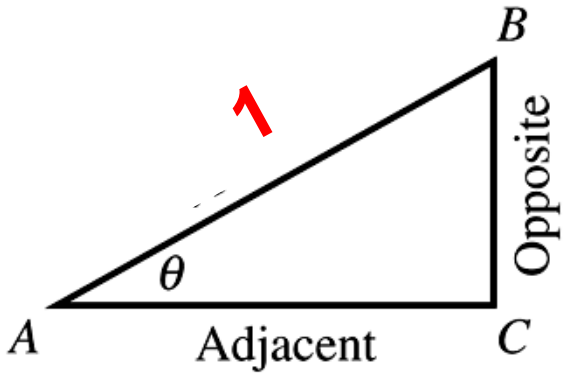
Math-3
Lesson 6-2

Exact Trigonometric Ratios for Nice Angles

hypotenuse = 1

Why is it “nice” to have a hypotenuse whose length is ‘1’?



Sin Θ= <u>opposite side</u>

Cos Θ= <u>adjacent side</u>

Tan Θ= <u>opp/adj</u>

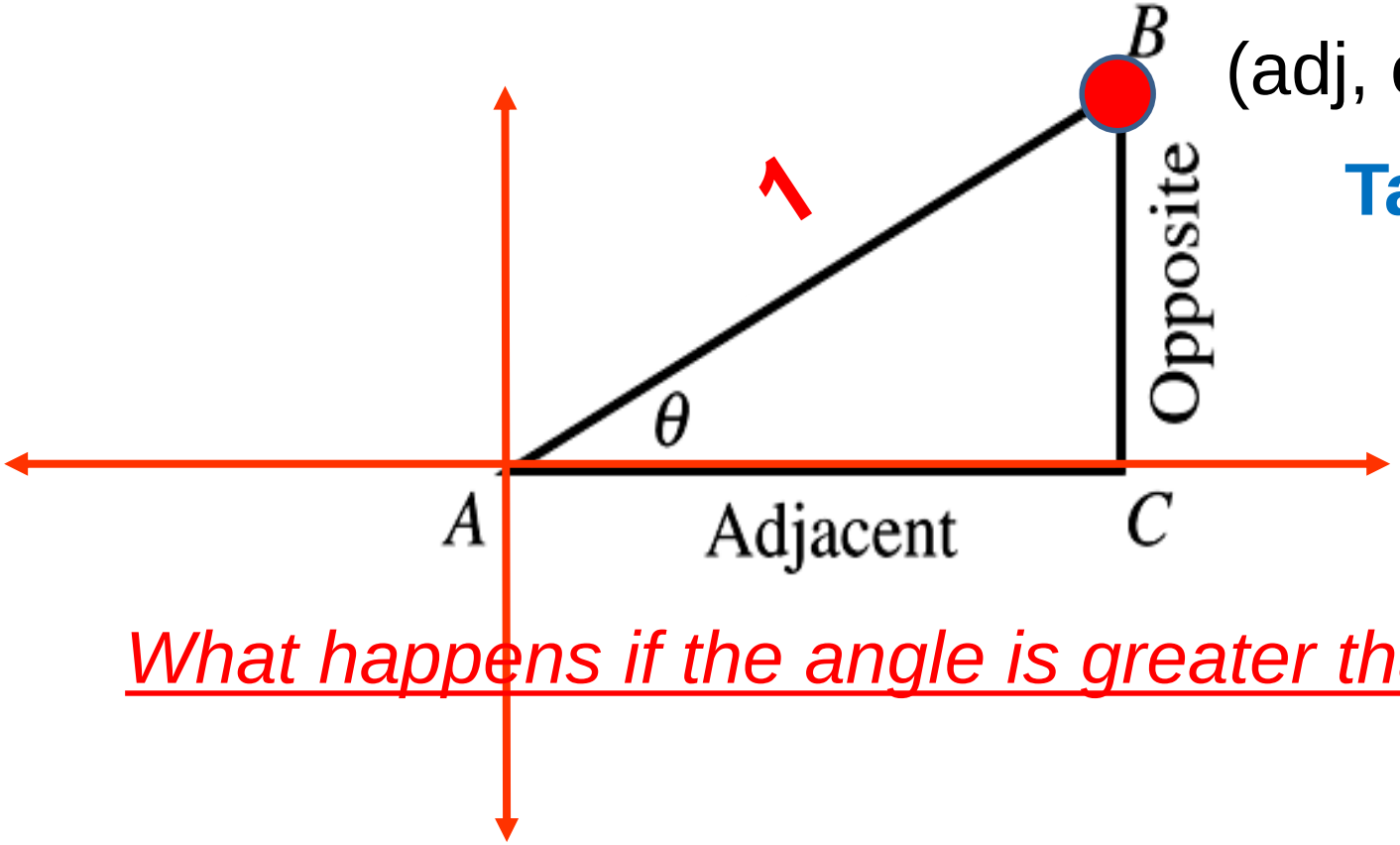
The length of the hypotenuse is <u>no longer in the ratio!</u>

What happens if the angle is greater than 90?

We must put the angle <u>on the x-y plane</u> with the vertex of the angle at (0, 0)

Trig ratios now must account for <u>sign (+/-)</u> of the x-y pair of the point.

Cos Θ = x
Sin Θ = y



(adj, opp) = (x, y)

