# Math-3
# Lesson 6-1

## Trigonometric Ratios for Right Triangles

Triangle Similarity: Same shape (not same size)

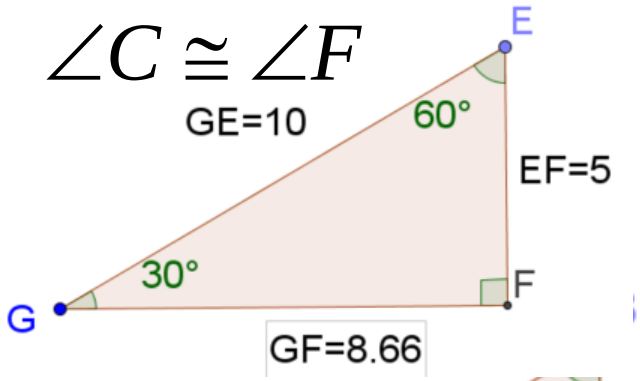
Shape results from three pairs of congruent angles.

$\angle A \cong \angle G$

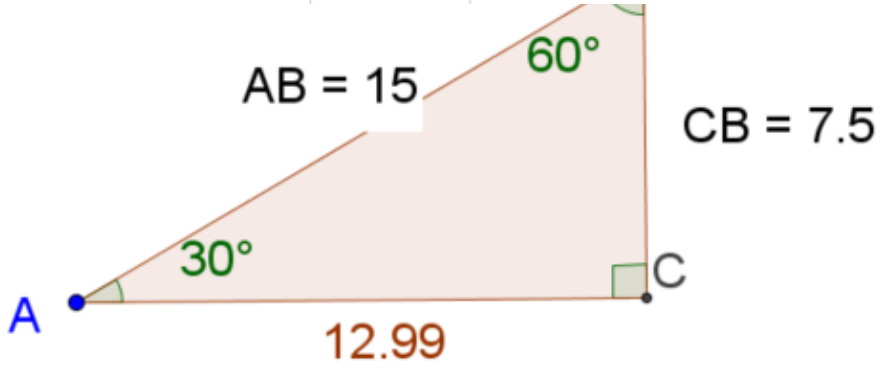
$\angle B \cong \angle E$

$\angle C \cong \angle F$

Similarity results in the ratios of corresponding sides always equaling the same number.




$$\frac{AB}{GE} = \frac{BC}{EF} = \frac{AC}{GF} = \frac{15}{10} = \frac{3}{2}$$




We call this number the "scale factor"

"Ratios" are decimal form (not in fraction form).

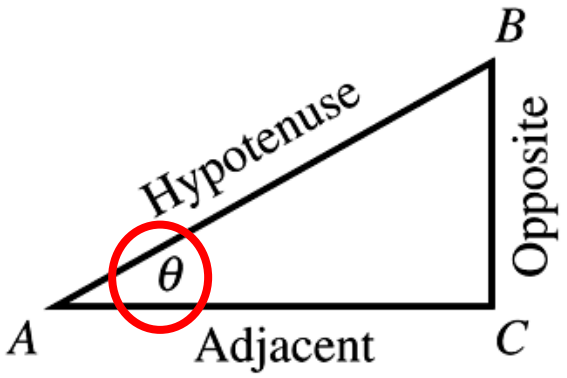
These "Ratios" are unique numbers for each angle; they are ***Properties of the angle.***

| Angle | $\frac{opp}{hyp}$ |
|---|---|
| $10^\circ$ | 0.1736 |
| $20^\circ$ | 0.3420 |
| $30^\circ$ | 0.5 |
| $43.9^\circ$ | 0.6934 |
| $60^\circ$ | 0.8660 |

| Radian | Degree | Sine | Cosine | Tan |
|---|---|---|---|---|
| 0.000 | 0 | 0.000 | 1.000 | 0.0 |
| 0.017 | 1 | 0.017 | 1.000 | 0.0 |
| 0.035 | 2 | 0.035 | 0.999 | 0.0 |
| 0.052 | 3 | 0.052 | 0.999 | 0.0 |
| 0.070 | 4 | 0.070 | 0.998 | 0.0 |
| 0.087 | 5 | 0.087 | 0.996 | 0.0 |
| 0.105 | 6 | 0.105 | 0.995 | 0.1 |
| 0.122 | 7 | 0.122 | 0.993 | 0.1 |
| 0.140 | 8 | 0.139 | 0.990 | 0.1 |
| 0.157 | 9 | 0.156 | 0.988 | 0.1 |
| 0.175 | 10 | 0.174 | 0.985 | 0.1 |
| 0.192 | 11 | 0.191 | 0.982 | 0.1 |
| 0.209 | 12 | 0.208 | 0.978 | 0.2 |

# What are the "code words" for the ratios?

SOH-CAH-TOA




Key point: sine of an angle
(measured in degrees or radians)

$$\sin 30^\circ = \frac{\text{opposite (length)}}{\text{hypotenuse (length)}}$$

$$\sin\theta = \frac{\text{opposite (length)}}{\text{hypotenuse (length)}}$$

$$\cos\theta = \frac{\text{adjacent (length)}}{\text{hypotenuse (length)}}$$

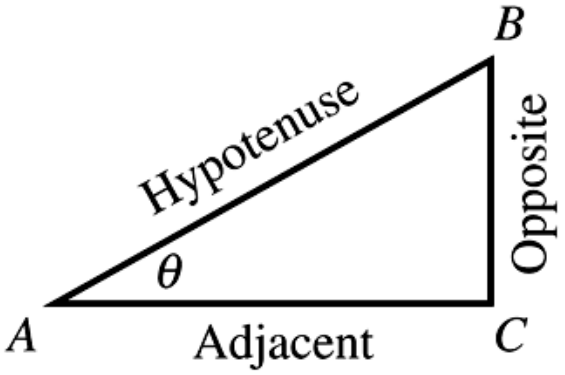
$$\tan\theta = \frac{\text{opposite (length)}}{\text{adjacent (length)}}$$

The ratio is a property of the angle. We must know the angle measure to find the correct ratio.

These only work for right triangles!!!

# Trigonometric Functions

*Shot your cow*:
"Sha – Cho – Cao"




Notice that these ratios are *reciprocals* of the sine, cosine, and tangent ratios.

