Math-3
Lesson 4-5
Polynomial and Rational Inequalities

Graph the general shape of the equation. $y = x^2 - 4x - 12$

→ Positive lead coefficient, even degree

→ 2nd degree polynomial (has a maximum of 2 x-intercepts)

→ Odd multiplicities (graph passes thru x-axis at each "zero.")

In Lesson 4-4 We learned the “Boundary Numbers” Method of solving a quadratic inequality.

$$28 \le x^2 - 12x$$

When solving quadratic equations, we first rearranged the equation to be in standard form.

When solving quadratic inequalities, we first rearrange the inequality into standard form.

$$0 \le x^2 - 12x - 28$$

Find the boundary numbers → solve the equation.

$$0 = x^2 - 12x - 28$$

$$0 = (x - 14)(x + 2)$$

x = 14, -2

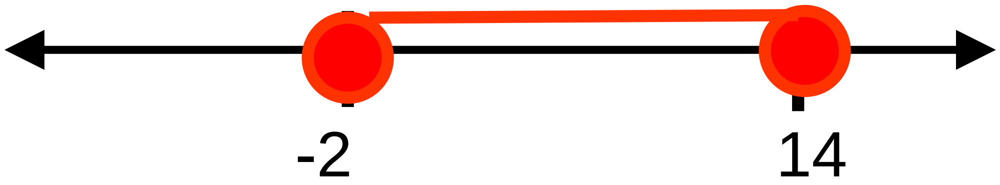
$$0 = x^2 - 12x - 28$$

$$0 = (x - 14)(x + 2)$$

x = 14, -2

For two boundary numbers, the solution is either:

1) Between the boundary numbers or



2) Outside of the boundary numbers

Test a value to see if it is a solution. Zero is often the best number to test.

$$0 < (0)^2 - 12(0) - 28 \qquad 0 \le -28$$

Is “0” a solution? (does x = 0 make the inequality true?)

The shaded part of the graph is the solution

→ we must pick the option that “shades” the number “0”.

$$(-\infty,\ -2] \cup [14,\ \infty)$$

Solve Single Variable *<u>Polynomial</u>* Inequalities

For 3rd degree and higher degree polynomials, there are often more than two zeroes. So we need to work harder to find the solution.

Method 1: "Sign (+/-) Table" (Required for Math-1050)

Method 2: "Sign (+/-) Chart" (Required for Math-1050

Method 1: "Sign (+/-) Table"

$$0 \le x^2 - 12x - 28$$

$0 <$ (*positive* numbers)

1. Find the "real" zeroes of the polynomial equation.

$0 = x^2 - 12x - 28 \quad 0 = (x-14)(x+2)$

$x = -2, 14$

2. Identify the intervals (between the zeroes)

$$(-\infty, -2] \cup [-2, 14] \cup [14, \infty)$$

3. Build a table to organize the information

intervals	Input	(x – 14)(x + 2)	output	Solution?
$(-\infty, -2]$	-3	$(-)(-)$	$(+)$	yes
$[-2, 14]$	0	$(-)(+)$	$(-)$	no
$[14, \infty)$	15	$(+)(+)$	$(+)$	yes

4. Write the Solution in interval notation

$x = (-\infty, -2] \cup [14, \infty)$

Method 2: "Sign (+/-) Chart"

