Math-3 Lesson 5-5

Properties of Logarithmic Functions (Product of Logs Log of a Power)

$$f(x) = 5^{2x+4} \quad \text{Find} \quad f^{-1}(x)$$

$$y = 5^{2x+4} \quad \text{Replace } f(x) \text{ with 'y'}$$

$$x = 5^{2y+4} \quad \text{exchange 'x' and 'y'}$$

$$\log_5 x = 2y + 4 \quad \text{Log is the exponent (remember how to convert between the two?)}$$

$$-4 + \log_5 x = 2y \quad \text{Solve for 'y'}$$

$$\frac{-4 + \log_5 x}{2} = y \quad y = -2 + \frac{1}{2} \log_5 x$$

$$2^3 * 2^2 = 2^5$$

The product of powers \rightarrow add the exponents

$$2^3 * 2^2 = 2^5$$

Logarithm: another way of writing the <u>exponent</u> Convert each exponent above into a log:

$$\log_2 8 + \log_2 4 = \log_2 32$$

 $3 + 2 = 5$

This is the logarithm equivalent of the multiply powers property of exponents.

Log of a Product Property $\log_2 15 = \log_2 (3*5)$ $\log_2 15 = \log_2 3 + \log_2 5$ $\log_b (RS) = \log_b R + \log_b S$ $\log_b of a product = sum of the logs of the factors.$

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

 $\log_3 xy \qquad \log_3 x + \log_3 y$

$$\log_{3} 45 \qquad \log_{3} 3 + \log_{3} 3 + \log_{3} 5$$
$$45 = 3*3*5 \qquad 2\log_{3} 3 + \log_{3} 5$$

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

$$\log(3xy^2) = \log 3 + \log x + \log y^2$$

$$= \log 3 + \log x + \log y + \log y$$

$$= \log 3 + \log x + 2\log y$$

$$\left|\log_4 6\right| = \log_4 3 + \log_4 2$$

$$|\ln 2xyw| = \ln 2 + \ln x + \ln y + \ln w$$

<u>Condense the Logarithm</u>: apply properties of logarithms to rewrite the log expression as a single log.

$$\log_2 7 + \log_2 5 = \log_2 (7*5) = \log_2 35$$
$$\log 5 + \log x = \log 5x$$

 $\log_7 5 + \log_5 7$ "unlike logs" \rightarrow can't condense

"Condense the Log"

$$\log_{5} 2 + \log_{5} 7 = \log_{5} 14$$

 $\log 9 + \log 4 = \log 36$

 $\log_5 6 + \log_8 4$ "unlike logs" \rightarrow can't condense

"Expand the Log"

$$log_2 9 = log_2(3*3)$$

= $log_2 3 + log_2 3$
= $2log_2 3$

Notice something interesting

$$\log_2 9 = \log_2 (3)^2 = 2 \log_2 3$$

"Expand the Product"

$$log_{3}16 = log_{3}(4*4)$$

= $log_{3}4 + log_{3}4$
= $2log_{3}4$

Notice something interesting

$$\log_3 16 = \log_3 (4)^2 = 2 \log_3 4$$

"Expand the Product"

 $\log_5 10^2$ <u>Log of a product is the sum of the logs</u> of the factors.

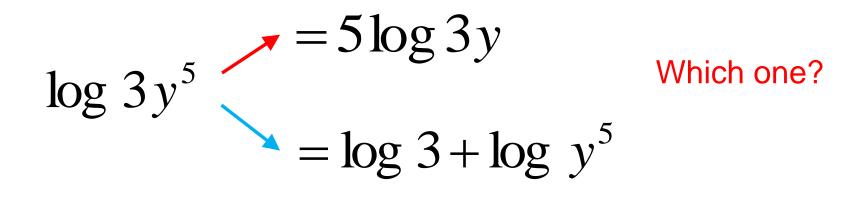
$$= \log_{5} 10 + \log_{5} 10$$
 Combine "like terms"
$$= 2 \log_{5} 10$$

$$\log_{5} 10^{2} = 2 \log_{5} 10$$

<u>New property</u>: "log of a power"