$\sin\theta \rightarrow \csc\theta$

$\cos\theta \rightarrow \sec\theta$

*"S" goes with "C"*

$\sec A = \frac{h}{a}$ $\quad \frac{\text{hypotenuse}}{\text{adjacent}}$

$\csc A = \frac{h}{o}$ $\quad \frac{\text{hypotenuse}}{\text{opposite}}$

$\cot A = \frac{a}{o}$ $\quad \frac{\text{adjacent}}{\text{opposite}}$

*The sine ratio is the reciprocal of the cosecant ratio.*

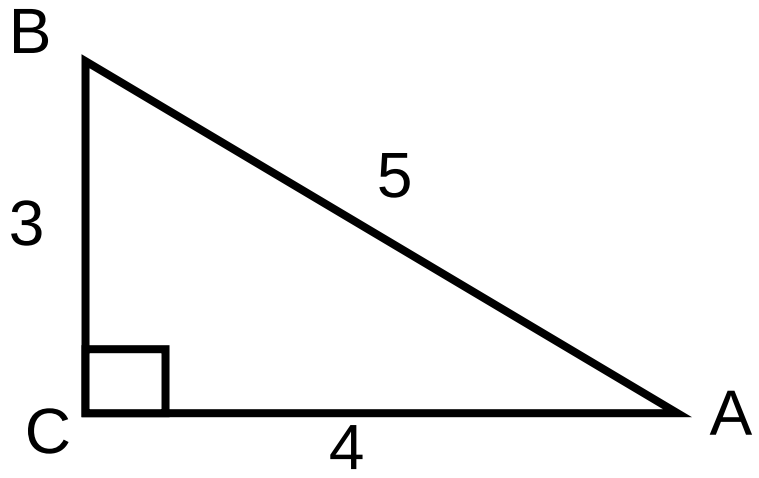
Trig Ratios

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec A = \frac{5}{4}$




$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

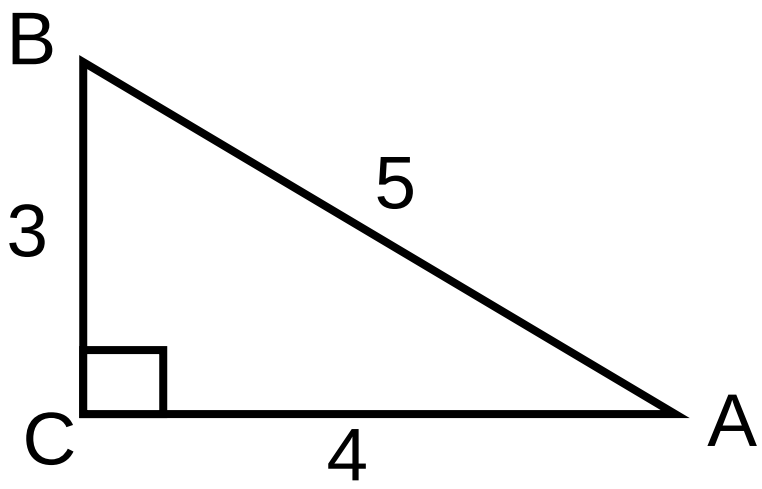
What patterns can you see?

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec A = \frac{5}{4}$




$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

**Cosine A and Secant A are reciprocals.**

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

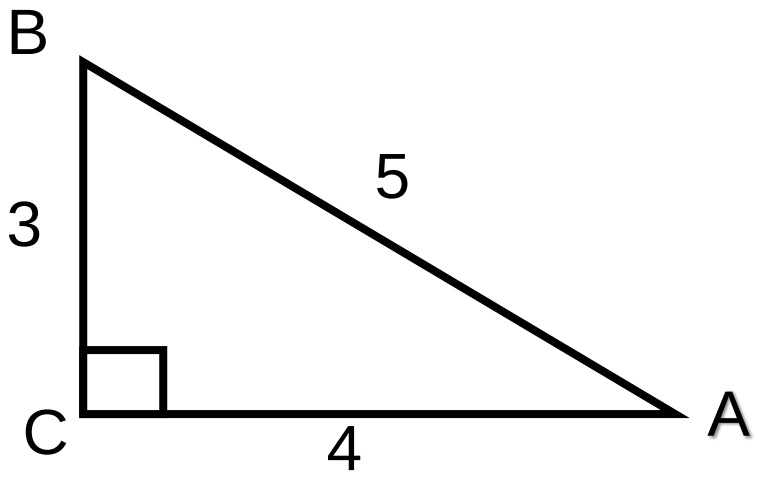
What patterns can you see?

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec A = \frac{5}{4}$




$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

**Sine B and Cosecant B are reciprocals.**

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

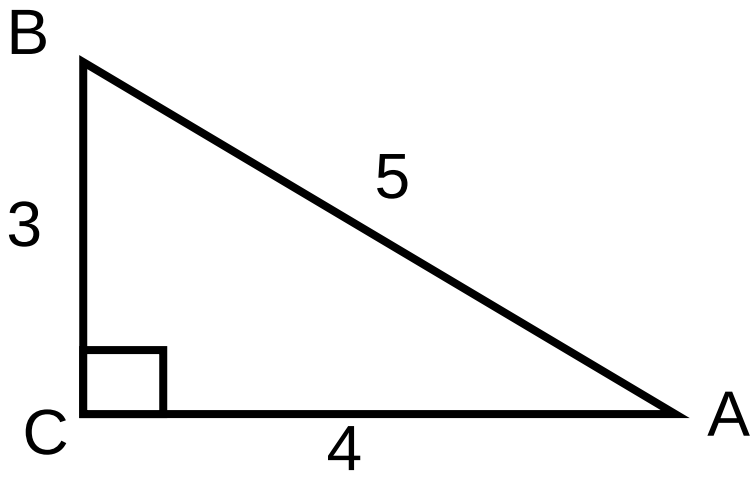
What patterns can you see?

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec B = \frac{5}{4}$




$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

**Cos A = Sin B**

**Cos 20 = Sin 70**

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

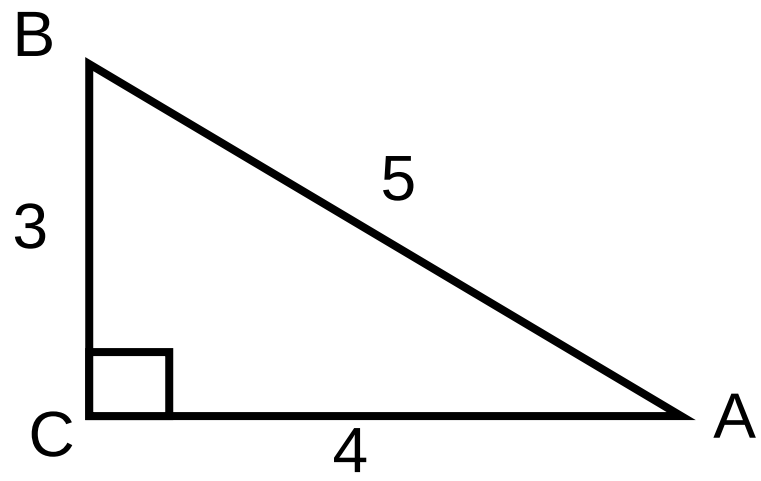
What patterns can you see?