Tan Θ = y/x

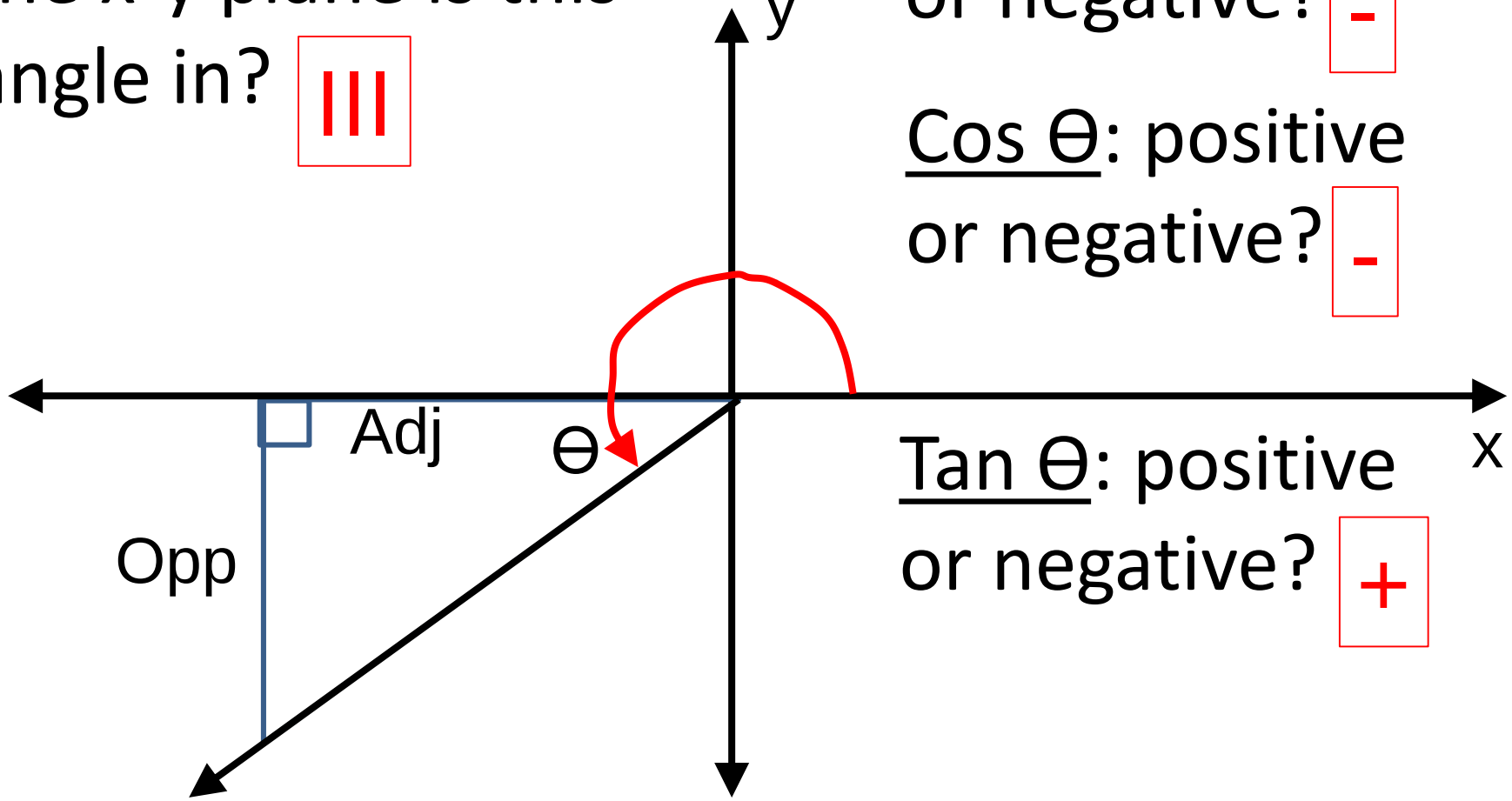
<u>What happens if the angle is greater than 90?</u>

Reference angle: The acute angle with the x-axis.

What quadrant of the x-y plane is this angle in? III

Sin Θ: positive or negative? -

Cos Θ: positive or negative? -



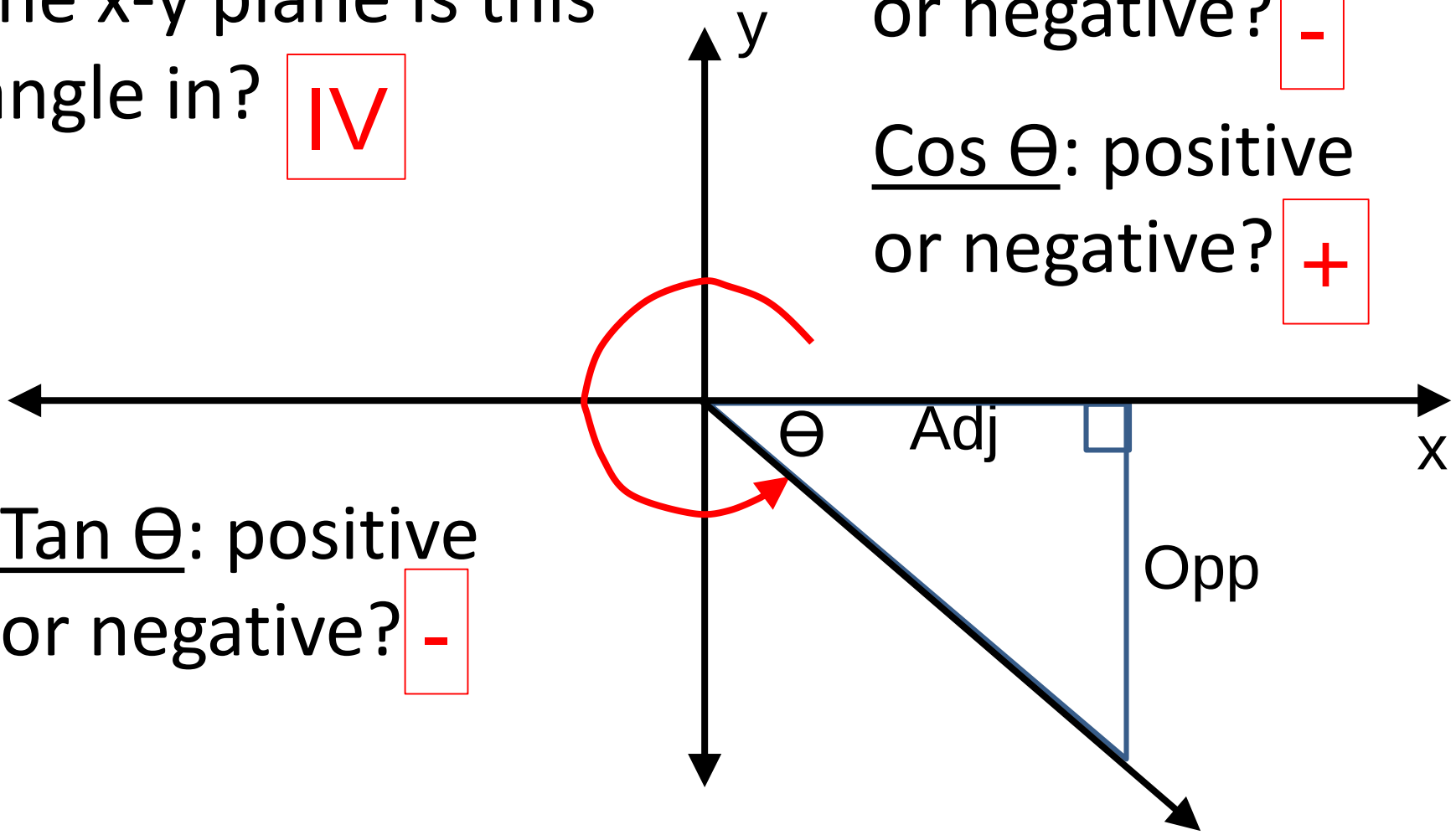
Tan Θ: positive or negative? +

Reference angle: The acute angle with the x-axis.