Use Log of a Power to expand the log $\log x^3 = 3\log x$ $\ln 8 = \ln 2^3 = 3 \ln 2$ $\log \sqrt{x} = \log x^{\frac{1}{2}} = \frac{1}{2}\log x$ $\log_3 x^2 y^3 \sqrt[4]{z} = \log_3 x^2 + \log_3 y^3 + \log_3 \sqrt[4]{z}$ $= 2\log_3 x + 3\log_3 y + \frac{1}{4}\log_3 z$

Gotcha'



$$5\log 3y = \log (3y)^5 = \log 3^5 y^5$$

$$c \log_b R^c \rightarrow c \log_b R$$

A potential error is this:

$$\log_2 6x^3 = 3\log_2 6x$$

What is the error ? '3' is an exponent of the base 'x' not '6x'

Correct the error.

$$\log_2 6x^3 = \log_2 6 + \log_2 x^3$$
$$= \log_2 3 + \log_2 2 + 3\log_2 x$$

More Practice

- 1. <u>Convert to a logarithm:</u> $7 = 2(3)^{x}$ $\log_{3}\left(\frac{7}{2}\right) = x$
- 2. Convert to an exponential;

$$3\log_5(x-6) = 6$$

$$(5)^2 = x - 6$$

- 3. What is the Domain and range? $f(x) = 3\log(x+2) - 5$ Domain: (-2, ∞)
 - Range: (-∞, ∞)

More Practice

4. What do we mean when we say: "A log is an exponent"? $log_5 25$

This example can be restated as "what exponent of 5 equals 25. $\log_5 25 = x$ $5^x = 25$

Notice something interesting...

Using substitution, replace 'x' in the second equation with the equivalent expression from the 1st equation.

$$5^{\log_5 25} = 25$$

This is a composition of a function with its inverse \rightarrow they undo each other.

5. Simplify:
$$(3)^{\log_3 x} = x$$

6a. What is the
$$f(x) = 2\log(2x-4)-6$$

logarand? $logarand = 2x - 4$

6b. What is the x =vertical asymptote?

$$x = 2$$

Eq. #1
$$3^5 = x$$

Convert to log form:

Eq. #2
$$\log_3 x = 5$$

Substitute Eq. #1 into Eq. #2

$$\log_3 3^5 = 5$$

When you compose a function with its inverse, they "undo" each other.

Properties of Logarithms: Log of a Product.

$$\log_{b}(RS) = \log_{b}R + \log_{b}S$$

$$\log_2 15 = \log_2 3 + \log_2 5$$

<u>log of a product</u> = $\underline{sum of the logs of the factors}$.

Logarithm: another way of writing the <u>exponent</u> Analogous to:

- \rightarrow Product of powers
- \rightarrow Add the exponents

$$x^5 x^2 = x^{5+2}$$

Properties of Logarithms Log of a Power

$$c \log_{b} R^{\circ} \rightarrow c \log_{b} R$$

$$\log_{2} 3^{4} \qquad \log 32 = \log 2^{5}$$

$$= 4 \log_{2} 3 \qquad = 5 \log 2$$

$$(x^5)^2 = x^{5*2} = x^{10}$$

Logarithm: another way of writing the exponent

$$\frac{x^5}{x^2} = x^5 x^{-2} = x^{5-2}$$

Quotient of powers: subtract denominator exponent from the numerator exponent. Logarithm: another way of writing the <u>exponent</u>

 $\log_{3}\left(\frac{5}{2}\right) = \log_{3}\left(5*2^{-1}\right) = \log_{3}(5) + \log_{3}(2)^{-1}$ Negative Log of a Product Exponent Property Property $= \log_3 5 + (-1) \log_3 2$ Log of a Power Property $= \log_3 5 - \log_3 2$ Definition of Subtraction: (adding a negative is subtraction)

