Math-3
Lesson 4-3
Inverse Functions

1. Graph the line: y = x

2. Plot the following points using stars

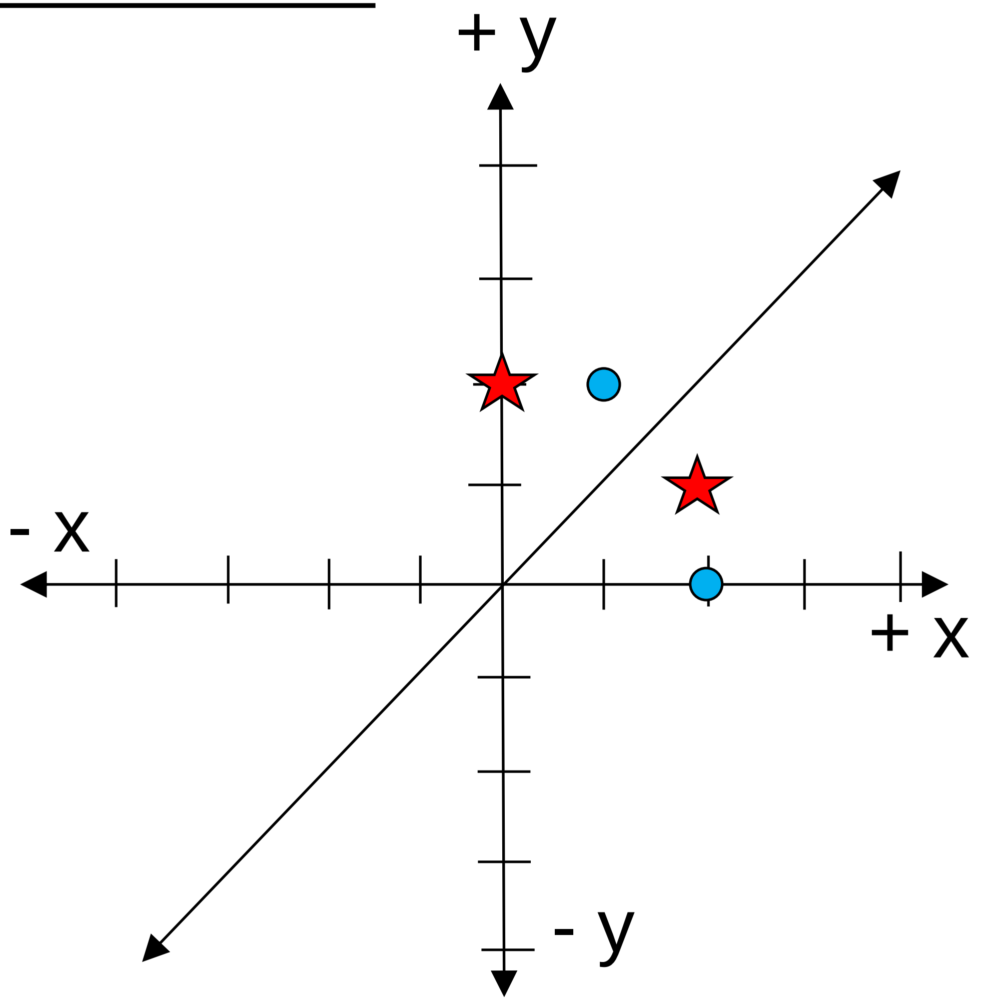
x	y
2	1
0	2

3. Exchange the x and y values in the table

x	y
1	2
2	0

4. Graph the new points using little circles.

5. What do you notice?



Relation: A pairing of input values to output values.

Inverse Relation: A relation that interchanges the input and output values of the original relation.

Relation: (-2, 5), (5, 6), (-2, 6), (7, 6)

Inverse Relation: (5, -2), (6, 5), (6, -2), (6, 7)

How to find the inverse relation:

Relation: $y = 1/2x + 2$

1. Exchange ‘x’ and ‘y’ in the original relation.

2. Solve for ‘y’ (get ‘y’ all by itself).

$$x = 1/2y + 2$$

$$-2 \qquad -2$$

$$x - 2 = 1/2y$$

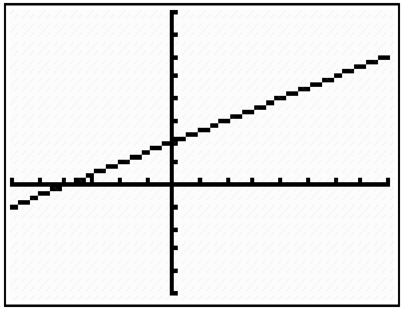
$$*2 \qquad *2$$

$$2(x-2) = \frac{2}{1} * \frac{1}{2} y$$

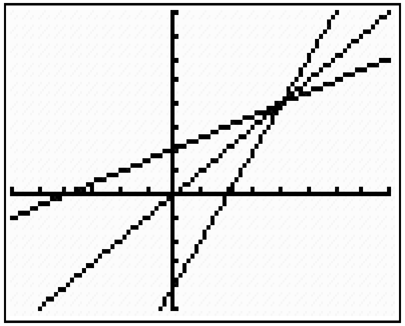
$$2x - 4 = y$$

$$y = 2x - 4$$

Graph of
$y = 2x - 4$
is a <u>reflection</u> of the graph of the line
$y = 1/2x + 2$
across the line
$y = x$.



$y = \frac{1}{2} x + 2$



$y = 2x - 4$

Find the inverse of: f(x) = 4x + 2 Exchange 'x' and 'y'

$$x = 4y + 2$$

This <u>IS</u> the inverse function (written as: "x as a function of y")

Rewrite it so that it is written as: "y as a function of x")

$$x - 2 = 4y$$

subtract '2' (left and right)

$$\frac{x}{4} - \frac{2}{4} = \frac{4y}{4}$$

Divide (all of the) left and right by 4

$$\frac{x}{4} - \frac{1}{2} = y$$

Reduce the fractions

Rearrange into "slope intercept form"

$$y = \frac{x}{4} - \frac{1}{2}$$

This is the inverse of: y = 4x + 2

Function Notation: “the inverse of f(x)”

$$f(x) \qquad\qquad f^{-1}(x)$$

$f^{-1}(x)$ means “the inverse of f(x)”

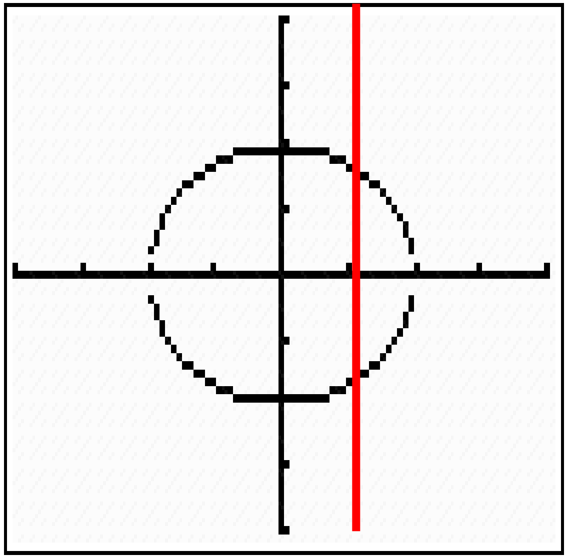
Do not confuse this notation with the negative exponent property:

$$x^{-1} = \frac{1}{x^{1}}$$

Negative exponent on a number or an expression means “flip the number” (the reciprocal of the number)”

The inverse of a function means “exchange ‘x’ and ‘y’ (then solve for ‘y’).”

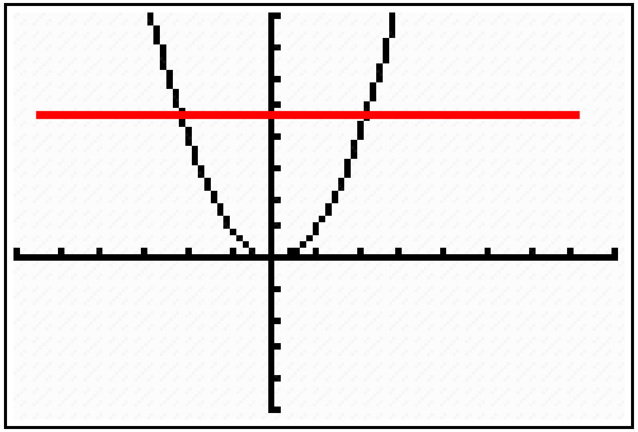
If you have the graph of a relation; <u>how can you tell</u> if the relation is a function?