What quadrant of the x-y plane is this angle in? IV

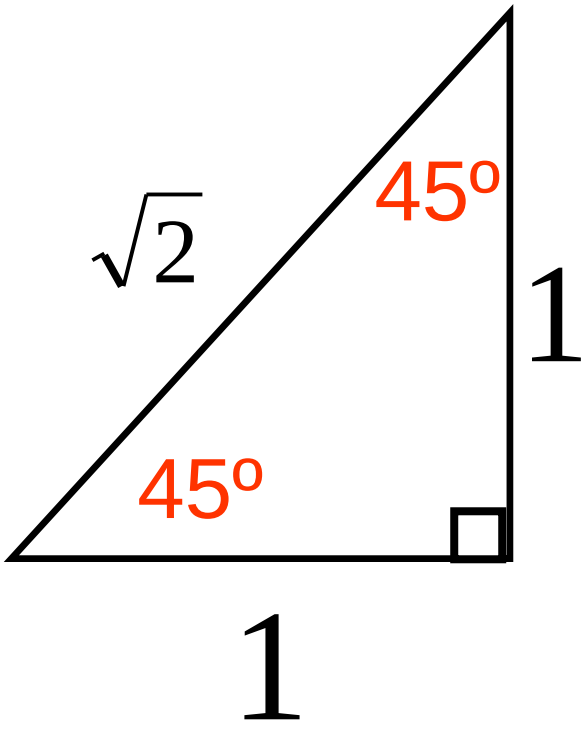
Sin Θ: positive or negative? -

Cos Θ: positive or negative? +

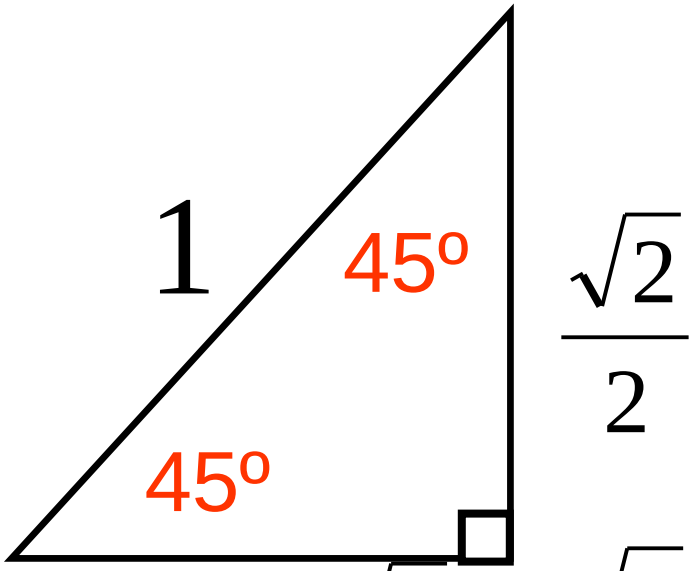


Tan Θ: positive or negative? -

Do you remember the side lengths for a 45-45-90 triangle?

