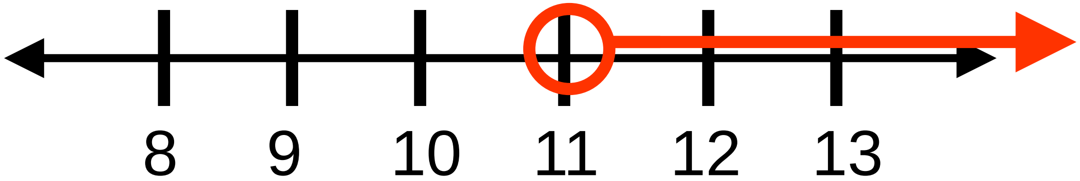
Math-3
Lesson 4-4
Quadratic Inequalities

What does the following inequality mean?

x > 11 "all the numbers that are greater than 11"

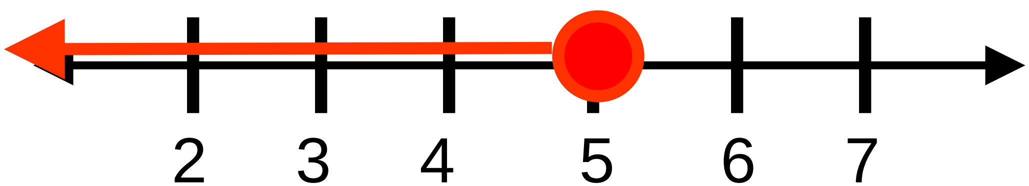
Number line equivalent: (shade all #'s that are solutions)



Interval notation equivalent: $x = (11, \infty)$

x ≤ 5 "all the number that are less than or equal to 5"

Number line equivalent:



Interval notation equivalent: $x = (-\infty, 5]$

<u>Property of Inequality</u>

If you perform the same mathematical operation to the left and right sides of the inequality (<, >, ≤, ≥) then the rewritten inequality is equivalent to the original inequality <u>Provided that</u> if you <u>multiply or divide by a negative number</u> you must switch the direction of the inequality. (Here's why:

$$x - 4 > 8$$

$$+4 \quad +4$$

$$x > 12$$

$$5 > 1$$

$$*(-1) \quad *(-1)$$

$$-5 > -1 \quad \text{(not true)}$$

$$-5 < -1 \quad \text{(true)}$$

Your turn: Solve the inequalities (one step-rewrite)

$2x + 2 \leq 6$

$2(x - 3) \geq 8$

$-14 < -5x + 6$

Draw the equivalent number line for each solution.

Solving inequalities (variable on both sides of a single inequality symbol)

$$3x + 1 \le 2x + 6$$

(Subtraction Property Of Inequality

$$\begin{array}{rcr} 3x + 1 & \le & 2x + 6 \\ -2x & & -2x \\ \hline x + 1 & \le & 6 \\ -1 & & -1 \\ \hline \end{array}$$

(Subtraction Property Of Inequality

$$x \le 5$$

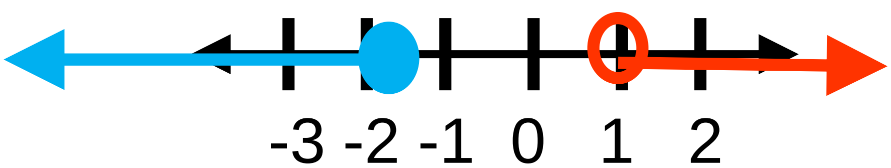
Compound Inequality: the result of combining two simple inequalities with the logical words “and” or “OR”.

“OR” type

x ≤ -2 or x > 1

Is -3 a solution ?

Or means: the numbers that satisfy either condition will make the compound inequality “true”.



$$x = (-\infty, -2] \;\; U \;\; (1, \infty)$$

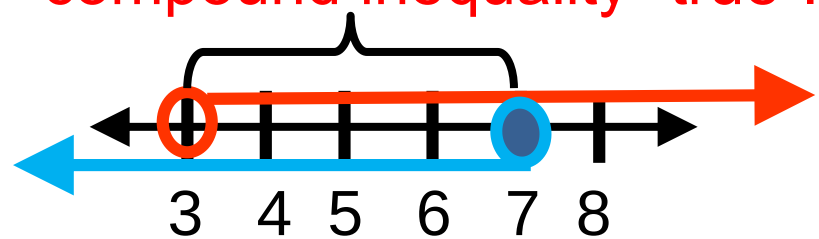
“union” symbol (“or”)

“AND” type

x > 3 and x ≤ 7

Is -3 a solution ?