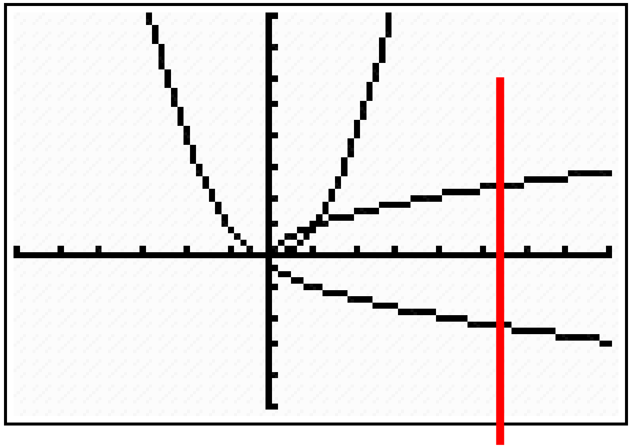
<u>Vertical Line Test</u> if the line intersects the graph more than once, it is <u>NOT</u> a function.

If you have a graph; <u>how can you tell</u> if the inverse of the graphed function is also a function?

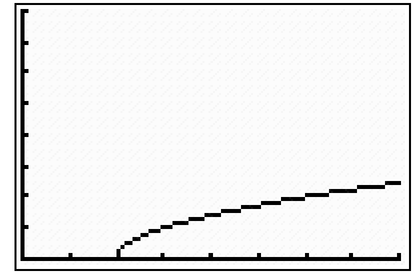
$$f(x) = x^2$$



<u>Horizontal Line Test</u>: if the line intersects the graph more than once, then the
<u>Inverse</u> of the function is <u>NOT</u> a function.



$$f(x)=\sqrt{x-2} \qquad f^{-1}(x)=?$$



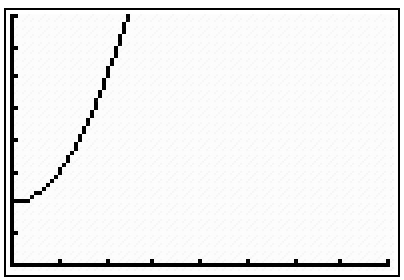
Exchange ‘x’ and ‘y’ in the original relation.

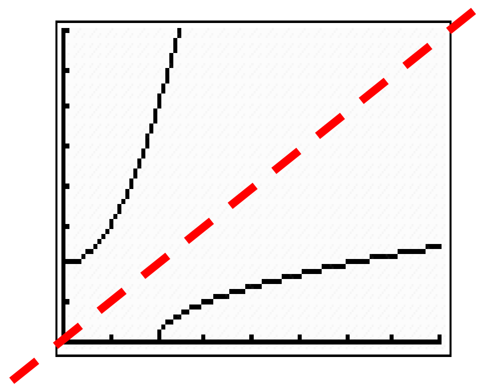
$$x=\sqrt{y-2}$$

This IS the inverse function (written as: “x as a function of y”)

Rewrite it so that it is written as: “y as a function of x”)

$$(x)^2=\left(\sqrt{y-2}\right)^2 \qquad x^2=y-2 \qquad y=x^2+2$$





Why do we only graph the right side of the parabola?

Since the x-y pairs of SQRT are all positive, then the x-y pairs of the inverse of the SQRT (square function) will be positive.

Find the inverse of: $f(x) = x^3 - 3$ Exchange ‘x’ and ‘y’

$$x = y^3 - 3$$

This IS the inverse function, but it is written in the form “x as a function of y”

Rewrite it so that it is written as: “y as a function of x”)

$$x + 3 = y^3$$

Add ‘3’ (left and right)

$$\sqrt[3]{x+3} = \sqrt[3]{y^3}$$

cubed root both sides

$$\sqrt[3]{x+3} = y$$

Simplify

$$f^{-1}(x) = \sqrt[3]{x+3}$$

Is the inverse of: $f(x) = x^3 - 3$

The temperature of a bowl of soup is 100 degrees.

Function A: heating by 10 degrees

Function B: cooling by 10 degrees

The temperature of a bowl of soup is 100 degrees. Apply function A then function B (in sequence) to the bowl of soup. What is the final temperature of the soup?

Temperature = 100 + 10 – 10

Composition of *inverse functions*

Function A and Function B are inverses of each other.

Function A: *“does something”* to the input.

Function B: *“undoes whatever function A* did to the input.