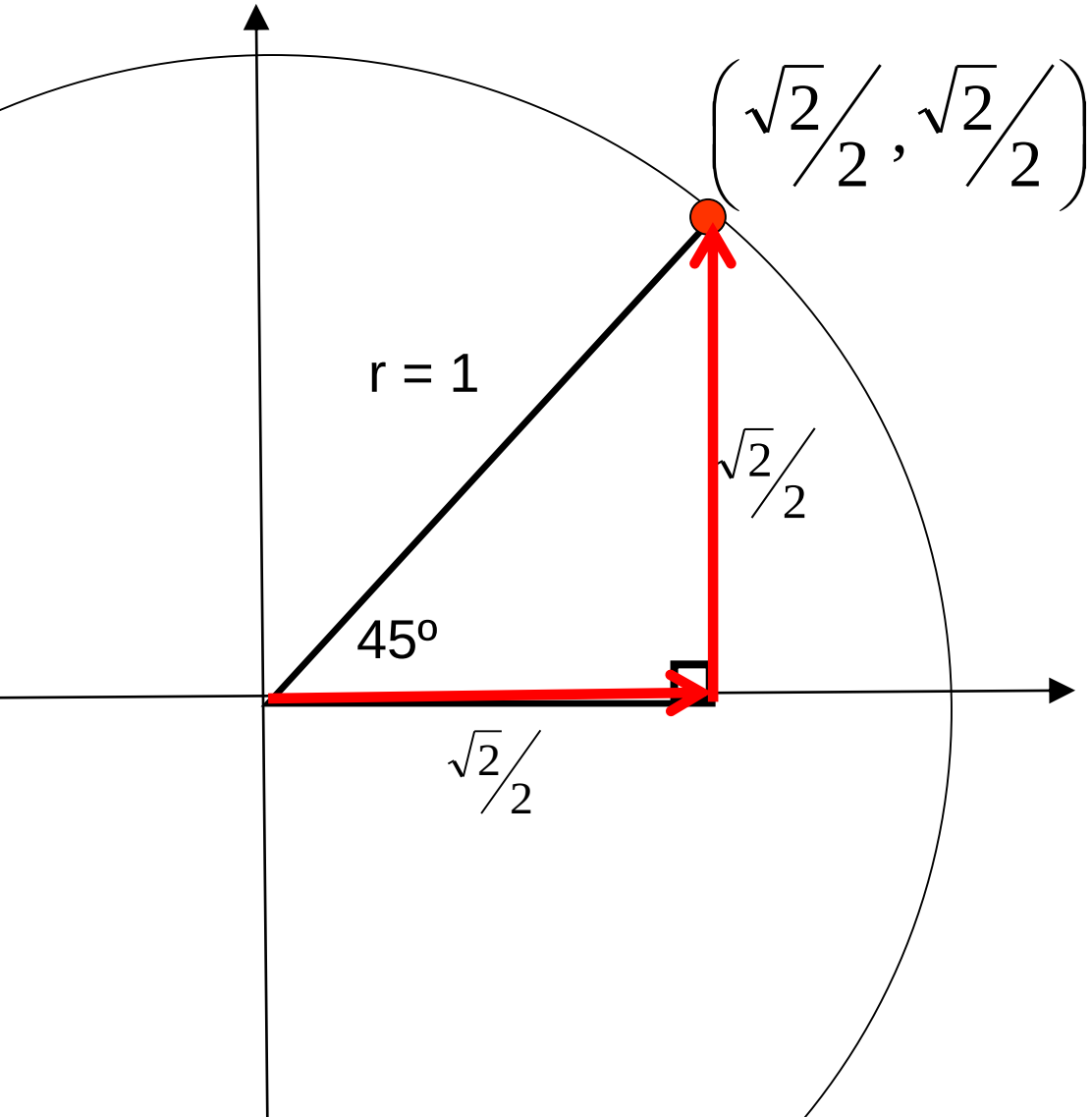


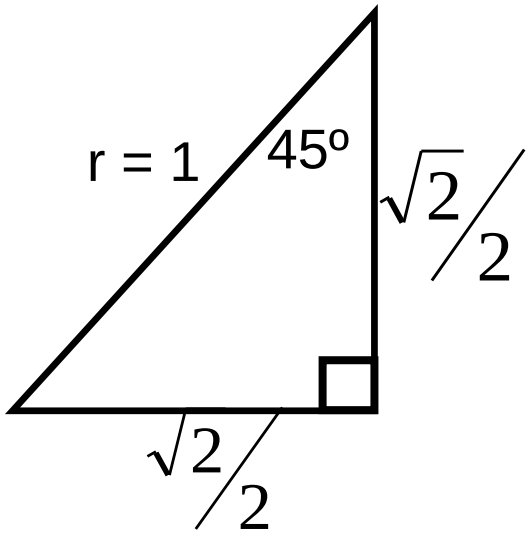
What are the leg lengths if the hypotenuse = 1?



$$\frac{1}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Let's put the triangle <u>on top of</u> a circle with radius = 1.