Log of a Quotient Property

$$\log_b\left(\frac{R}{S}\right) = \log_b R - \log_b S$$

$$\log_3\left(\frac{5}{2}\right)$$

"expand the quotient"
$$\log_3 5 - \log_3 2$$

 $\ln 8 - \ln 3$ "condense the quotient" $\ln \frac{8}{3}$

"Negative Log" \rightarrow denominator of the logarand

Expand the Quotient

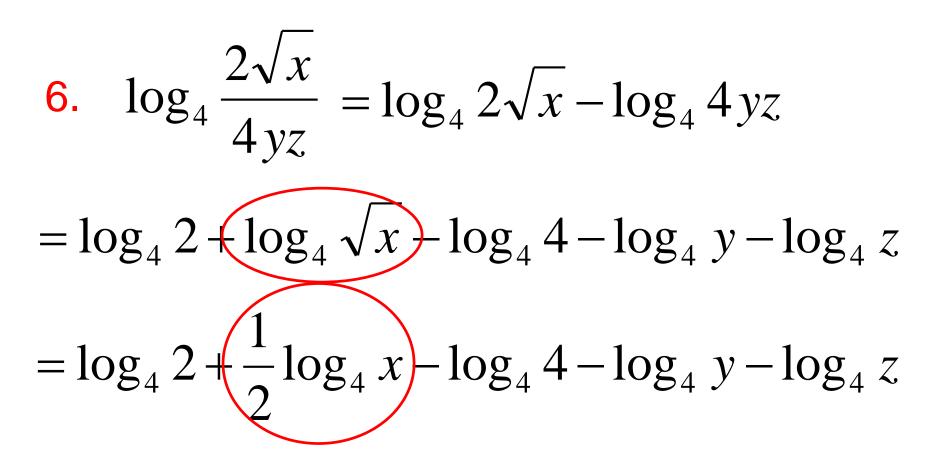
1. $\log \frac{4}{5} = \log 4 - \log 5 = \log 2 + \log 2 - \log 5$ $= 2 \log 2 - \log 5$ 2. $\ln \frac{3}{7} = \ln 3 - \ln 7$ Condense the quotient $\log_4 5 - \log_4 2 = \log_4 \frac{5}{2}$ 3. 4. $\log_5 8 - \log_5 16 = \log_5 \frac{8}{16} = \log_5 \frac{1}{2}$

Expand the Logarithm

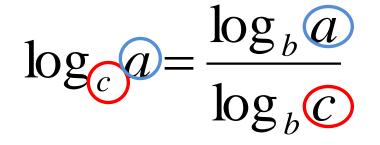
5.
$$\log\left(\frac{2x}{3y^5}\right) = \log 2x - \log 3y^5$$
 The
denominator
is a product!
 $= \log 2x - (\log 3 + 5\log y)$
Distributive property!
 $= \log 2x - \log 3 - 5\log y$
 $= \log 2 + \log x - \log 3 - 5\log y$

Logs of factors in the numerator will be positive. Logs of factors in the denominator will be negative.

Expand the quotient

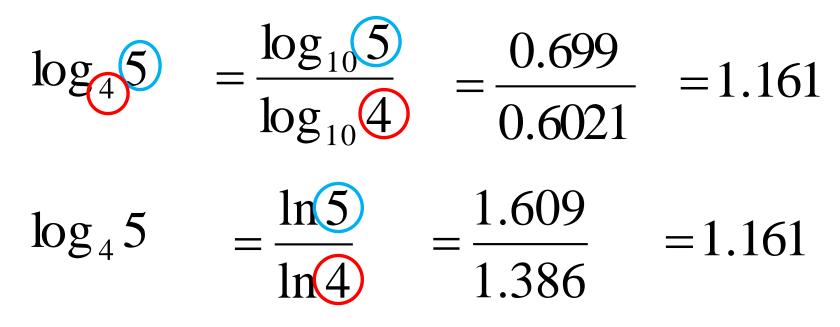


Change-of-Base Formula for Logarithms



Change to log <u>base 10</u> or <u>base 'e'</u> (your calculator can do these).

Convert to base 10.



Simplify

 $\log_2 2$ $\log_2 2 = x$ $2^{x} = 2$ x = 1 $5\log_3 27$ $5\log_{3}3^{3}$ $(3*5)\log_3 3$

Using Change of base:

$$\log_2 2 = \frac{\log 2}{\log 2} = 1$$

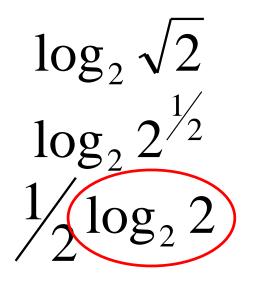
Possible Error (which one is true?)

$$\log \frac{3}{5} = \frac{\log 3}{\log 5}$$
 $\log \frac{3}{5} = \log 3 - \log 5$

15

Simplify:

 $\log_4 16$ $\log_4 4^2$ 2(1) = 2



(1)

Or: "4 raised to what power equals 16?"

<u>Or:</u> "2 raised to what exponent equals the square root of 2?"