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec A = \frac{5}{4}$




$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

**Tan A= Cot B**

**Tan 30 = Cot 60**

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

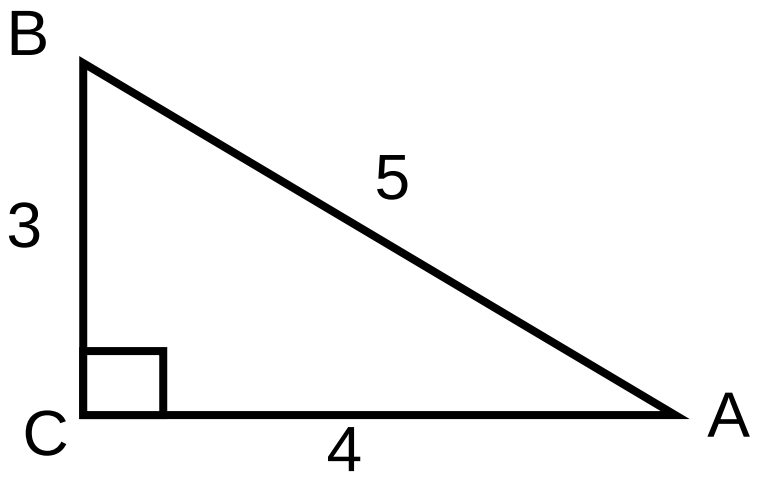
What patterns can you see?

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec A = \frac{5}{4}$




$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

**Tan A and Tan B are reciprocals.**

$\tan B = ?$

$\tan B = \frac{4}{3}$

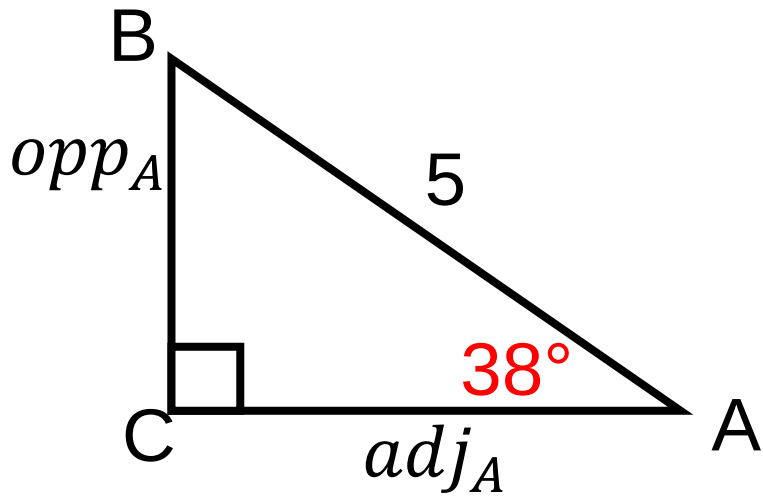
$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

Solve the triangle.




1) $a^2 + b^2 = c^2$

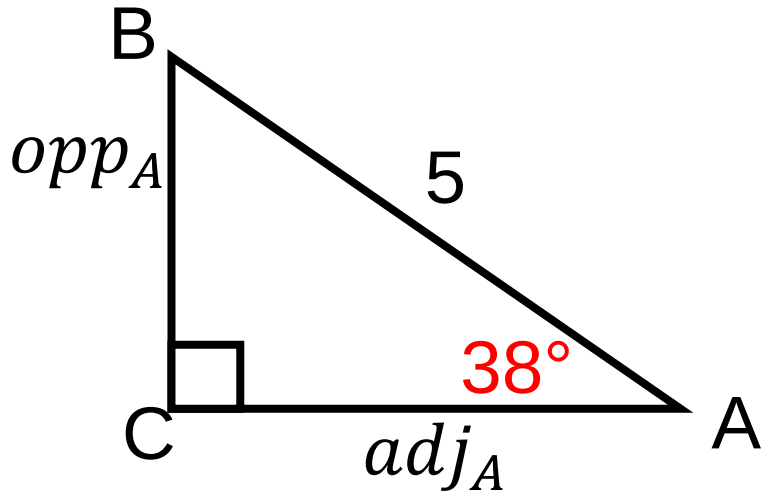
2) $\sin A = \frac{o}{h}$

3) $\cos A = \frac{a}{h}$

4) $\tan A = \frac{o}{a}$

5) $m\angle A + m\angle B + M\angle C = 180^0$ → $m\angle B = 52^0$

Solve the triangle.




1) $a^2 + b^2 = c^2$

2) $\sin A = \frac{o}{h}$ → $\sin 38^\circ = \frac{opp}{5}$

$0.616 = \frac{opp}{5}$

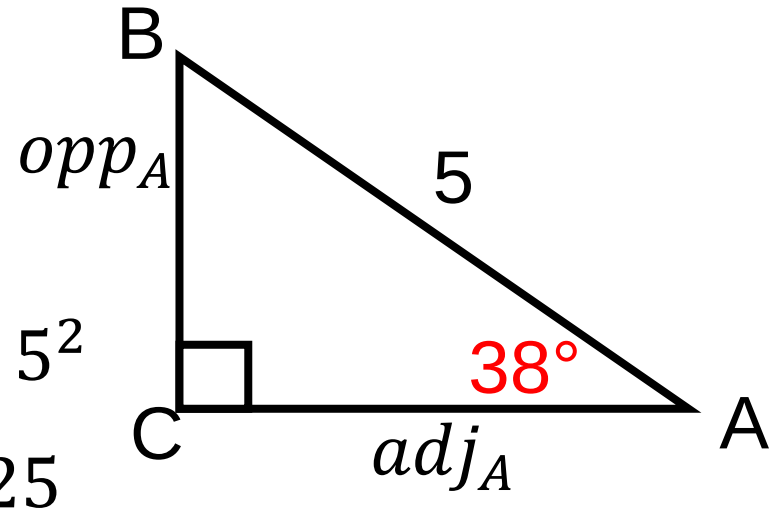
3) $\cos A = \frac{a}{h}$

$5(0.616) = \text{opp} = 3.08$

4) $\tan A = \frac{o}{a}$

5) $m\angle A + m\angle B + M\angle C = 180^0$

Solve the triangle.