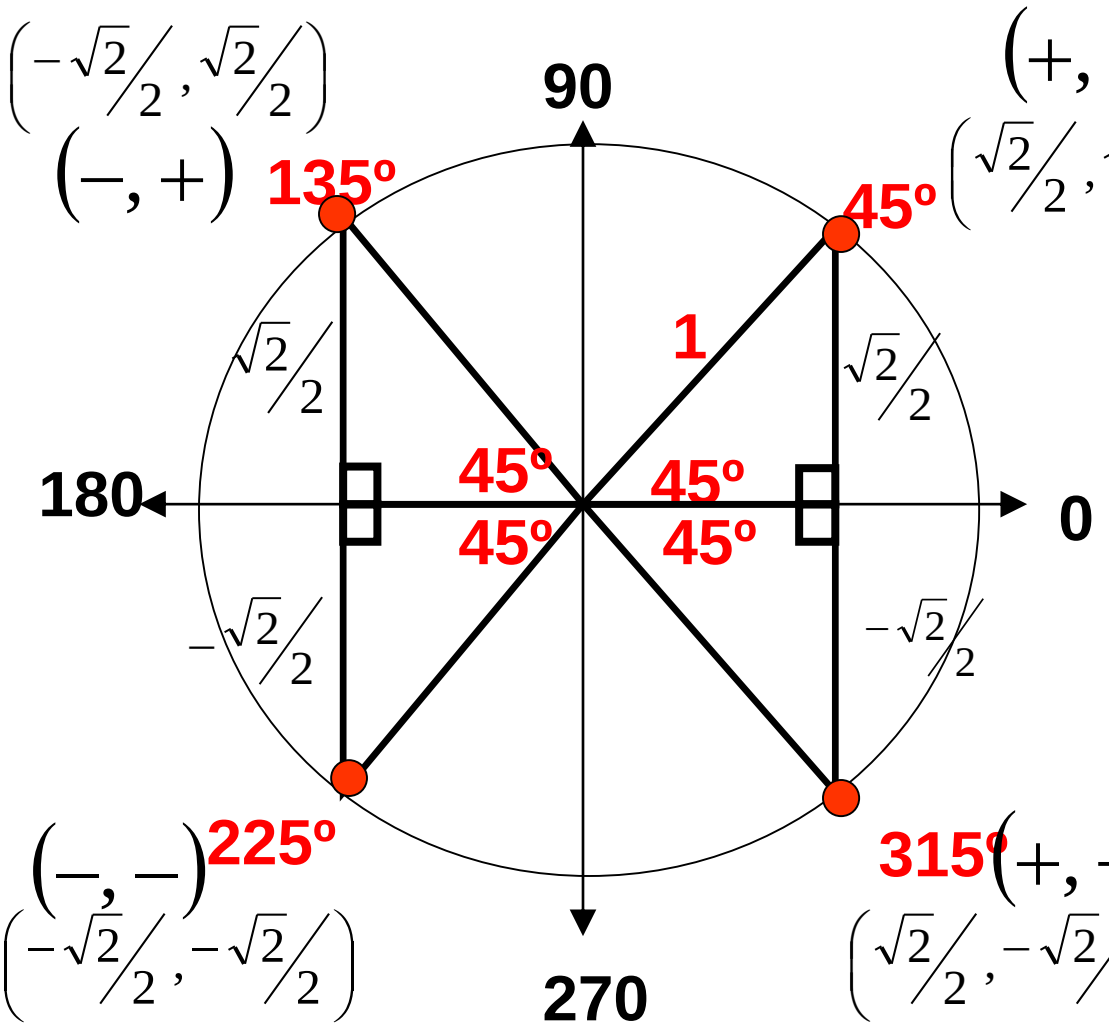


Sin Θ = y

Cos Θ = x

Tan Θ = y/x

We can use a 45° reference angle 4 times

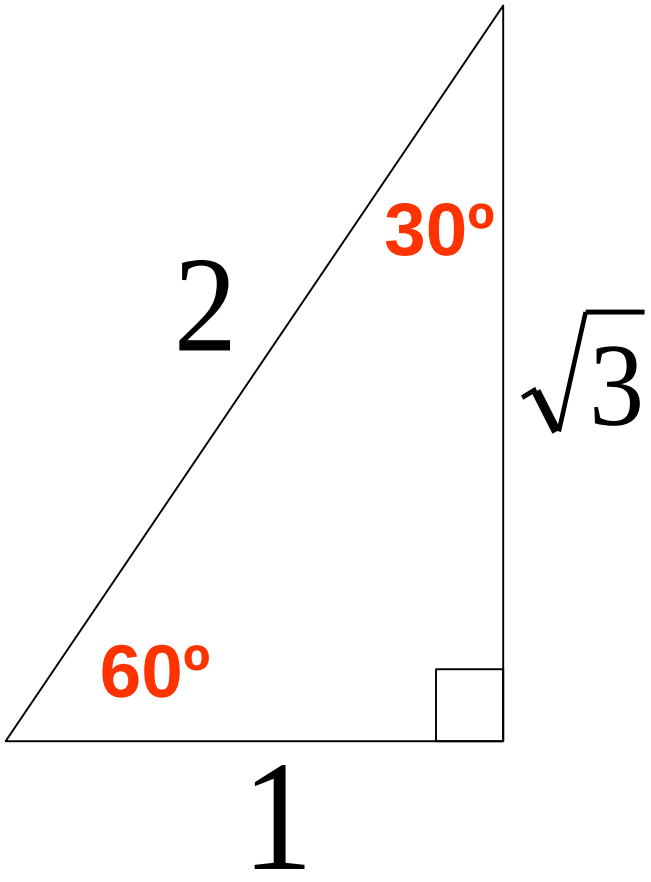


Sin Θ = y

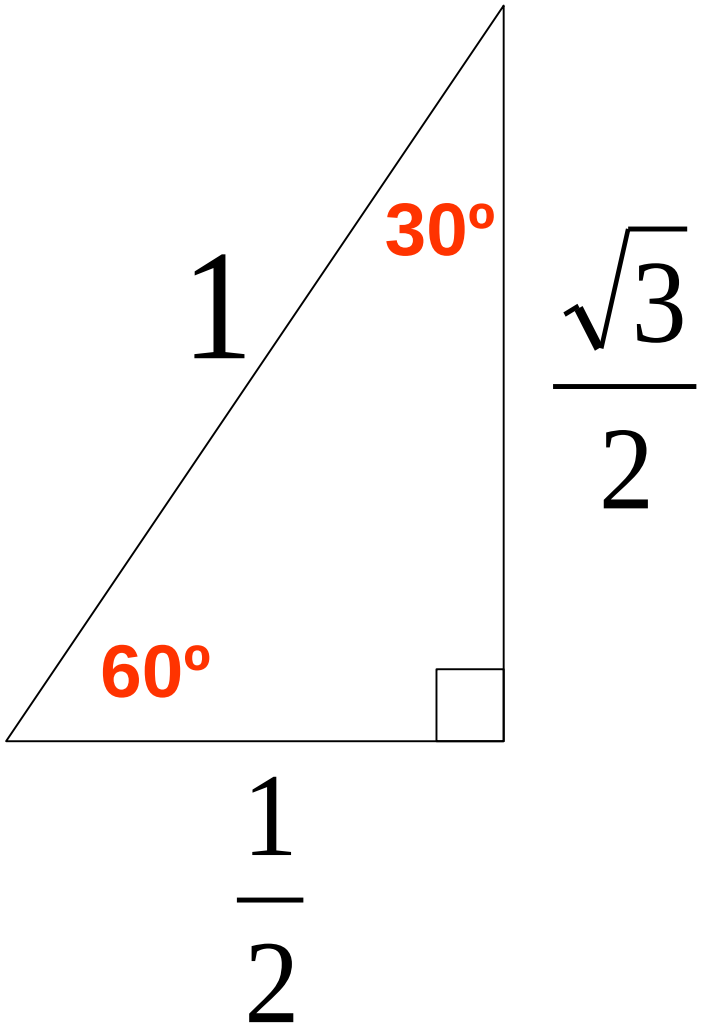
Cos Θ = x

Tan Θ = y/x

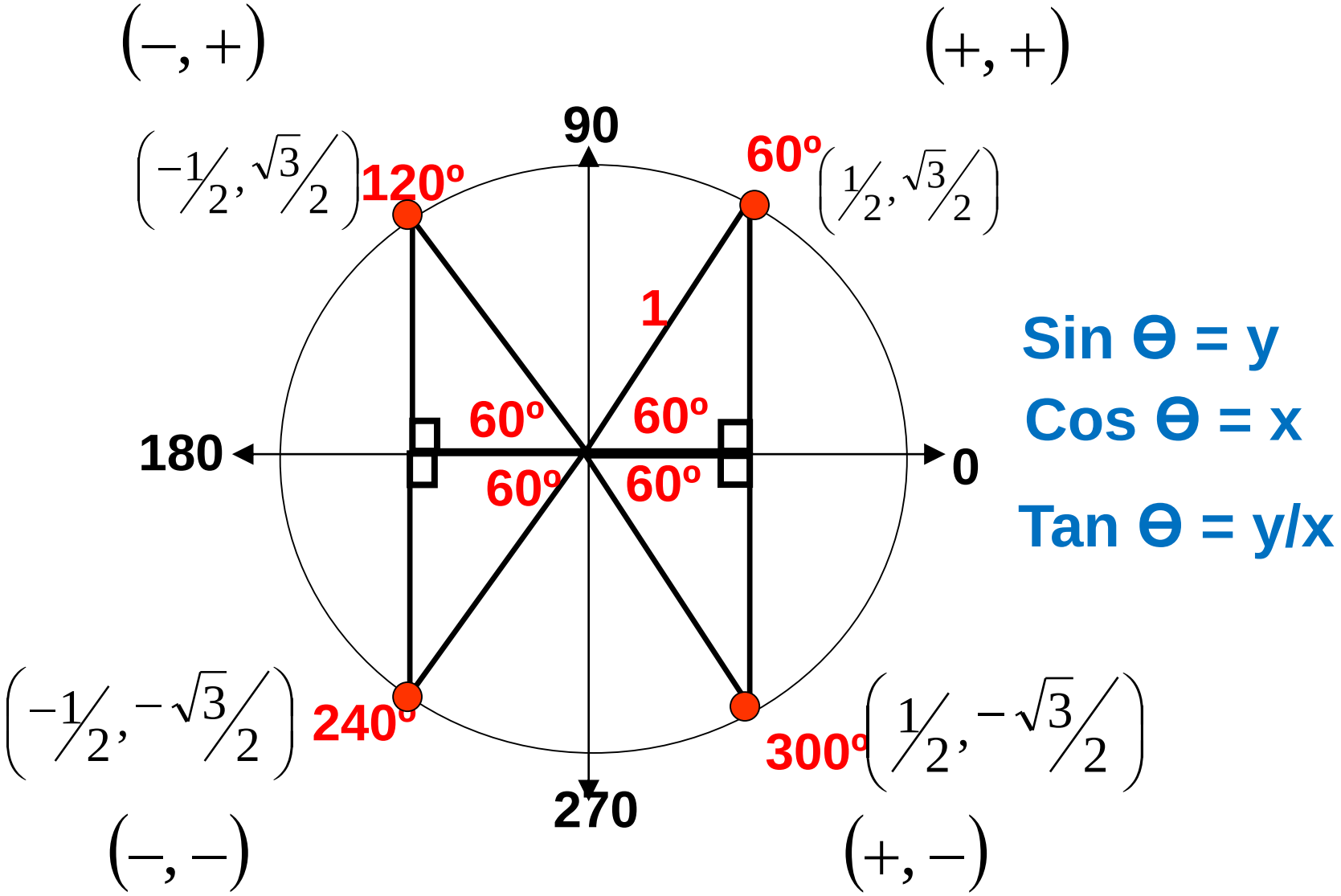
Do you remember the side lengths for the 30-60-90 triangle?