$$0 \le x^2 - 12x - 28$$

$$0 \le (\textit{positive} \quad \text{numbers})$$

$$0 \le (+)$$

$$0 = x^2 - 12x - 28$$

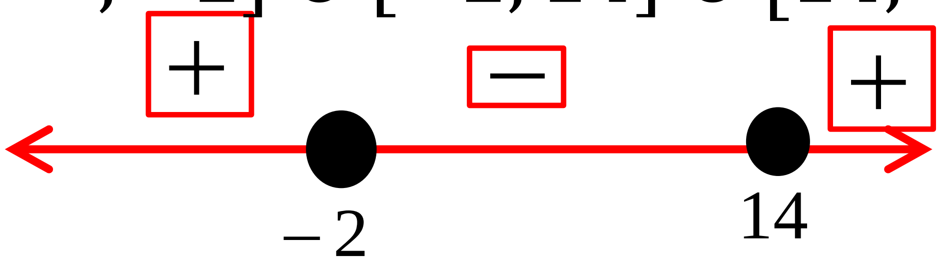
$$0 = (x-14)(x+2)$$

1. Find the "real" zeroes of the polynomial equation.

$$x = -2, 14$$

2. Divide the x-axis into intervals using the zeroes

$$(-\infty, -2] \cup [-2, 14] \cup [14, \infty)$$



3. Determine the sign (+/-) for each interval

$$(-)(-) = (+) \qquad (-)(+) = (-) \qquad (+)(+) = (+)$$

$$0 \le (x-14)(x+2)$$

4. Write the interval of x-values that make the inequality true (positive intervals).

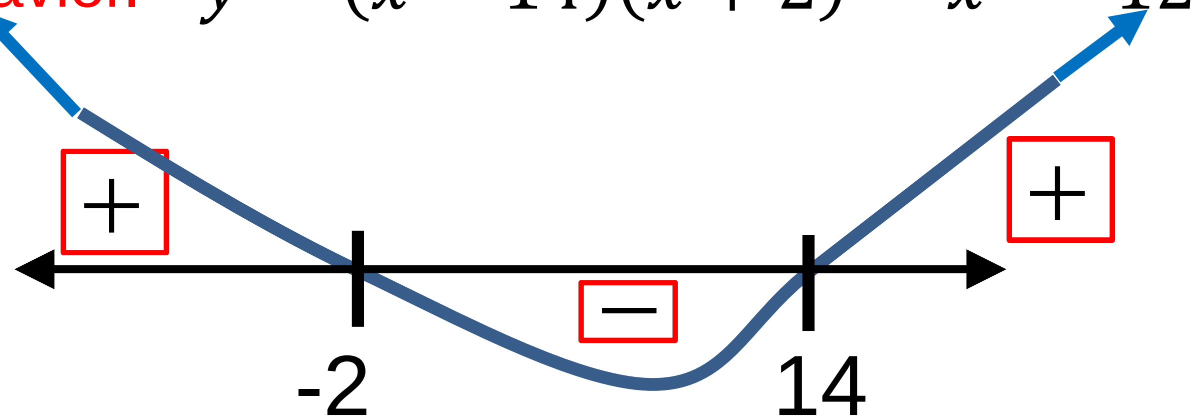
$$(-\infty, \ -2] \cup [14, \ \infty)$$

Method 2: "Sign (+/-) Chart"

$0 \le x^2 - 12x - 28$ $\quad$ $0 \le (\textit{positive} \quad \text{numbers})$ $\quad$ $0 \le (+)$

$0 = x^2 - 12x - 28$ $\quad$ $0 = (x-14)(x+2)$ $\quad$ $x = -2, 14$

You can build the *sign chart* by replacing '0' with 'y' then drawing a rough graph using the zeroes (and their multiplicities) and the end-behavior. $y = (x-14)(x+2) = x^2 - 12x - 28$



$(-\infty, -2] \cup [-2, 14] \cup [14, \infty)$

The expression will be positive for regions where the graph is above the x-axis (and negative below)

$$x = (-\infty, -2] \cup [14, \infty)$$

Solve using the "sign change method" (show your work!!)

$0 > (x+5)(x-6)$ $0 > (negative\ numbers)$

$0 \geq (-)$ Solution: x-values that make the expression negative

intervals	Input	(x+5)(x-6)	output	Solution?
$(-\infty, -5)$	-6	$(-)(-)$	$(+)$	no
$(-5, 6)$	0	$(+)(-)$	$(-)$	yes
$(6, \infty)$	7	$(+)(+)$	$(+)$	no

$0 > (x+5)(x-6)$ $\rightarrow x = (-5, 6)$

Solve using the "Sign Chart or table" method (show your work!!)