1) $a^2 + b^2 = c^2$ →

2) $\sin A = \frac{o}{h}$

3) $\cos A = \frac{a}{h}$

4) $\tan A = \frac{o}{a}$

$(adj)^2 + (3.08)^2 = 5^2$

$(adj)^2 + 9.49 = 25$

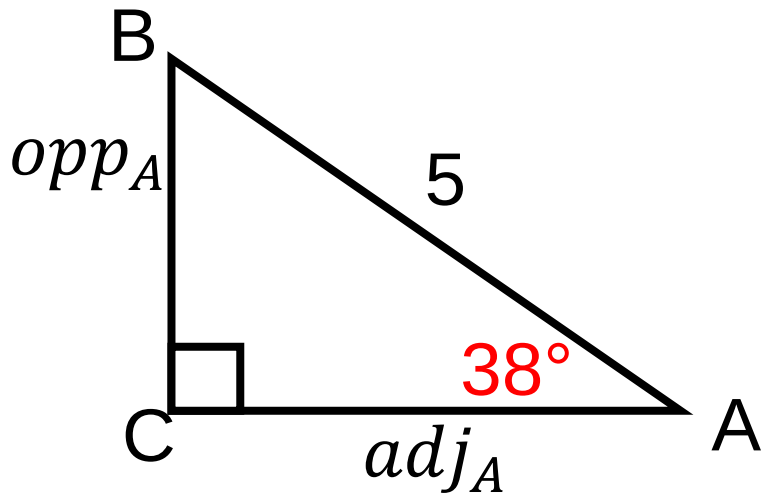
$(adj)^2 = 25 - 9.49$

$(adj)^2 = 15.51$

$adj = \sqrt{15.51} = 3.94$

5) $m\angle A + m\angle B + M\angle C = 180^0$

Solve the triangle.




1) $a^2 + b^2 = c^2$

2) $\sin A = \frac{o}{h}$

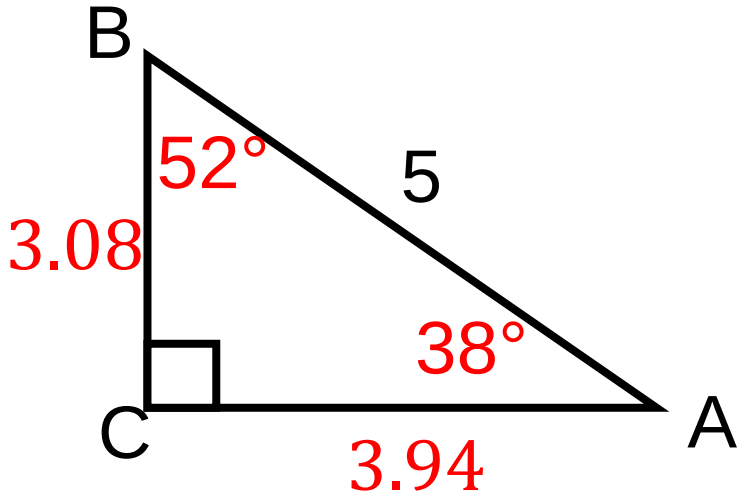
3) $\cos A = \frac{a}{h}$ → $\cos 38^\circ = \frac{adj}{5}$

$0.788 = \frac{adj}{5}$

$5(0.788) = \text{adj}$ $= 3.94$

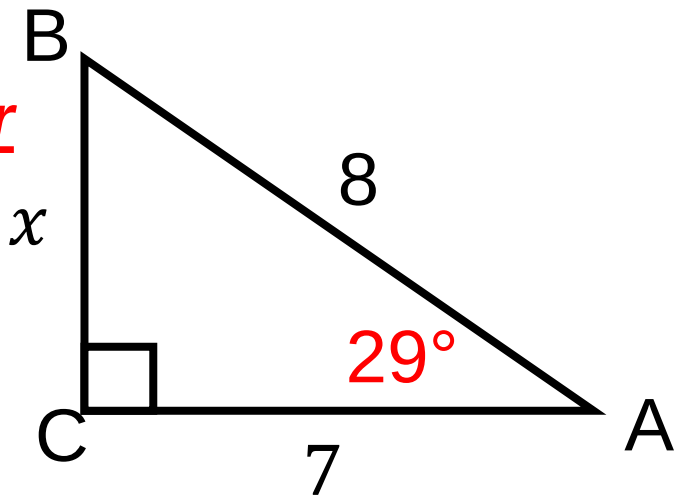
4) $\tan A = \frac{o}{a}$

5) $m\angle A + m\angle B + M\angle C = 180^0$



Usually, the problem only asks you to solve for 'x'

***Change the numbers on your slide for this problem.***




1) $a^2 + b^2 = c^2$

2) $\sin A = \frac{o}{h}$ → $\sin 29^\circ = \frac{x}{8}$

3) $\cos A = \frac{a}{h}$ $\quad 0.485 = \frac{x}{8}$

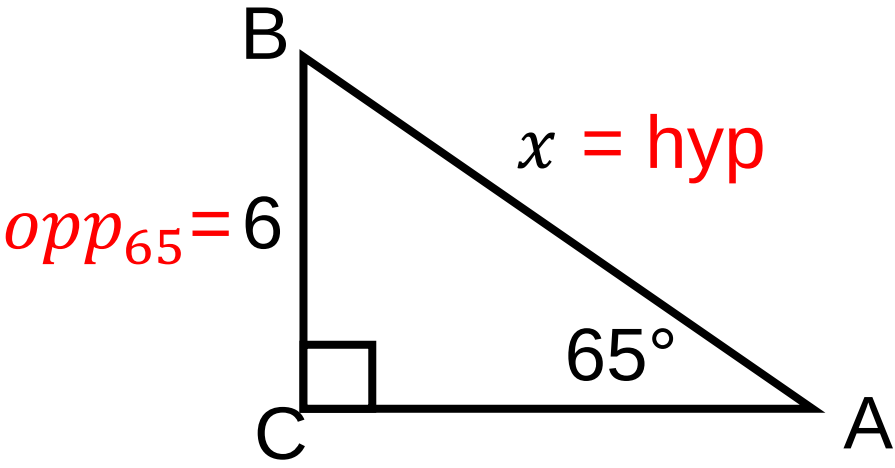
$8(0.485) = \text{opp} \quad = 3.9$

4) $\tan A = \frac{o}{a}$

5) $m\angle A + m\angle B + M\angle C = 180^0$

## The Hardest problem

$\sin A = \frac{o}{h}$ $\sin 65^\circ = \frac{6}{x}$




$$0.906 = \frac{6}{x}$$

'x' is in the denominator!

$$x(0.906) = 6$$

*"undo"* division by 'x'

$$x = \frac{6}{0.906} = 6.6$$

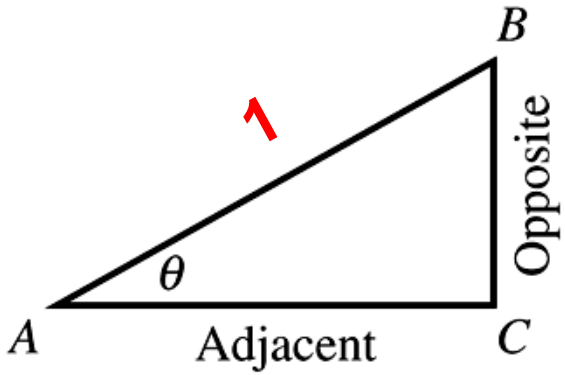
hypotenuse = 1

Why is it “nice” to have a hypotenuse whose length is ‘1’?