What are the leg lengths if the hypotenuse = 1?



We can use a 60° reference angle 4 times

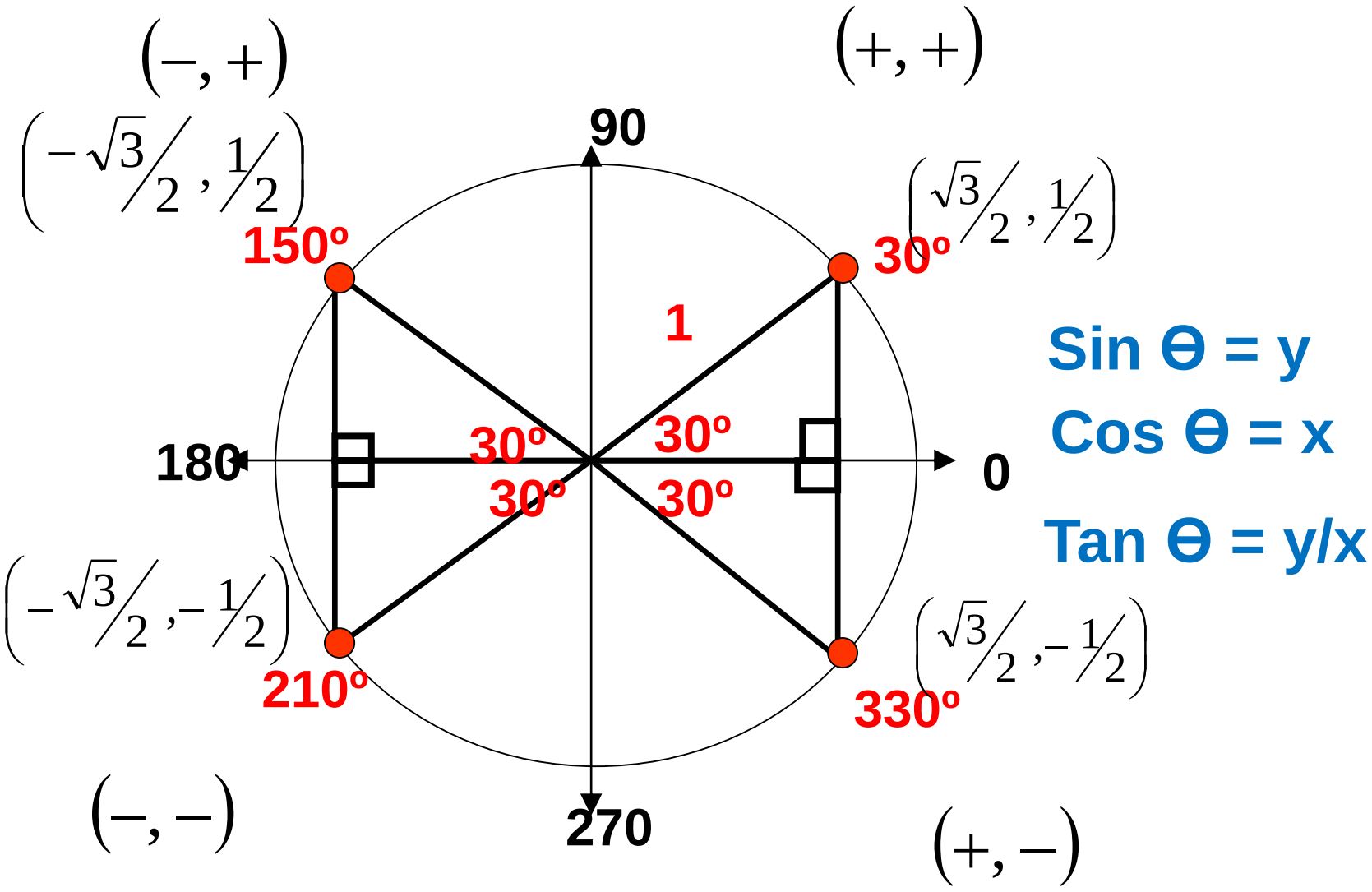


Sin Θ = y

Cos Θ = x

Tan Θ = y/x

We can use a 30° reference angle 4 times



We know the exact ratios for the following angles.

Angle	Sine	Cosine	Tangent
30	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
150	$1/2$	$-\sqrt{3}/2$	$-\sqrt{3}/3$
210	$-1/2$	$-\sqrt{3}/2$	$\sqrt{3}/3$
330	$-1/2$	$\sqrt{3}/2$	$-\sqrt{3}/3$

Reference Angle: 30°

$$\text{Tan } 30° = y/x = \frac{1/2}{\sqrt{3}/2} = \frac{1}{2} * \frac{2}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

We know the exact ratios for the following angles.