$$0 \leq x^2 - 4 \qquad 0 \leq positive\ \#'s\ and\ zero$$

$$0 \leq (+), 0 \qquad 0 \leq (x+2)(x-2)$$

<u>Solution</u>: x-values that make the expression <u>positive</u> or <u>zero</u>

intervals	Input	(x + 2)(x – 2)	output	Solution?
$(-\infty, -2)$	-3	$(-)(-)$	$(+)$	yes
$(-2, 2)$	0	$(+)(-)$	$(-)$	no
$(2, \infty)$	3	$(+)(+)$	$(+)$	yes

$$0 \leq x^2 - 4 \qquad \rightarrow x = (-\infty, -2) U (2, \infty)$$

$$0 < (x-1)(x+1)(x-2) \qquad 0 < (\textit{positive} \text{ numbers})$$

1. Find the "real" zeroes of the polynomial equation.

$$0 = (x+1)(x-1)(x-2) \qquad x = -1,\ 1\ ,2$$

2. Build "Sign (+/-) Table"

intervals	Input	(x – 1)(x + 1)(x – 2)	output	Solution?
$(-\infty,-1)$	-2	$(-)(-)(-)$	$(-)$	no
$(-1,1)$	0	$(-)(+)(-)$	$(+)$	yes
$(1,2)$	1.5	$(+)(+)(-)$	$(-)$	no
$(2,\infty)$	3	$(+)(+)(+)$	$(+)$	yes

3. Write the solution.

$$x = (-1,1) \cup [2,\infty)$$

$0 \geq (\textit{negative} \text{ numbers})$

$0 \geq (-)$

$0 \leq (\textit{positive} \text{ numbers})$

$0 \leq (+)$

Expect the following types of polynomais:

1. "Nice Pattern" → these are the numbers in the box

$$0 \geq x^3 + 2x^2 - 4x - 8$$

2. "Quadratic form" → factor using "m-substitution"

$$0 < x^4 - 10x^2 + 9$$

3. "'x' is a common factor" → factor

$$0 \geq x^3 + 6x^2 - 16x$$

Solving Single Variable Rational Inequalities

We will solve inequalities similar to the following:

$$0 < \frac{3(x+6)(x-2)}{(x+2)(x+1)}$$

$$0 \leq (\textit{positive} \text{ numbers})$$

$$0 \leq (+)$$

Solution: x-values that make the rational expression positive

$$0 > \frac{x^2+7x+10}{x^2+6x-16}$$

$$0 \geq (\textit{negative} \text{ numbers})$$

$$0 \geq (-)$$

Solution: x-values that make the rational expression negative

1) Zeroes of the numerator are x-intercepts

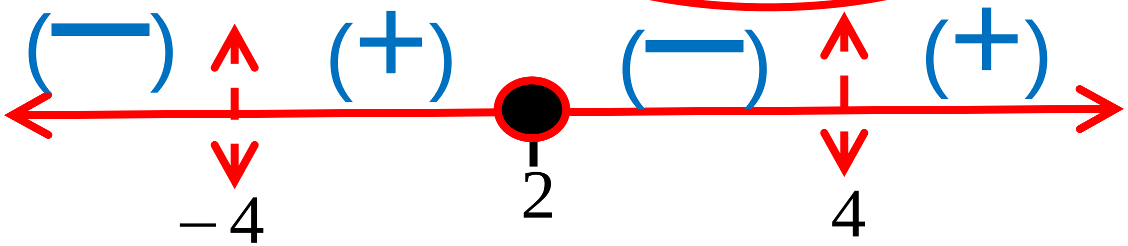
2) Zeroes of the denominator that *do not disappear due to simplification* are vertical asymptotes.

3) Zeroes of the denominator that *disappear due to simplification* are holes.

$$0 < \frac{3x-6}{x^2-16} \qquad y = \frac{3(x-2)}{(x+4)(x-4)}$$

$0 <$ (*positive* numbers)

$0 < (+)$



<u>Solution</u>: x-values that make the rational expression <u>positive</u>

1) <u>Zero of the numerator</u>: x-intercept: (<u>Passes thru</u> at x = 2)

<u>Passes thru</u> means opposite sign on each side of the zero.