AND means: the numbers must that satisfy both conditions will make the compound inequality “true”.



$$x = (3, 7]$$

Solve and graph the compound inequality:

Solve each <u>simple inequality</u> separately.

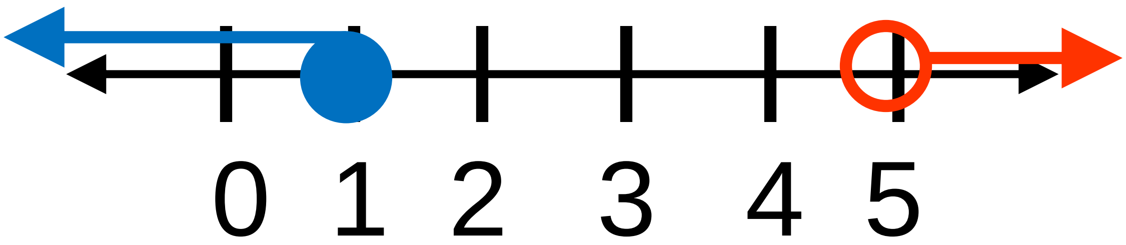
$2x + 3 \le 5$ <u>or</u> $x - 3 > 2$

$-3 \quad -3 \qquad\qquad +3 \quad +3$

$2x \le 2$ <u>or</u> $x > 5$

$\div 2 \quad \div 2$

$\boxed{x \le 1 \quad \underline{\text{or}} \quad x > 5}$



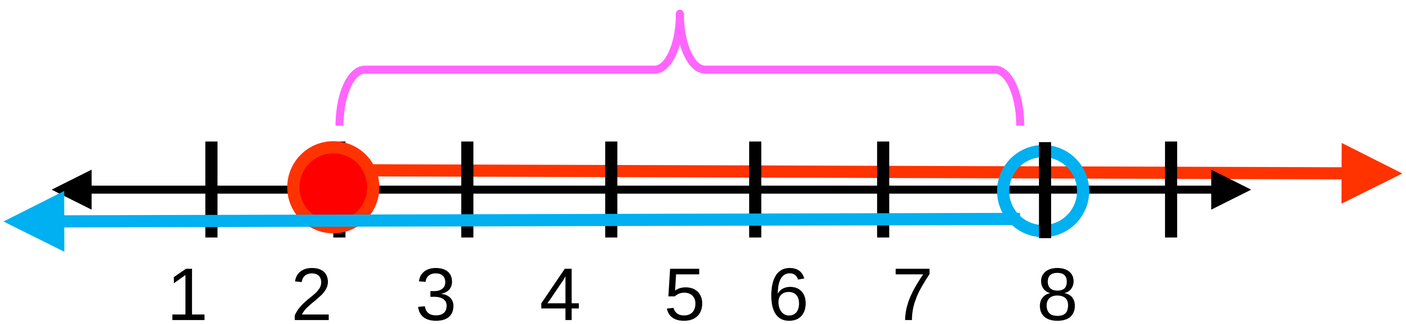
Solve: $x - 4 < 3$ and $x + 3 \geq 5$

To solve: find the values of the variable that make the both inequalities true (“and” inequality).