Angle	Sine	Cosine	Tangent
45	$\sqrt{2}/2$	$\sqrt{2}/2$	1
135	$\sqrt{2}/2$	$-\sqrt{2}/2$	-1
225	$-\sqrt{2}/2$	$-\sqrt{2}/2$	1
315	$-\sqrt{2}/2$	$\sqrt{2}/2$	-1

Reference Angle: 45°

$$\text{Tan } 45° = y/x = \frac{\sqrt{2}/2}{\sqrt{2}/2} = 1$$

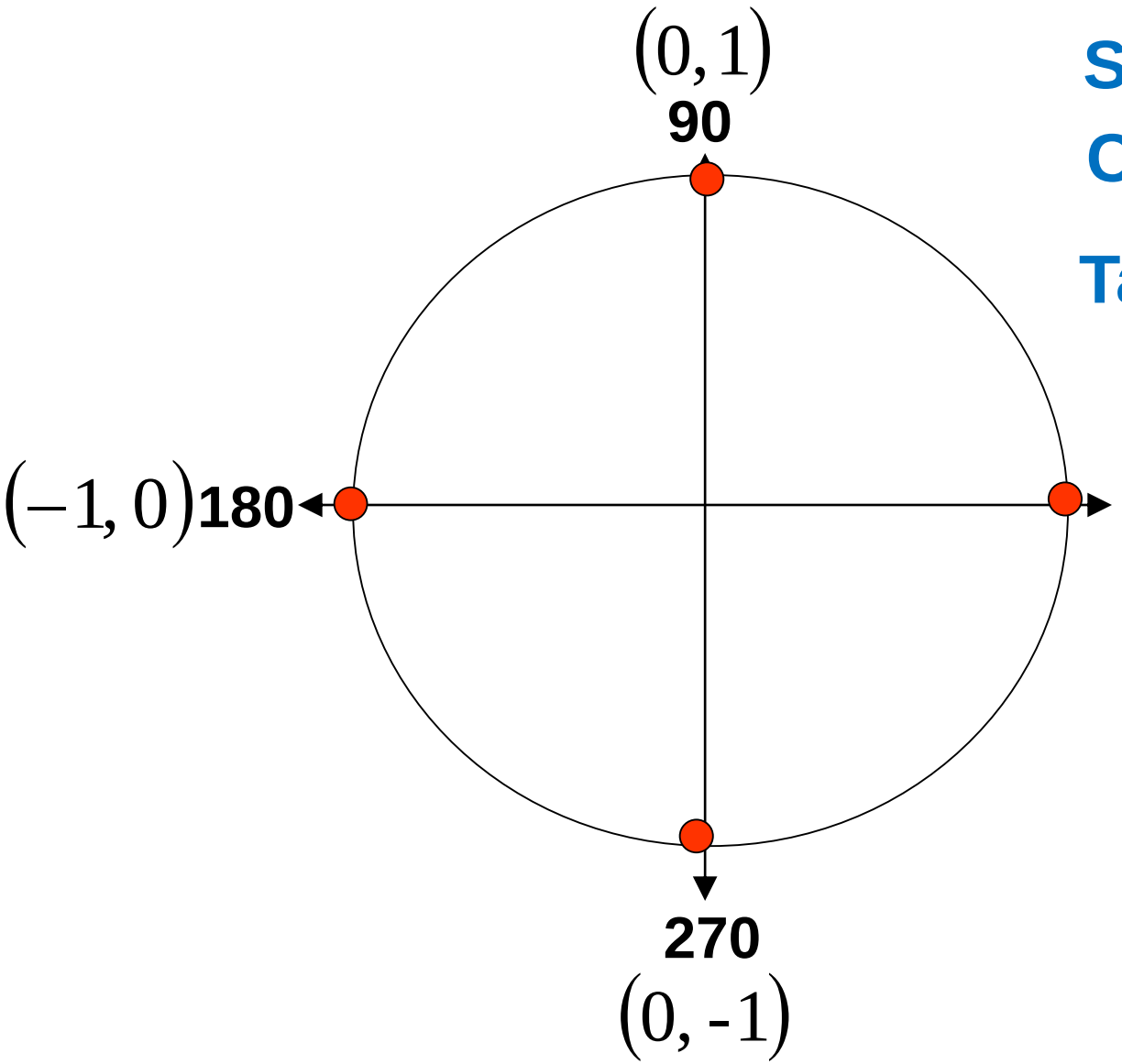
We know the exact ratios for the following angles.

Angle	Sine	Cosine	Tangent
60	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
120	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$
240	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}$
300	$-\sqrt{3}/2$	$1/2$	$-\sqrt{3}$

Reference Angle: 60°

$$\text{Tan } 60° = y/x = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2} * \frac{2}{1} = \sqrt{3}$$

What about the “cardinal angles”?



We know the exact ratios for the following angles.

Angle	Sine	Cosine	Tangent
0	0	1	0
90	1	0	$undef$
180	0	-1	0
270	-1	0	$undef$

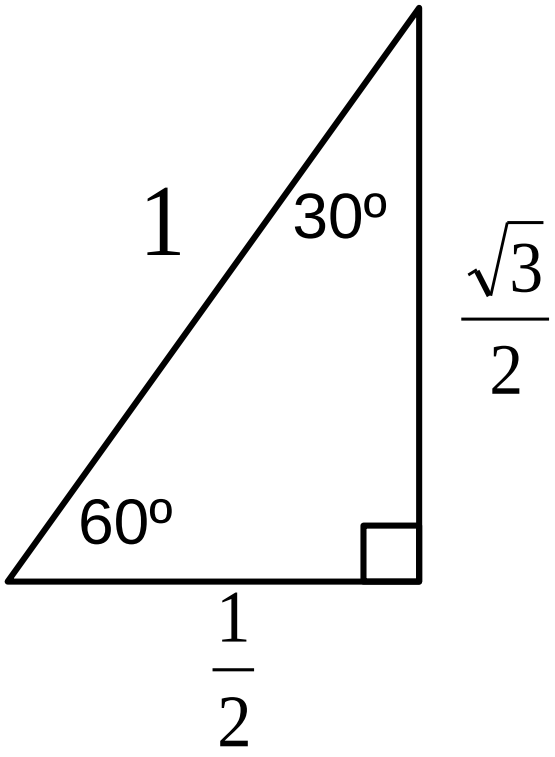
Tan 0° $= y/x = \frac{0}{1} = 0$

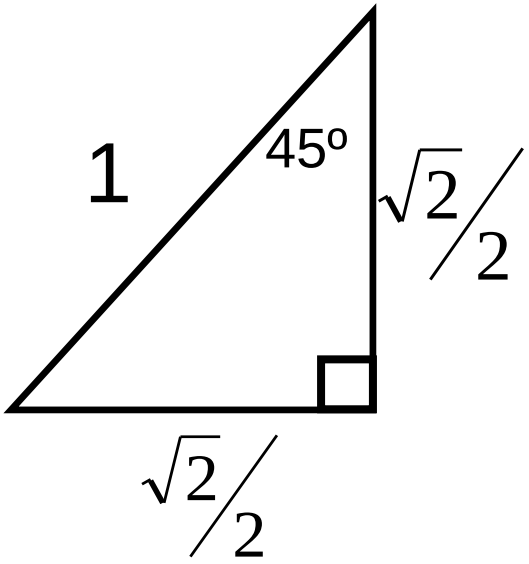
Tan 90° $= y/x = \frac{1}{0} = undefined$

The tangent function does **NOT** have a domain of "all real numbers".