Sin Θ= opposite side

Cos Θ= adjacent side

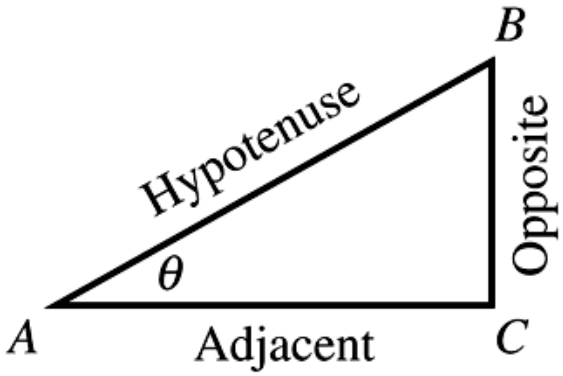
Tan Θ= opp/adj




The length of the hypotenuse is no longer in the ratio!

# Trig Ratios of Acute Angles

What shape is used to define trig ratios? right triangle.



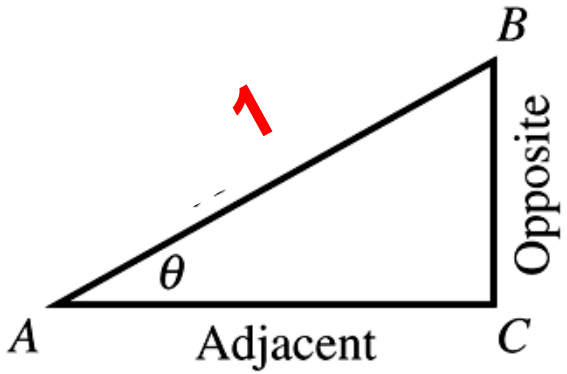

Using these definitions we can't have angles > 90!!!

