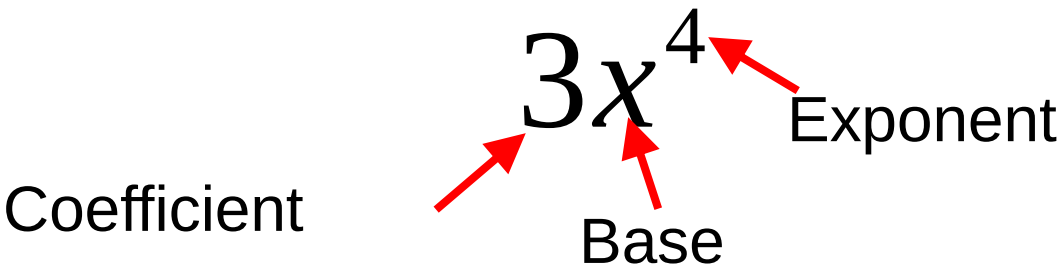
# Math-3

## 5-1: Properties of Exponents

# Properties of Exponents

What is a power?

Power: An expression formed by repeated multiplication of the base.




The exponent applies to the number or variable immediately to its left, not to the coefficient !!!

No Exponent? $3x = 3^1 x^1$

Usually, we don't write the exponent '1' (saves ink).

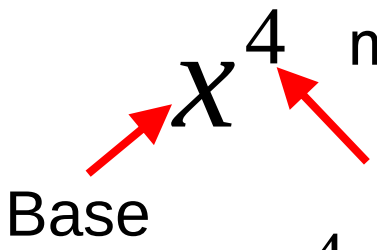
No Coefficient? $x^3 = 1 * x^3 = 1^1 * x^3$

Usually, we don't write the coefficient '1' (saves ink).

Negative? $-x^2 = (-1) * x^2 = (-1)^1 * x^2$

Usually, we don't write the coefficient '-1', we just put the "negative symbol" (saves ink).

Factor: a number that is being multiplied.




means “base x used as a factor 4 times”

$$x^4 = x * x * x * x$$

Power: is repeated multiplication $x^4 = x * x * x * x$

multiplication: is repeated addition $3x = x + x + x$

(adding two terms)

$$3x+4x \quad =(x+x+x)+(x+x+x+x)$$

$$3x+4x=7x$$

$$2x^2+3x^2 \ =(x^2+x^2)+(x^2+x^2+x^2)$$

$$2x^2+3x^2=5x^2$$

(multiplying two terms)

$$x^2*x^3=(x*x)(x*x*x)$$

$$x^2*x^3=x^5$$

# Exponents

$$\left(\frac{x}{2}\right)^2 = ? = \left(\frac{x}{2}\right)\left(\frac{x}{2}\right) = \frac{x^2}{4}$$

Remember how to multiply fractions?

$$\left(\frac{2}{3x}\right)^3 = ? = \left(\frac{2}{3x}\right)\left(\frac{2}{3x}\right)\left(\frac{2}{3x}\right) = \frac{8}{27x^3}$$

# Your Turn:

**1.** $(4y)^2 = ?$

$= 16y^2$

**2.** $2(5x)^2 = ?$

$= 50x^2$

**3.** $\left(\frac{-2}{x}\right)^4 = ?$ $= \frac{16}{x^4}$

**4.** $\left(\frac{x}{2}\right)^3 = ?$ $= \frac{x^3}{8}$

## Multiply Powers Property

$$(x^2)(x^3) = (x * x)(x * x * x)$$

This is ‘x’ used as a factor how many times?

$$(x^2)(x^3) = x^2 x^3 = x^{2+3} = x^5$$

‘x’ used as a factor five times

When you multiply powers having the same base, you add the exponents.

## Exponent of a Power Property $(x^2)^3$

$$(x^2)^3 = (x*x)(x*x)(x*x)$$

This is ‘x’ used as a factor how many times?

$$(x^2)^3 = \quad = x^6$$

‘x’ used as a factor six times

$$(x^2)^3 = x^{2*3} = x^6$$

**you multiply the exponents.**

# Exponent of a Product Property

$$(xy)^2 = (xy)(xy) = x*y*x*y = x*x*y*y$$
$$= x^2y^2$$

$$(xy)^m = x^m y^m$$

This makes it seem like you can "distribute" in the exponent. This only works with the power of a product!!

$$(x-y)^2 \neq x^2 - y^2$$

$$(x-y)^2 = (x-y)(x-y)$$

$$= x^2 - 2xy + y^2$$

Combination of
1. Power of a Product
2. Power of a Power

$$(3x^3y^4)^2 = (3^1x^3y^4)^2$$
$$= 3^2x^6y^8$$

Constants (integer, etc.) usually have an exponent of '1'.

'x' is a number, we just don't know what it is. You treat all numbers the same (whether they are variables or constants).

$$3x^2(4x^3) = ? \quad = 3*4*(x^2)(x^3) = 12x^5$$

You can re-arrange the order of multiplication.

Coefficients of the powers are handled separately from the base and the exponent.

$$(x^2)^5 = ? \quad = x^{10}$$

$$(5x^2)(2x^3) = ? \quad = 10x^5$$

$$(2x)\left(\frac{1}{2}x^3\right) = ? \quad = x^4$$

$$5(x)^3 x^4 = ? \quad = 5x^7$$

$$(2y^5)^3 = ? \quad = 8y^{15}$$

What is the difference between?