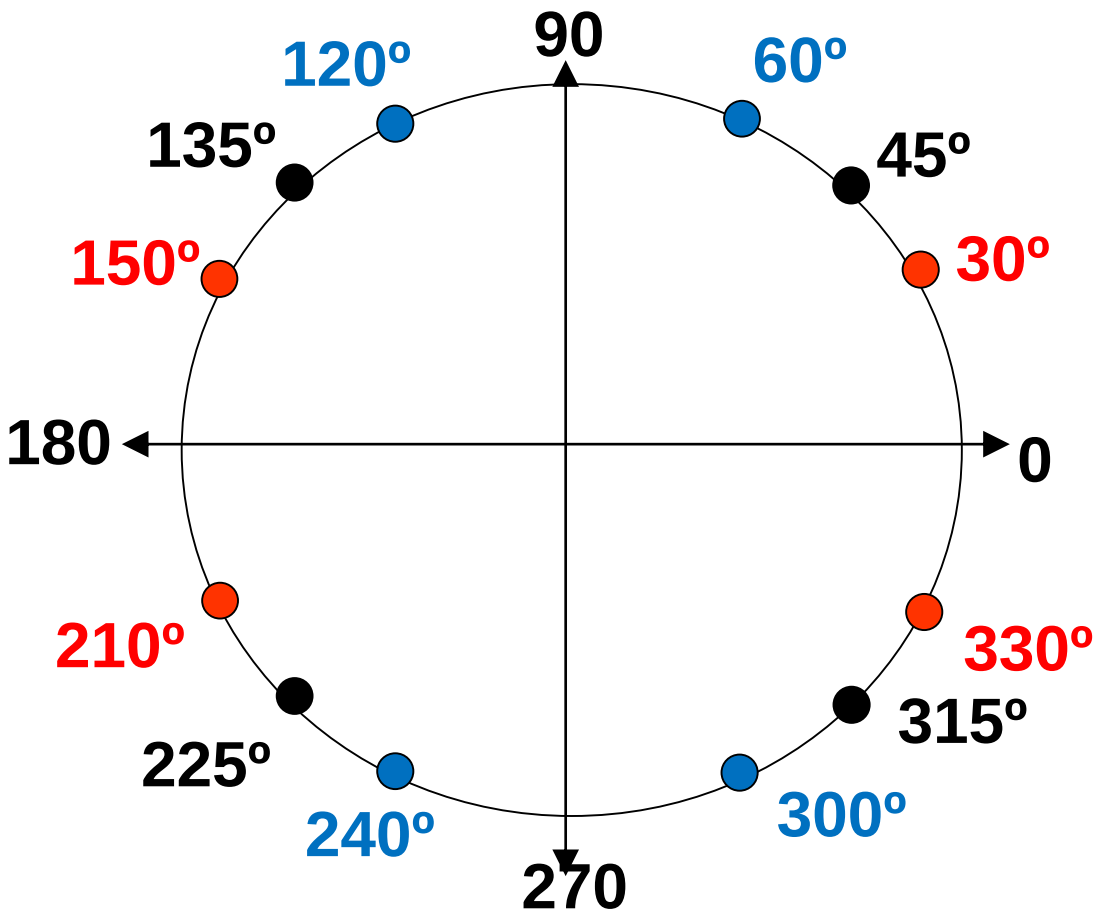
The “numbers” for “nice angles.”

$$0 \qquad \frac{1}{2} \qquad \frac{\sqrt{2}}{2} \qquad \frac{\sqrt{3}}{2} \qquad \frac{\sqrt{3}}{3} \qquad \sqrt{3} \qquad 1$$





Can you quickly come up with the exact ratio?



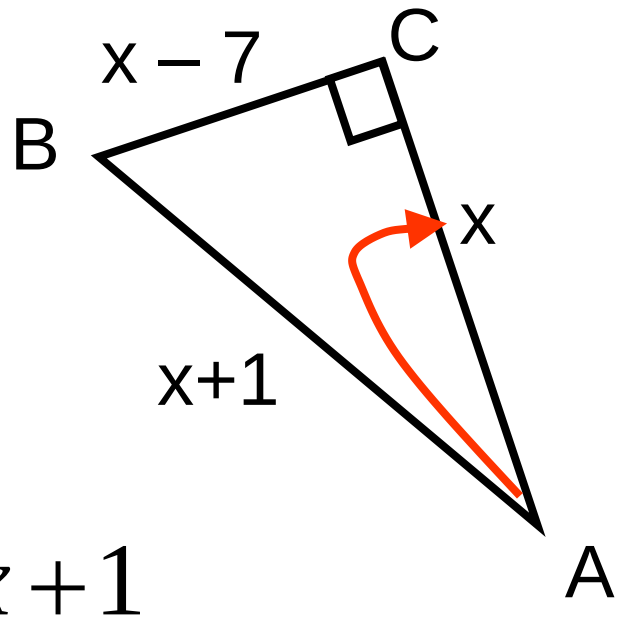
$\sin \theta = y$

$\cos \theta = x$

$\tan \theta = y/x$

Your turn:

What is the <u>length</u> of the leg adjacent to angle A?



x

$$(x-7)^2+(x)^2=(x+1)^2$$

$$x^2-14x+49+x^2=x^2+2x+1$$

$$2x^2-14x+49=x^2+2x+1$$

$$x^2-16x+48=0$$

$$(x-12)(x-4)=0$$

$$x=12,\ 4$$

Only x = 12 works
(since x – 7 yields a side length of -3).