# Trig ratios for obtuse angles: we need acute angles!!

Sin Θ= opposite side

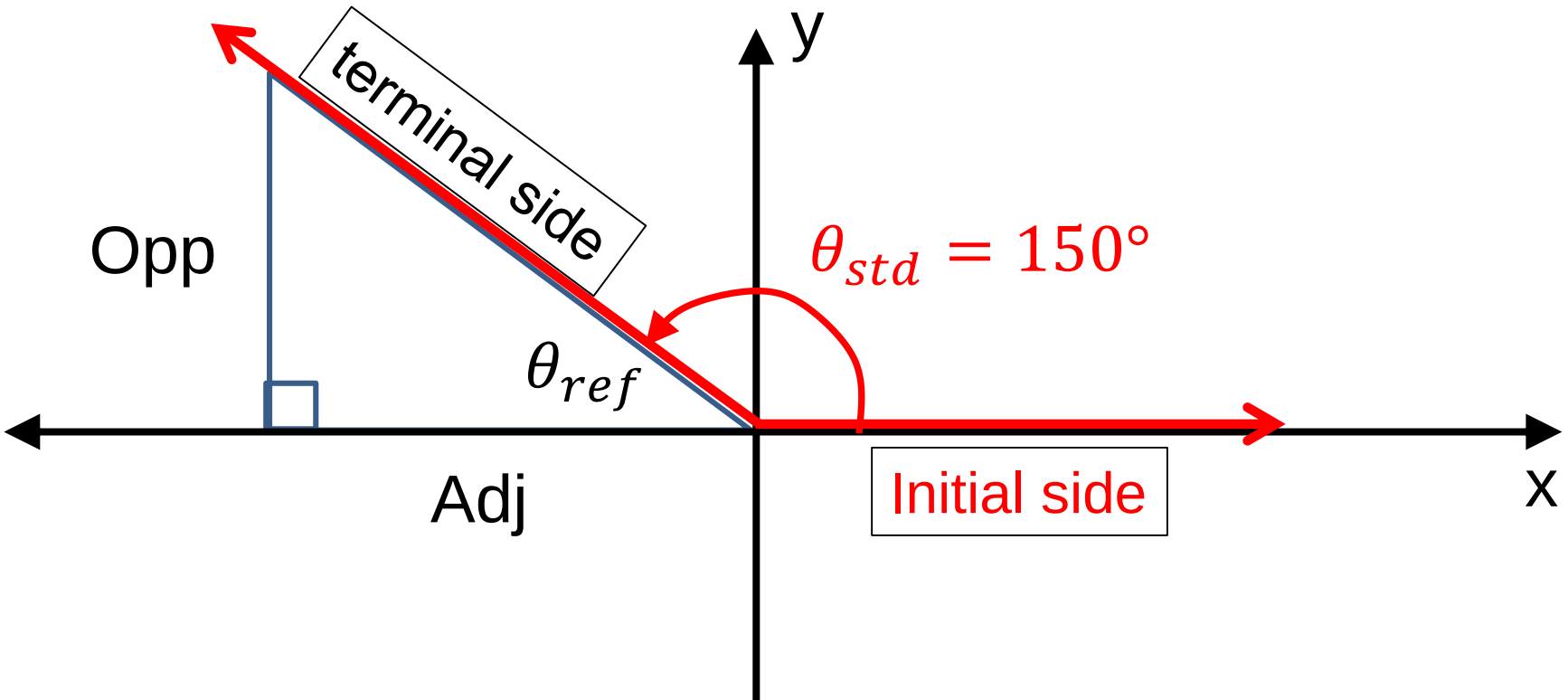
Cos Θ= adjacent side

Tan Θ= opp/adj



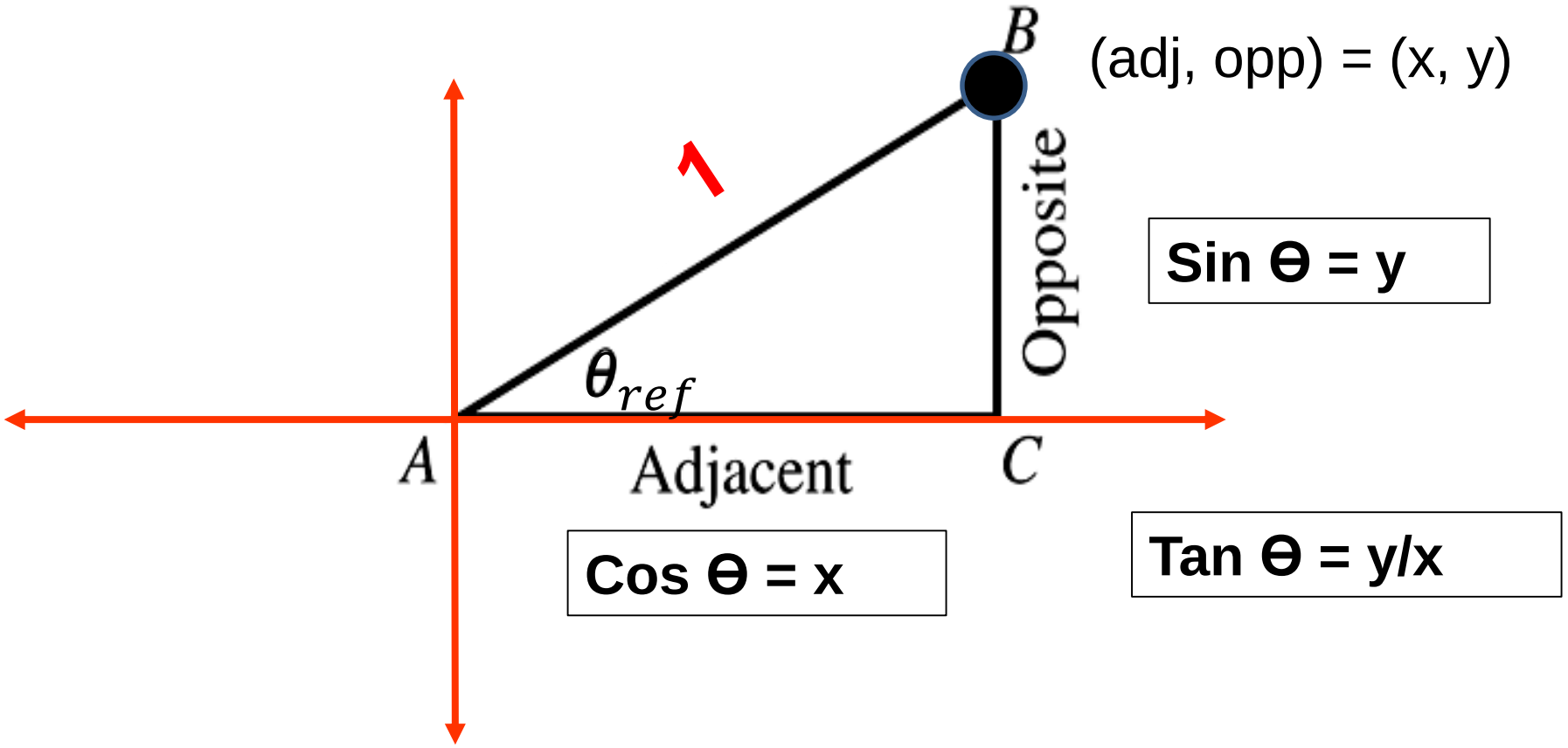
Trig ratios *only work for right triangles!* *If angle is greater than 90…*

1. Build a standard position angle on the x-y plane, with vertex at (0, 0)
2. Initial side of the angle: always points along the positive x-axis.
3. Terminal side of the angle: points outward from (0, 0).
4. We build a right triangle, hypotenuse is the terminal side.
5. We use the reference angle for our trig ratios.



Reference angle: the acute angle between the terminal side of a standard position angle and the x-axis.

With a *right triangle* with a hypotenuse =1 on the x-y plane, by using the *reference angle* for our trig ratios, the x-y pair at the end of the hypotenuse gives us *adjacent side length* and *opposite side length* of the right triangle.



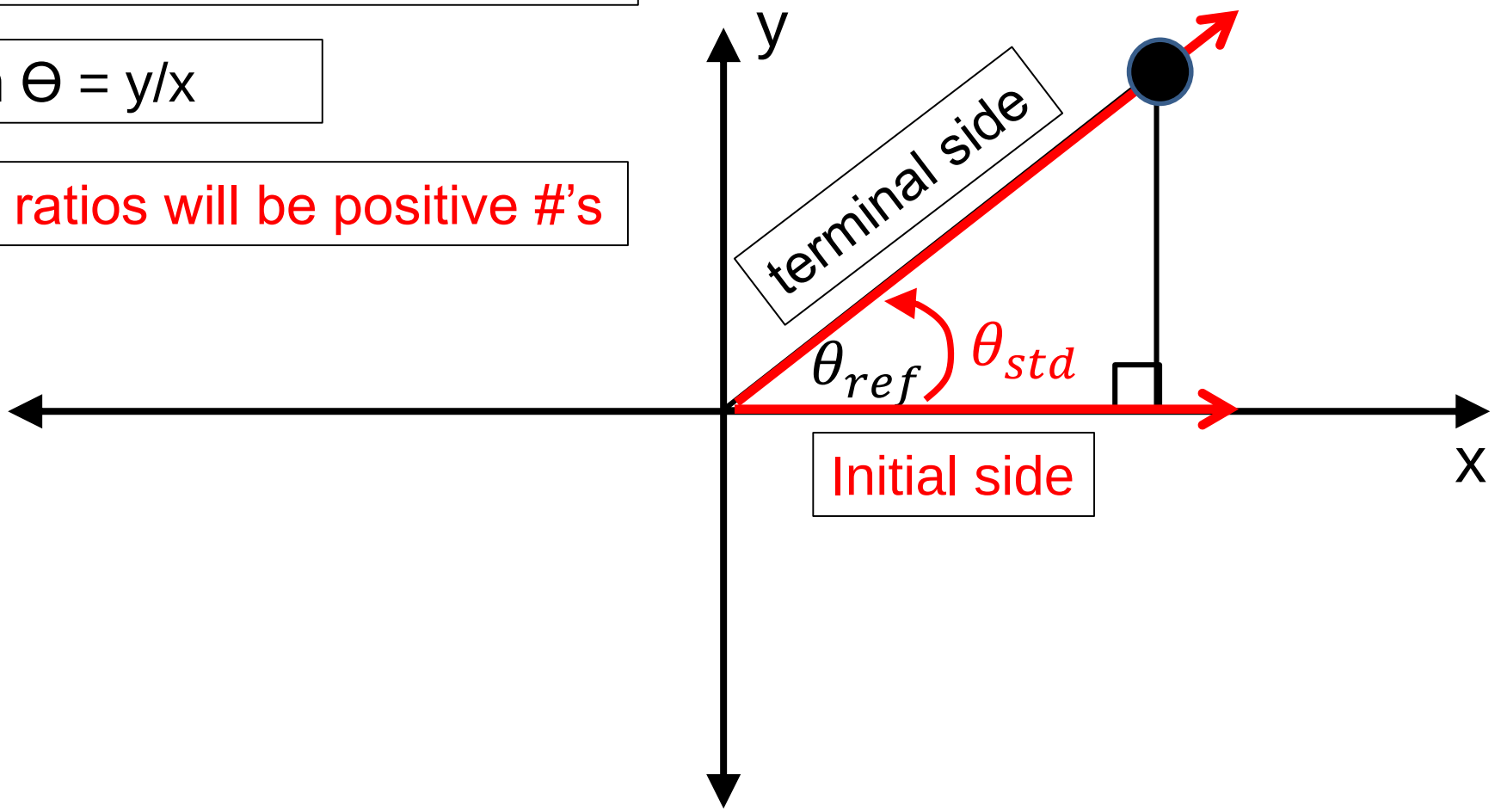
For Quadrant I $\theta_{ref} = \theta_{std}$