$x - 4 < 3$ and $x + 3 \geq 5$

$+4 \quad +4$ $\qquad$ $-3 \quad -3$

$x < 7$ and $x \geq 2$



Three Ways to write the solution to an inequality:

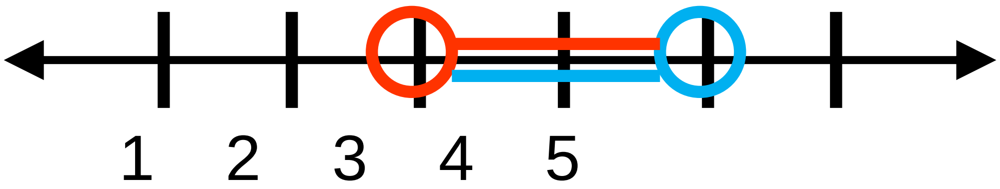
1. Simplified inequality

$x > 3$ and $x < 5$

Another way to write this is:

$3 < x < 5$

2. Graph (number line for a single variable inequality)



3. Interval notation (brackets/parentheses)

$(3, 5)$

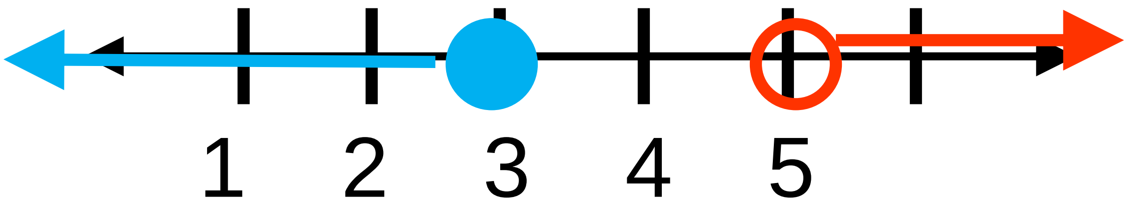
Solve: $2x - 3 \le 3$ or $2x - 5 > 5$

$x \le 3$ or $x > 5$

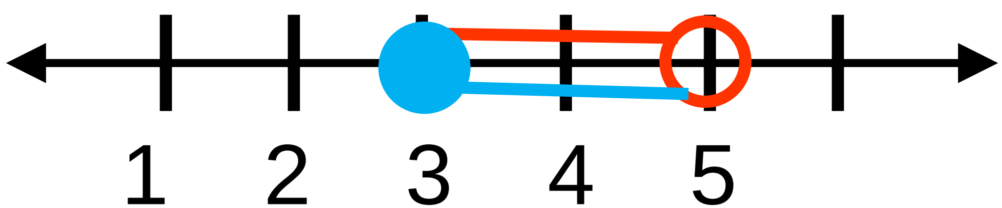
The "boundary numbers" $x = 3$ $x = 5$
separate the solution from the non-solution.

The solution is usually either:

1) Between the boundary numbers or
2) Outside of the boundary numbers



$x \le 3$ or $x > 5$



$3 \le x < 5$

The shaded part of the graph is the solution.

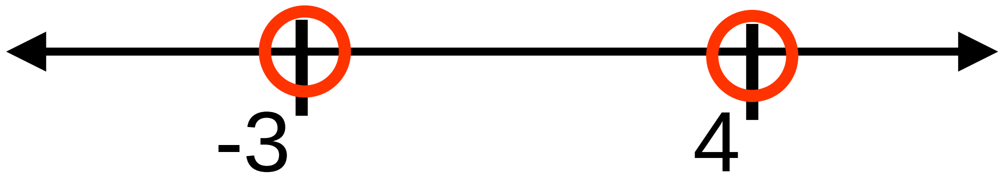
1. Find the boundary numbers: (Solve the equation)