$(x)^4 \; and \; \mathrm{x}^4$

$(x^2)^3 \text{ and } (x^3)^2$

$x^4 x^3 \text{ and } x^3 x^4$

$(x+1)^2 \text{ and } (x+1)(x+1)$

Watch the negatives!

$$(-x^3y^4)^2$$

$$= ((-1)^1 x^3 y^4)^2$$ Turn negative signs into multiplication by -1.

$$= (-1)^2 x^6 y^8$$

This way you will be able to tell if the simplified version is positive or negative.

$$= x^6 y^8$$

$$(-2x^2y^6)^3$$ Negative coefficients have an exponent of '1'.

$$= ((-2)^1 x^2 y^6)^3$$

$$= (-2)^3 x^6 y^{18}$$

A negative number raised to an odd exponent remains negative.

$$= -8x^6 y^{18}$$

simplify

$(-2x^2y^4z)^3 \quad = -8x^6y^{12}z^3$

$2(-m^4x^3)^5 \qquad -2m^{20}x^{15}$

$-3(-2x^2yz^3)^4 \quad -48x^8y^4z^{12}$

# Negative Exponent Property "Grab and drag"

$$x^{-2} = \frac{1 * x^{-2}}{1} = \frac{1}{x^2}$$

When you "Grab and drag" the base and its exponent across the "boundary line" between numerator and denominator, you just change the sign of the exponent.

$$x^2 y^{-2} = \frac{x^2}{y^2}$$

$$\left(\frac{1}{x^3}\right)^{-2} = \frac{1}{x^{-6}} = \frac{1}{x^{-6}} = x^6$$

# Negative Exponent Property

Possible errors

$$4x^{-2} = \frac{4 * x^{-2}}{1} = \frac{4}{x^2}$$

When you “Grab and drag” the base and its exponent across the “boundary line” between numerator and denominator, you just change the sign of the exponent.

DO NOT GRAB the coefficient!

$$\frac{4 * x^{-2}}{1} \neq \frac{1}{4x^2}$$

# Zero Exponent Property

Any base raised to the zero power simplifies to one.

$10^3 = 1000$

$10^2 = 100$

$10^1 = 10$

$10^0 = 1$

$2^0 = 1$

$(2x)^0 = 1$

$2x^0 = 2*1 = 2$

Combination: (1) Negative Exponent, (2) Product of Powers, (3) Power of a Power, (4) Power of a Quotient

$$\left(\frac{3x^2}{2x^{-4}y}\right)^2 = \left(\frac{3x^2x^4}{2y}\right)^2 = \left(\frac{3x^6}{2y}\right)^2 = \left(\frac{3^1x^6}{2^1y^1}\right)^2$$

$$= \frac{3^{1*2}x^{6*2}}{2^{1*2}y^{1*2}} = \frac{3^2x^{12}}{2^2y^2} = \frac{9x^{12}}{4y^2}$$

$$(w^{-2})^5 = \frac{1}{w^{10}}$$

$$\frac{1}{2}(3x^{-3})^2 = \frac{9}{2x^6}$$

$$\left(\frac{2x^2}{3y^{-2}z^3}\right)^{-2} \quad \frac{9z^6}{4x^4y^4}$$

$$\left(\frac{5x^4}{3y^{-2}}\right)^{-1} = \frac{3}{5x^4y^2}$$

$$\frac{32x^{10}}{x^{2}y^{17}} = \frac{32x^{10}x^{-2}}{y^{17}} = \frac{32x^{10-2}}{y^{17}}$$

"Grab and drag"

Product of powers: add the exponents of same based powers

$$= \frac{32x^{8}}{y^{17}}$$

$$\frac{2x^3}{4x^5} = \frac{1}{2x^2}$$

$$\frac{(x^{-2})^4}{2x^{-3}} = \frac{1}{2x^5}$$

$$\frac{9(2x)^4}{2x} = 72x^3$$

$$\frac{(-2y^2x^{-3})^4}{2yx^{-3}} = \frac{8y^7}{x^9}$$

Do you "grab and drag (up or down)??

**It doesn't matter!!!!**

$$\frac{3x^2}{2x^{-4}y} = \frac{3x^2x^4}{2y} = \frac{3x^{2+4}}{2y} = \boxed{\frac{3x^6}{2y}}$$

Product of powers property: add the exponents of like-based powers

$$\frac{3x^2}{2x^{-4}y} = \frac{3}{2x^{-4}x^{-2}y} = \frac{3}{2x^{-4-2}y} = \frac{3}{2x^{-6}y} = \boxed{\frac{3x^6}{2y}}$$

Product of powers property: add the exponents of like-based powers

**Make sure when you're all done, there are NO NEGATIVE EXPONENTS remaining.**

$$\left(\frac{5x^2}{25x^4}\right)^2$$

$$\left(\frac{2x^2yz^{-2}}{6x^4y^3z^3}\right)^2$$

$$\left(\frac{2yx^3}{xz}\right)^4$$

$$\frac{\left(yx^3\right)^4}{yx^{12}z^0}$$