2

Function A “does something” to input value 2

Function B “undoes (whatever A did) to the input value 2

What is the output of function B? **2**

$$f(x) = (x+1)^{2/3} \qquad f^{-1}(x) = ?$$

$$x = (y+1)^{2/3}$$

$$x^{3/2} = \left((y+1)^{2/3} \right)^{3/2}$$

$$x^{3/2} = y+1$$

$$y = x^{3/2} - 1$$

What is the inverse function?

1. $f(x) = \{x^4,\ \ x = [0, \infty)\}$ $\qquad f^{-1}(x) = \sqrt[4]{x}$

2. $g(x) = x^{2/3}$ $\qquad g^{-1}(x) = x^{3/2}$

3. $h(x) = x^{4/5}$ $\qquad h^{-1}(x) = ? \quad = x^{5/4}$

4. $k(x) = x^5$ $\qquad k^{-1}(x) = ? \ = x^{1/5} = \sqrt[5]{x}$

Identify the pairs that cannot be inverses. Why can't they be inverses?

1. $y = x^4 - 3$ $\qquad$ $y = \sqrt[4]{x+3}$

2. $y = (x-5)^{2/3}$ $\qquad$ $y = x^{3/2} + 5$

3. $y = (x+6)^{4/5}$ $\qquad$ $y = x^{5/4} + 5$

4. $y = (x+2)^5 - 3$ $\qquad$ $y = (x+3)^{1/4} - 2$

Are f(x) and g(x) inverses of each other ?

$$g(x) = \frac{x+1}{4}$$

$$f(x) = 4x - 1$$

Are f(x) and g(x) inverses of each other ?

$$g(x) = \frac{(x-1)^2}{5}$$

$$f(x) = 1 + \sqrt{5x}$$

$$f(x) = \frac{2}{x-3} + 4$$

$$f^{-1}(x) = ?$$

$$x = \frac{2}{y-3} + 4$$

$$y - 3 = \frac{2}{(x-4)}$$

$$x - 4 = \frac{2}{y-3}$$

$$\boxed{y = \frac{2}{(x-4)} + 3}$$

$$(y-3)(x-4) = 2$$

$$f(x) = \frac{3x}{x+1} + 6$$

$$f^{-1}(x) = ?$$

$$x = \frac{3y}{y+1} + 6$$

$$x - 6 = \frac{3y}{y+1}$$

$$(y+1)(x-6) = 3y$$

multiply this out!

$$xy - 6y + x - 6 = 3y$$

$$xy - 6y - 3y = -x + 6$$

$$xy - 9y = -x + 6$$

$$y(x-9) = -x + 6$$

$$y = \frac{-x+6}{(x-9)}$$

We use compositions of inverse functions to solve equations.

$$(x-3)^2 + 4 = 40$$

-4 -4

"Isolate the square, undo the square".

$$(x-3)^2 = 36$$

$$\sqrt[2]{(x-3)^2} = \sqrt{36}$$

"undo the square" means "inverse function" of the square

$$x - 3 = \pm 6$$

$$x = 3 + 6 = 9$$

$$x = 3 - 6 = -3$$

Solve the following equation

$23 = 3x^3 - 1$ Isolate the power:

$$24 = 3x^3$$

$8 = x^3$ undo the power

$$\sqrt[3]{8} = \sqrt[3]{x^3}$$

$$2 = x$$

Solve $13 = x^4 - 3$

$$16 = x^4$$

$$\pm\sqrt[4]{16} = x$$

$$x = \pm 2$$

Solve $\sqrt{2x+1} = 3$

$$\left(\sqrt{2x+1}\right)^2 = 3^2$$

$$2x+1=9$$

$$2x=8$$

$$x=4$$

Solve:

Check your solution.

$$\sqrt{x+3}+5=0$$
$$\sqrt{x+3}=-5$$
$$x+3=25$$
$$x=22$$

$$\sqrt{22+3}+5=0$$
$$\sqrt{25}+5=0$$
$$5+5\neq 0$$

Extraneous solution.

$$\sqrt{2-x}=-x$$
$$2-x=(-x)^2$$
$$2-x=x^2$$
$$0=x^2+x-2$$
$$0=(x+2)(x-1)$$

$$x=-2,\ 1$$

Check your solutions.

$$\sqrt{2-(-2)}=-(-2)$$
$$\sqrt{4}=2$$

Checks.

$$\sqrt{2-(1)}=-(1)$$
$$\sqrt{1}\neq -1$$

Extraneous solution.

$$x=-2$$
$$x\neq 1$$