$$0 > x^2 - x - 12$$

$$0 = x^2 - x - 12$$

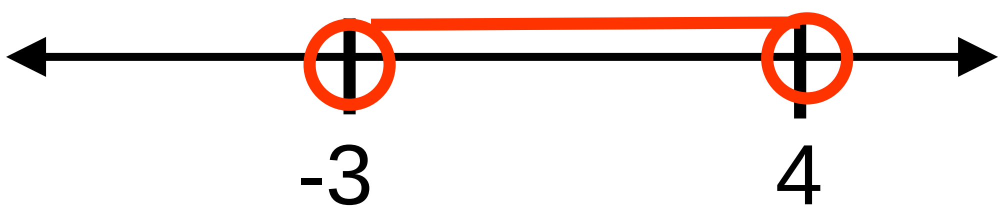
$0 = (x - 4)(x + 3)$

$x = \quad 4, \quad -3$



2. The solution is usually either:

1) Between the boundary numbers or

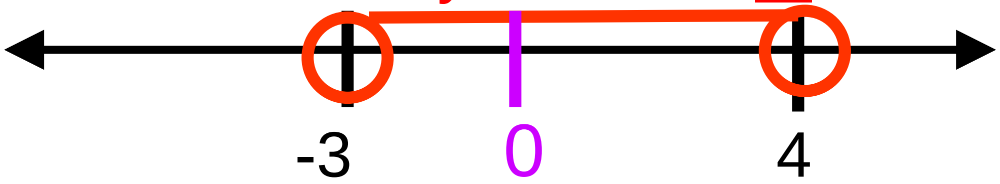


2) Outside of the boundary numbers

The solution is <u>usually</u> either:

$$0 > x^2 - x - 12$$

1) Between the boundary numbers <u>or</u>



2) Outside of the boundary numbers

3. Test a value to see if it is a solution. Zero is <u>often</u> the best number to test.

$$0 > (0)^2 - (0) - 12$$

$$0 > -12$$

Is “0” a solution? (does it make the inequality true?

The <u>shaded part</u> of the graph is the <u>solution</u>
→ we must pick the option that “shades” the number “0”.

$$-3 < x < 4$$

Steps to solve the Inequality $0 > x^2 - x - 12$

1. Find the boundary numbers: (Solve the equation)

2. The solution is usually either:
   a) Between the boundary numbers or
   b) Outside the boundary numbers

3. Test a number to see if it is a solution of the inequality:
   If a solution, pick the number line that shades this number
   If not a solution, pick the number line that doesn't shade

4. Answer the question
   a) Graph (if asked)
   b) Write solution in simplified inequality form (if asked)
   c) Write solution in interval form (if asked).

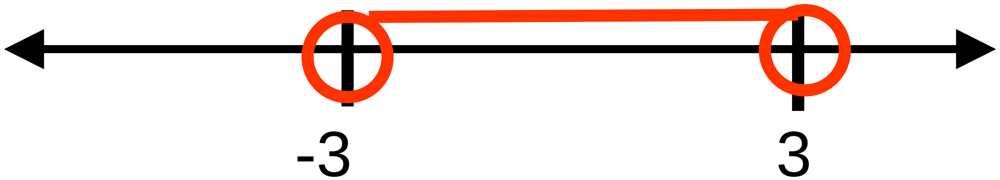
Solve $0 < x^2 - 9$

1. Find the boundary numbers: (solve equation)

$0 = (x - 3)(x + 3)$ $\qquad\qquad$ $x = -3, 3$

2. The solution is either:

a) Between the boundary numbers or



b) Outside of the boundary numbers

3. Test a number (→ "0") $\qquad$ $(0)^2 - 9 > 0$

"0" is not a solution.

