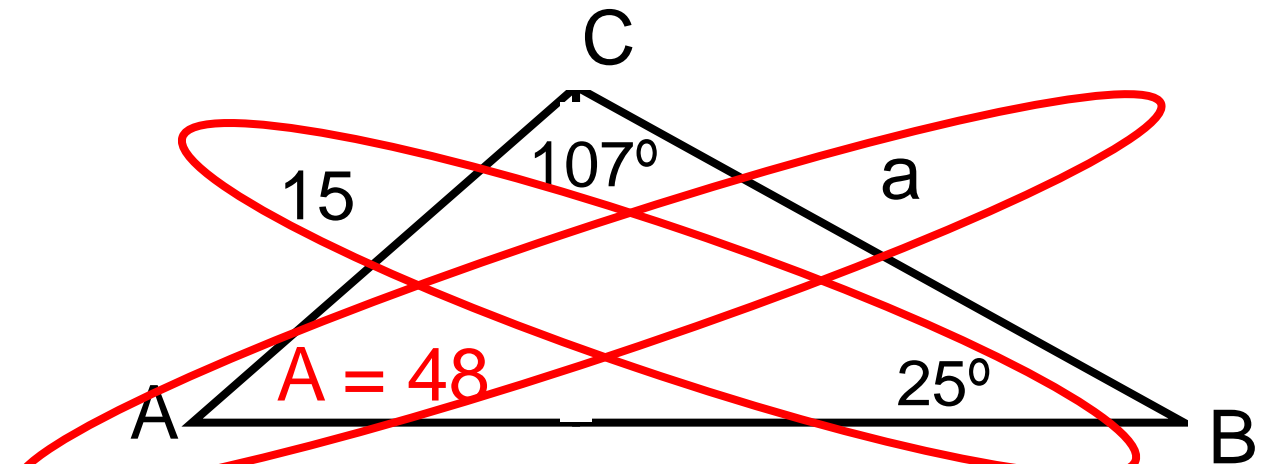
$$\frac{a}{\sin 48} = \frac{15}{\sin 25}$$

$$a = \frac{15 \sin 48}{\sin 25}$$

$$a = 26.4$$



$$c = 33.9$$



$$c = 33.9$$

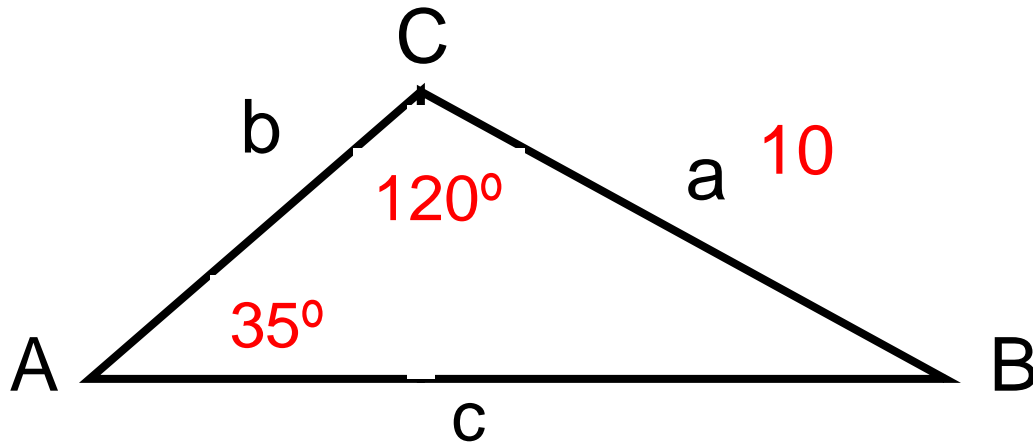
Another way a problem is given, is that they just give you some measurements.