Sin Θ = y-value of the point

Cos Θ = x-value of the point

Tan Θ = y/x

Trig ratios will be positive #'s



For Quadrant II $\theta_{ref} = 180 - \theta_{std}$

Sine ratio is a positive number

Cosine ratio is a negative number

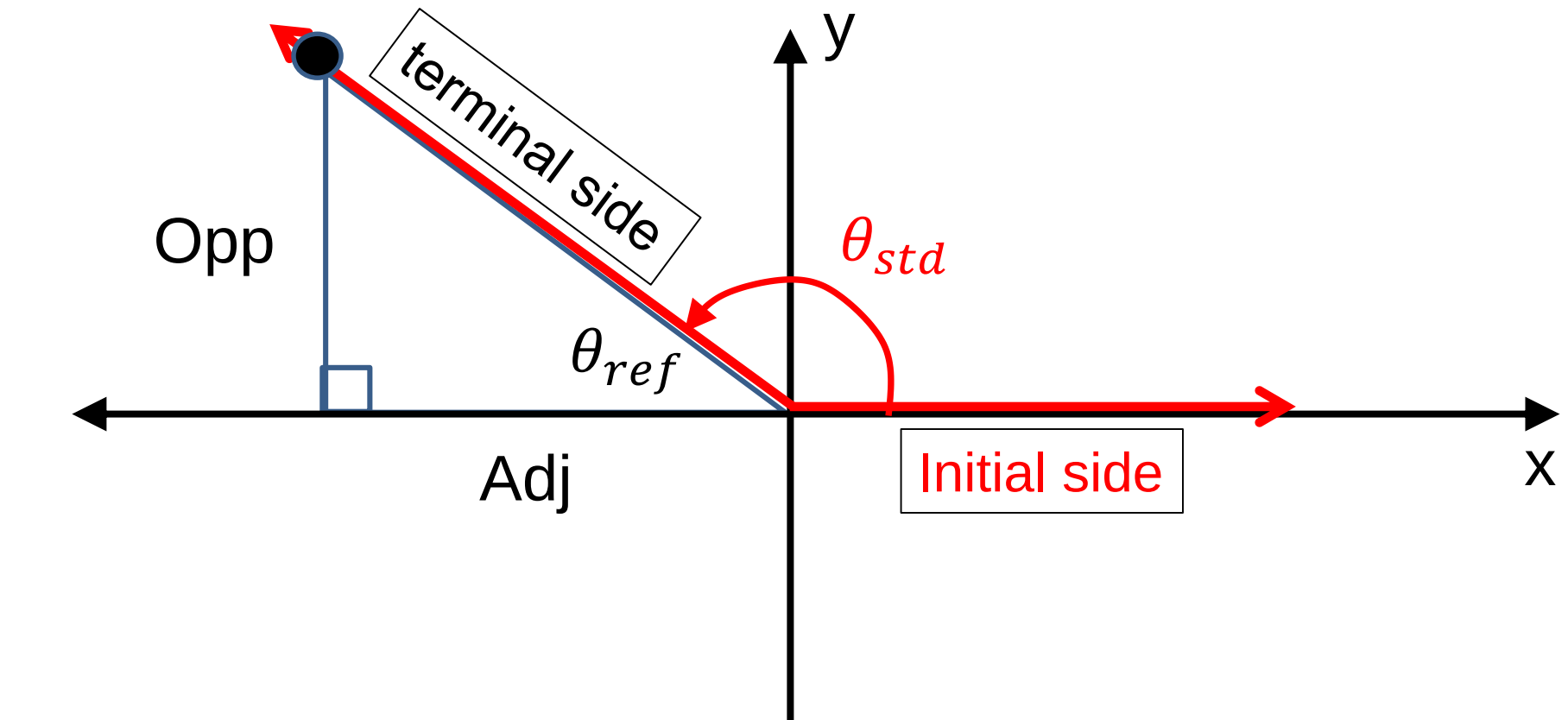
Tangent ratio is a negative number

**Sin Θ = y**

**Cos Θ = x**

**Tan Θ = y/x**

(adj, opp) = (x, y)