4. Solution is: $\qquad$ (-∞, -3) U (3, ∞)

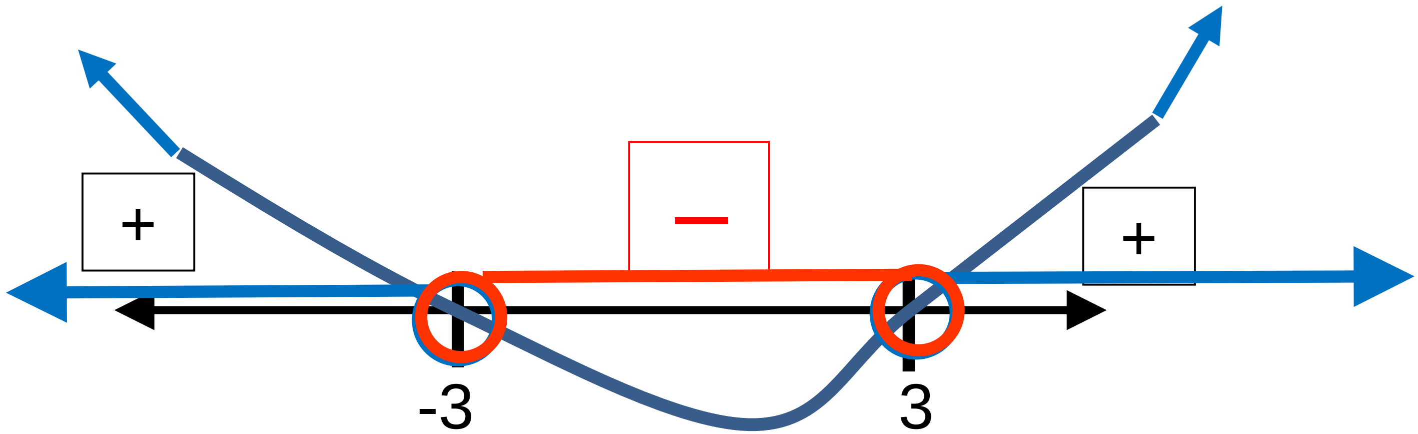
Solve $0 < x^2 - 9$

Graph the general shape of the equation. $y = x^2 - 9$

→ Positive lead coefficient, even degree

→ Up on left and right

→ No even multiplicity zeroes $y = (x - 3)(x + 3)$



Where is the graph “positive”?

Where is the graph “negative”?

→ you could solve the inequality by looking at the sign of ‘y’ from the graph!!!

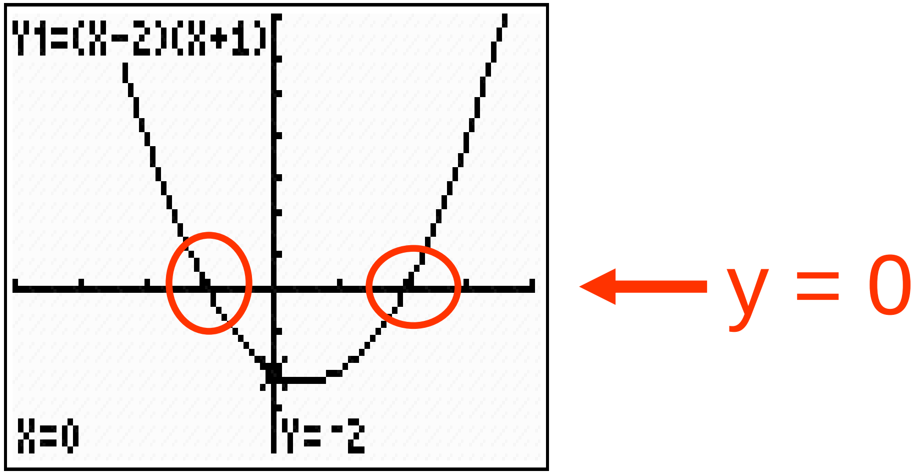
Where is the function <u>positive</u>? (where is f(x) > 0 ?)

$$f(x) = x^2 - x - 2$$

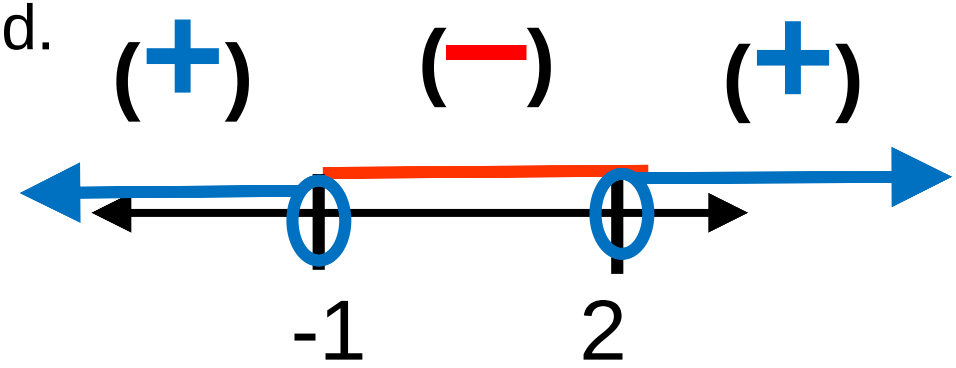
$$f(x) = (x+1)(x-2)$$

$$0 = (x+1)(x-2)$$

x = -1 x = +2



<u>Sign Chart</u>: a number line labeled so that the output value (+/-) is identified.



Where is f(x) > 0 ?

$$f(x) > 0 \text{ for } x = (-\infty, -1) \cup (2, \infty)$$