Math-3
Lesson 5-5

Properties of Logarithmic Functions
(Product of Logs
Log of a Power)

$f(x)=5^{2x+4}$ Find $f^{-1}(x)$

$y=5^{2x+4}$ Replace f(x) with ‘y’

$x=5^{2y+4}$ exchange ‘x’ and ‘y’

$\log_5 x = 2y+4$ <u>Log</u> is the <u>exponent</u> (remember how to convert between the two?)

$-4+\log_5 x = 2y$ Solve for ‘y’

$$\frac{-4+\log_5 x}{2}=y$$

$$\boxed{y=-2+\tfrac{1}{2}\log_5 x}$$

$$2^3 * 2^2 = 2^5$$

The <u>product of powers</u> → <u>add the exponents</u>

$$2^3 * 2^2 = 2^5$$

<u>Logarithm</u>: another way of writing the <u>exponent</u>

Convert each exponent above into a log:

$$\log_2 8 + \log_2 4 = \log_2 32$$

$$3 + 2 = 5$$

This is the logarithm equivalent of the multiply powers property of exponents.

Log of a Product Property

$$\log_2 15 = \log_2 (3*5)$$

$$\log_2 15 = \log_2 3 + \log_2 5$$

$$\log_b (RS) = \log_b R + \log_b S$$

log of a product = sum of the logs of the factors.

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

$$\log_3 xy \qquad \log_3 x + \log_3 y$$

$$\log_3 45 \qquad \log_3 3 + \log_3 3 + \log_3 5$$

$$45 = 3*3*5 \qquad 2\log_3 3 + \log_3 5$$

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

$$\boxed{\log(3xy^2)} = \log 3 + \log x + \log y^2$$

$$= \log 3 + \log x + \log y + \log y$$

$$= \log 3 + \log x + 2\log y$$

$$\boxed{\log_4 6} = \log_4 3 + \log_4 2$$

$$\boxed{\ln 2xyw} = \ln 2 + \ln x + \ln y + \ln w$$

Condense the Logarithm: apply properties of logarithms to rewrite the log expression as a single log.

$$\boxed{\log_2 7 + \log_2 5} \quad = \log_2 (7 * 5) \quad = \log_2 35$$

$$\boxed{\log 5 + \log x} \quad = \log 5x$$

$$\log_7 5 + \log_5 7$$ "unlike logs" → can't condense

"Condense the Log"

$$\log_5 2 + \log_5 7 \quad = \log_5 14$$

$$\log 9 + \log 4 \quad = \log 36$$

$$\log_5 6 + \log_8 4$$ "unlike logs" → can't condense

“Expand the Log”

$$\log_2 9 = \log_2 (3 * 3)$$

$$= \log_2 3 + \log_2 3$$

$$= 2 \log_2 3$$

Notice something <u>interesting</u>

$$\log_2 9 = \log_2 (3)^2 = 2 \log_2 3$$

"Expand the Product"

$$\log_3 16 \qquad = \log_3(4*4)$$

$$= \log_3 4 + \log_3 4$$

$$= 2\log_3 4$$

Notice something <u>interesting</u>

$$\log_3 16 \qquad = \log_3(4)^2 = 2\log_3 4$$

“Expand the Product”

$$\log_5 10^2$$

Log of a product is the sum of the logs of the factors.

$$= \log_5 10 + \log_5 10$$

Combine “like terms”

$$= 2\log_5 10$$

$$\log_5 10^{2} = 2\log_5 10$$

New property: “log of a power”

Use Log of a Power to expand the log

$$\log x^3 = 3\log x$$

$$\ln 8 = \ln 2^3 = 3\ln 2$$

$$\log\sqrt{x} = \log x^{1/2} = \frac{1}{2}\log x$$

$$\log_3 x^2 y^3 \sqrt[4]{z} = \log_3 x^2 + \log_3 y^3 + \log_3 \sqrt[4]{z}$$

$$= 2\log_3 x + 3\log_3 y + \frac{1}{4}\log_3 z$$

Gotcha'

$$\log 3y^5 = 5\log 3y$$

$$\log 3y^5 = \log 3 + \log y^5$$

Which one?

$$5\log 3y = \log (3y)^5 = \log 3^5 y^5$$

Log of a Power

$$\log_b R^c \rightarrow c\log_b R$$

A potential error is this:

$$\log_2 6x^3 = \cancel{3\log_2 6x}$$

What is the error ? ‘3’ is an exponent of the base ‘x’ not ‘6x’

Correct the error.

$$\log_2 6x^3 = \log_2 6 + \log_2 x^3$$

$$= \log_2 3 + \log_2 2 + 3\log_2 x$$

More Practice

1. Convert to a logarithm: $7 = 2(3)^x$

$$\log_3\left(\frac{7}{2}\right) = x$$

2. Convert to an exponential; $3\log_5(x-6) = 6$

$$(5)^2 = x - 6$$

3. What is the Domain and range? $f(x) = 3\log(x+2) - 5$

Domain: (-2, ∞)
Range: (-∞, ∞)

More Practice

4. What do we mean when we say: "A log is an exponent"? $\log_5 25$

This example can be restated as "what exponent of 5 equals 25.

$$\boxed{\log_5 25 = x}$$

$$\boxed{5^x = 25}$$

Notice something interesting…

Using substitution, replace 'x' in the second equation with the equivalent expression from the 1st equation.

$$\boxed{5^{\log_5 25} = 25}$$

This is a composition of a function with its inverse→ they undo each other.