For Quadrant III

$\theta_{ref} = \theta_{std} - 180$

**Sin Θ = y**

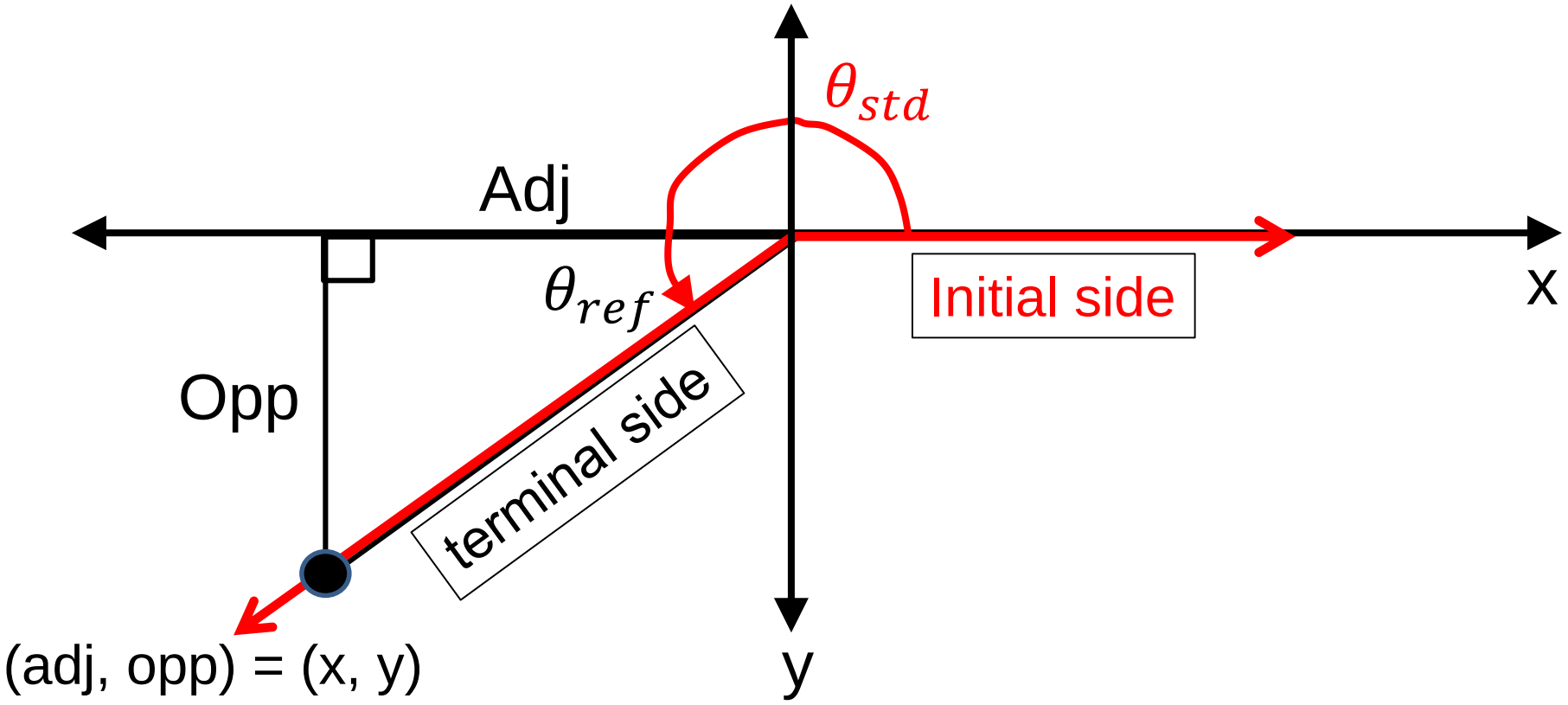
**Cos Θ = x**

**Tan Θ = y/x**

Sine ratio is a negative number

Cosine ratio is a negative number

Tangent ratio is a positive number



For Quadrant IV $\theta_{ref} = 360 - \theta_{std}$

**Sin Θ = y**

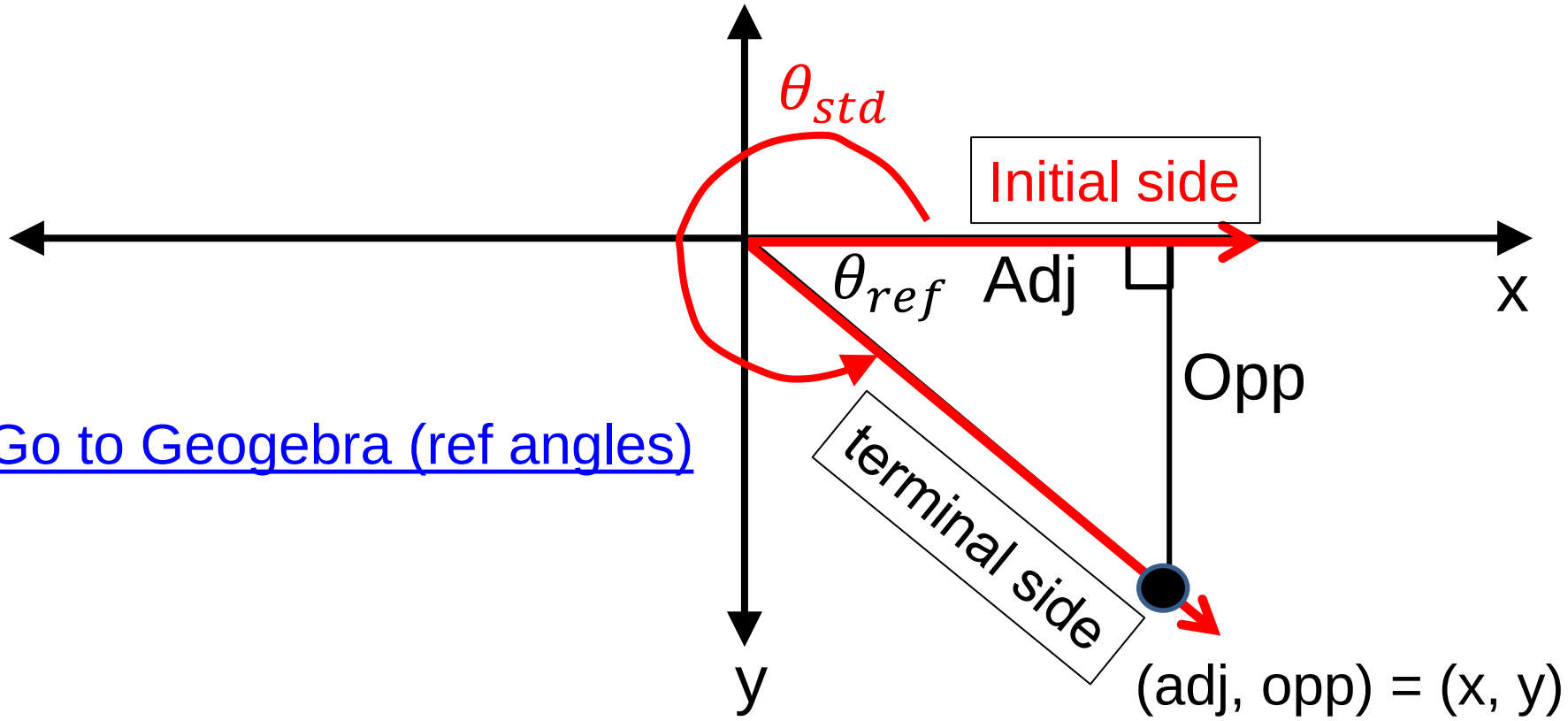
**Cos Θ = x**

**Tan Θ = y/x**

Sine ratio is a negative number

Cosine ratio is a positive number

Tangent ratio is a negative number




Go to Geogebra (ref angles)