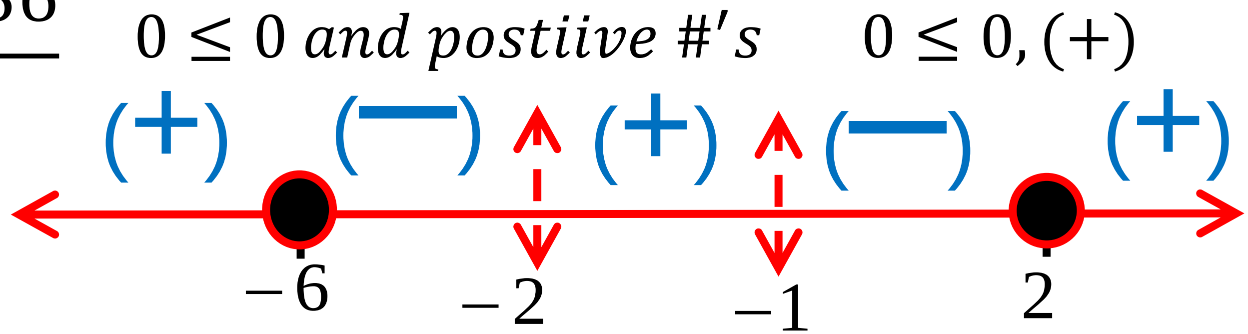
2) <u>Zero of the denominator</u>: (Either vertical asymptote OR hole).

No holes and VA at x = -4, 4

interval	Input	(x – 2)(x + 4)(x – 4)	output	Solution?
$(-\infty, -4)$	-5	$(-)(-)(-)$	$(-)$	no
$(-4, 2)$	0	$(-)(+)(-)$	$(+)$	yes
$(2, 4)$	3	$(+)(+)(-)$	$(-)$	no
$(4, \infty)$	5	$(+)(+)(+)$	$(+)$	yes

$$x = (-4, 2) \cup [4, \infty)$$

$$0 \le \frac{3x^2 + 12x - 36}{x^2 + 3x + 2}$$



$$y = \frac{3(x^2 + 4x - 12)}{(x+2)(x+1)}$$

$$y = \frac{3(x+6)(x-2)}{(x+2)(x+1)}$$

X-intercepts: $x = 2, -6$

(No holes), VA's are: x = -2, -1

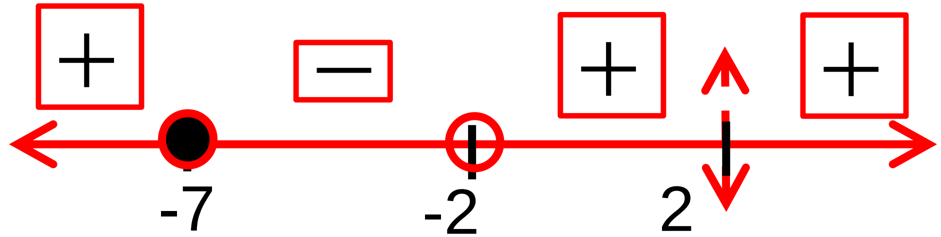
Solution: $\boldsymbol{x = (-\infty, -6] \cup (-2, -1) \cup [2, \infty)}$

interval	Input	$(x+6)(x-2)(x+2)(x+1)$	output	Solution?
$(-\infty, -6]$	-7	$(-)(-)(-)(-)$	$(+)$	yes
$[-6, -2)$	-3	$(+)(-)(-)(-)$	$(-)$	no
$(-2, -1)$	-1.5	$(+)(-)(+)(-)$	$(+)$	yes
$(-1, 2]$	0	$(+)(-)(+)(+)$	$(-)$	no
$[2, \infty)$	3	$(+)(+)(+)(+)$	$(+)$	yes

$$0 > \frac{2x^2 + 10x - 28}{x^2 - 4}$$

$$0 > \frac{2(x+7)(x-2)}{(x+2)(x-2)}$$

$$0 > \frac{2(x+7)}{(x+2)}$$



1. X-intercepts: x = -7

2. Vertical asymptote: x = -2

3. Hole: x = 2

interval	Input	$(x+7)(x+2)$	output	Solution?
$(-\infty, -7]$	-8	$(-)(-)$	$(+)$	no
$[-7, -2)$	-3	$(+)(-)$	$(-)$	yes
$(-2, 2)$	0	$(+)(+)$	$(+)$	no
$(2, \infty)$	3	$(+)(+)$	$(+)$	no