$$A = 35^\circ$$

$$a = 10$$

$$C = 120^\circ$$

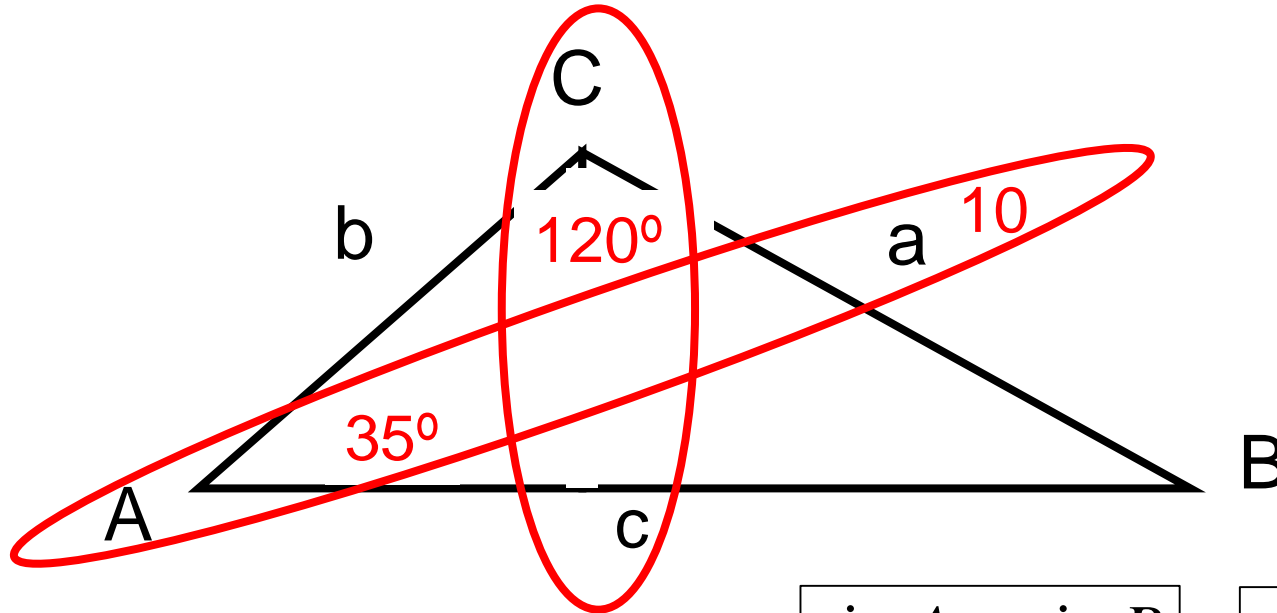
Draw the general triangle that has capital letters for angles and lower case letters for the lengths of the sides opposite the angles.



Label the triangle with values given in the problem.

$$A = 35^\circ \quad a = 10 \quad C = 120^\circ$$

Draw the "loops" to check if Law of Sines will work.



Write the Law of Sines equation

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

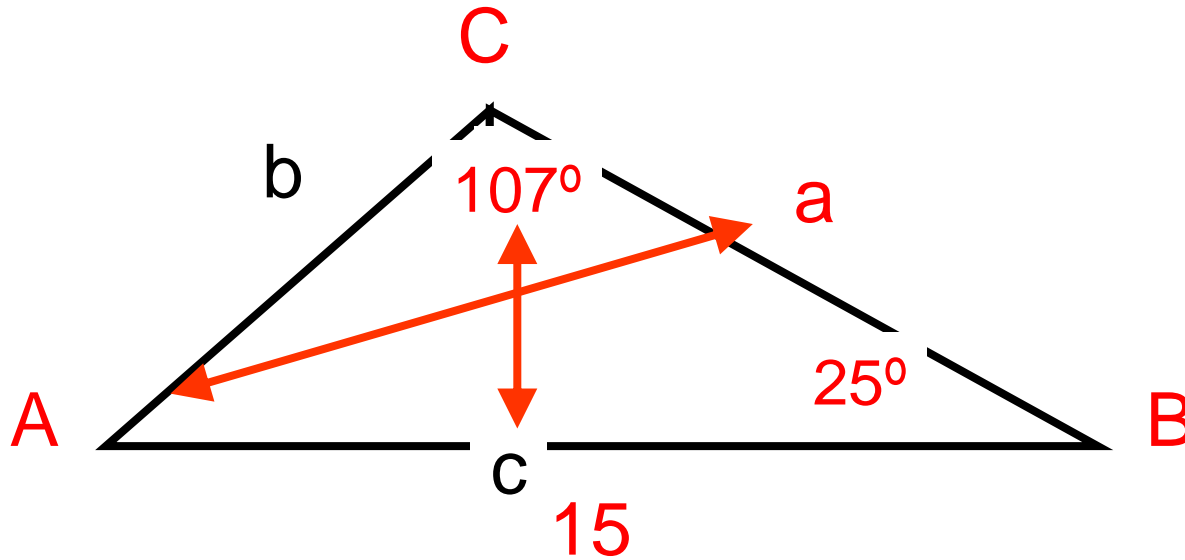
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Pick the version that puts the unknown variable in the numerator!