More Practice

5. Simplify: $(3)^{\log_3 x} = x$

6a. What is the logarand? $f(x) = 2\log(2x-4) - 6$

$$logarand = 2x - 4$$

6b. What is the vertical asymptote? $x = 2$

Eq. #1 $\boxed{3^5 = x}$

Convert to log form:

Eq. #2 $\boxed{\log_3 x = 5}$

Substitute Eq. #1 into Eq. #2

$$\boxed{\log_3 3^5 = 5}$$

When you compose a function with its inverse, they "undo" each other.

Properties of Logarithms: Log of a Product.

$$\log_b(RS) = \log_b R + \log_b S$$

$$\log_2 15 = \log_2 3 + \log_2 5$$

log of a product = sum of the logs of the factors.

Logarithm: another way of writing the exponent
Analogous to:
→ Product of powers
→ Add the exponents

$$x^5 x^2 = x^{5+2}$$

Properties of Logarithms

Log of a Power

$$\log_b R^c \rightarrow c \log_b R$$

c

$$\log_2 3^4$$

$$= 4 \log_2 3$$

$$\log 32 = \log 2^5$$

$$= 5 \log 2$$

$$\left(x^5\right)^2 = x^{5*2} = x^{10}$$

Logarithm: another way of writing the exponent

$$\frac{x^5}{x^2} = x^5 x^{-2} = x^{5-2}$$

Quotient of powers: subtract denominator exponent from the numerator exponent.

Logarithm: another way of writing the exponent

$$\log_3\left(\frac{5}{2}\right) = \log_3\left(5 * 2^{-1}\right) = \log_3(5) + \log_3(2)^{-1}$$

Negative Exponent Property

Log of a Product Property

$$= \log_3 5 + (-1)\log_3 2$$

Log of a Power Property

$$= \log_3 5 - \log_3 2$$

Definition of Subtraction: (adding a negative is subtraction)

Log of a Quotient Property

$$\log_b\left(\frac{R}{S}\right)=\log_b R-\log_b S$$

$\log_3\left(\frac{5}{2}\right)$ "expand the quotient" $\log_3 5-\log_3 2$

$\ln 8-\ln 3$ "condense the quotient" $\ln\frac{8}{3}$

"Negative Log"→ denominator of the logarand

Expand the Quotient

1. $\log\frac{4}{5} = \log 4 - \log 5 = \log 2 + \log 2 - \log 5$

$= 2\log 2 - \log 5$

2. $\ln\frac{3}{7} = \ln 3 - \ln 7$

Condense the quotient

3. $\log_4 5 - \log_4 2 = \log_4 \frac{5}{2}$

4. $\log_5 8 - \log_5 16 = \log_5 \frac{8}{16} = \log_5 \frac{1}{2}$

Expand the Logarithm

5. $$\log\left(\frac{2x}{3y^5}\right) = \log 2x - \log 3y^5$$

The denominator is a product!

$$= \log 2x - (\log 3 + 5\log y)$$

Distributive property!

$$= \log 2x - \log 3 - 5\log y$$

$$= \log 2 + \log x - \log 3 - 5\log y$$

Logs of factors in the numerator will be positive.

Logs of factors in the denominator will be negative.

Expand the quotient

6. $$\log_4 \frac{2\sqrt{x}}{4yz} = \log_4 2\sqrt{x} - \log_4 4yz$$

$$= \log_4 2 + \log_4 \sqrt{x} - \log_4 4 - \log_4 y - \log_4 z$$

$$= \log_4 2 + \frac{1}{2}\log_4 x - \log_4 4 - \log_4 y - \log_4 z$$

Change-of-Base Formula for Logarithms

$$\log_c a = \frac{\log_b a}{\log_b c}$$

Change to log base 10 or base 'e' (your calculator can do these).

Convert to base 10.

$$\log_4 5 = \frac{\log_{10} 5}{\log_{10} 4} = \frac{0.699}{0.6021} = 1.161$$

$$\log_4 5 = \frac{\ln 5}{\ln 4} = \frac{1.609}{1.386} = 1.161$$

Simplify

$$\log_2 2$$

$$\log_2 2 = x$$

$$2^x = 2$$

$$x = 1$$

$$5\log_3 27$$

$$5\log_3 3^3$$

$$(3*5)\log_3 3$$

$$15$$

Using Change of base:

$$\log_2 2 = \frac{\log 2}{\log 2} = 1$$

Possible Error (which one is true?)

$$log\frac{3}{5} = \frac{\log 3}{\log 5} \qquad log\frac{3}{5} = log3 - \log 5$$

Simplify:

$$\log_4 16$$

$$\log_4 4^2$$

$$2\log_4 4$$

$$2(1) = 2$$

Or: "4 raised to what power equals 16?"

$$\log_2 \sqrt{2}$$

$$\log_2 2^{1/2}$$

$$\tfrac{1}{2}\log_2 2$$

$$\tfrac{1}{2}(1)$$

$$\tfrac{1}{2}$$

Or: "2 raised to what exponent equals the square root of 2?"