$$\frac{c}{\sin 120^\circ} = \frac{10}{\sin 35^\circ} \quad c = \sin 120^\circ \left( \frac{10}{\sin 35^\circ} \right) = 15.1$$

What if they given you two angles but not the two that you need?

$B = 25^\circ$     $C = 107^\circ$     $c = 15$    Find "little" 'a'



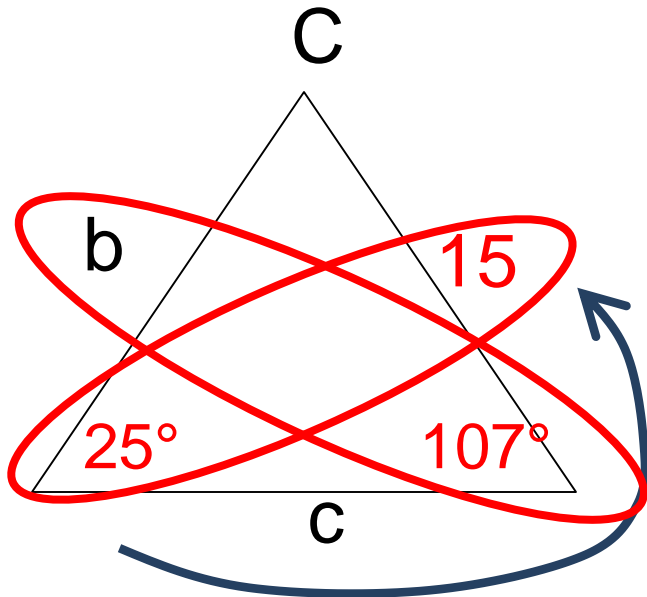
Using the Triangle Sum Theorem (angle in a triangle always add up to  $180^\circ$ ):  $m\angle A = 180 - (107 + 25) = 48$

Now solve using the Law of Sines.



## Triangle Review

“Walk around the block”



Start at the first side or angle that is known then list the order of the known items.

Angle, angle, side → AAS

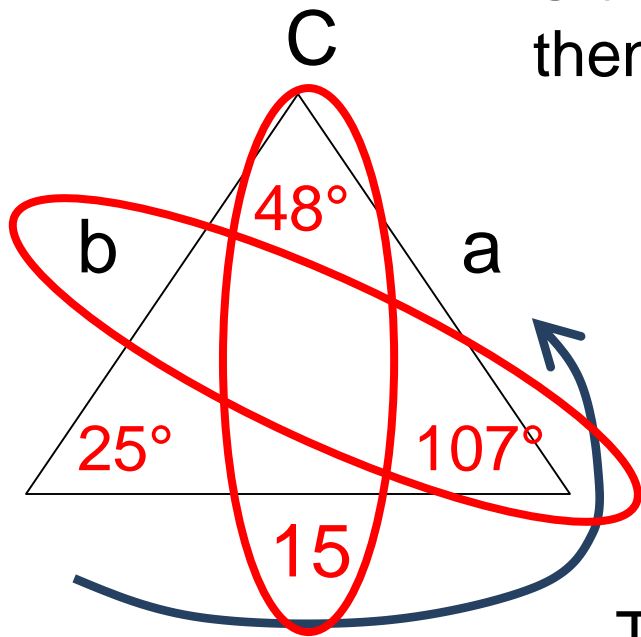
Or, in the opposite direction:  
Side, angle, angle → SAA

This means: “the measures of two angles and the non-included side are known.

Law of Sines will work for AAS or SAA.

If the following information is given “Walk around the block”

Start at the first side or angle that is known then list the order of the known items.



Angle, side, angle → ASA

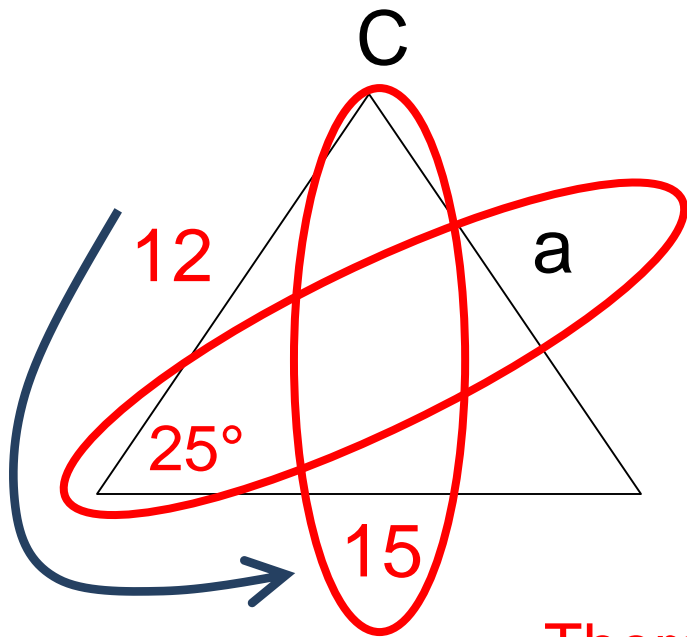
(same thing in the opposite direction: ASA)

This means: “the measures of two angles and the included side are known.

If you have two angles of a triangle, you can find the 3<sup>rd</sup> angle.

Law of Sines will work for ASA.

If the following information is given “Walk around the block”



Start at the first side or angle that is known then list the order of the known items.

Side, Angle, Side → SAS

This means: “the measures of two sides and the included angle are known.

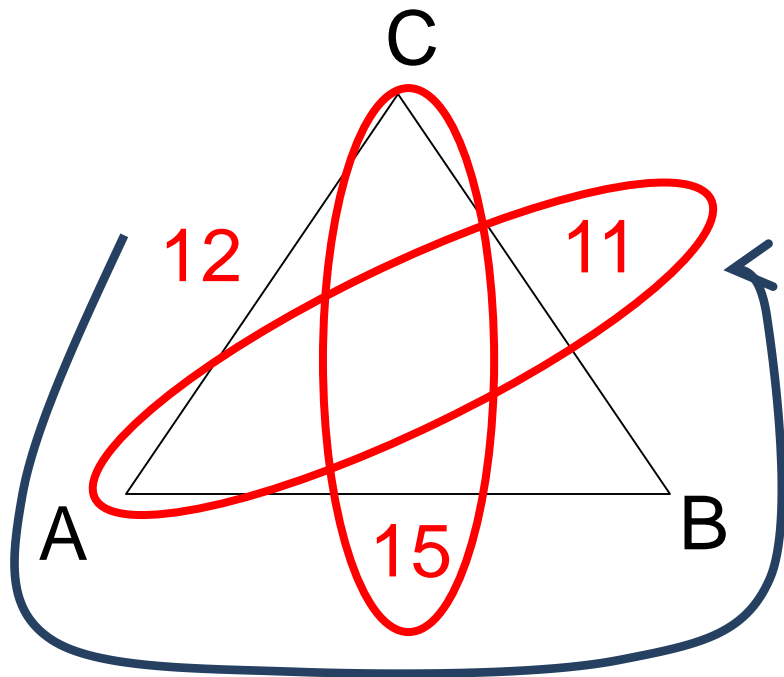
There are two unknown values in the “loops.”

You cannot solve a single equation that has two unknown values!

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Law of Sines will NOT work for SAS

If the following information is given: **“Walk around the block”**



Start at the first side or angle that is known then list the order of the known items.

Side, Side, Side → SSS

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Law of Sines will NOT work for SSS.