

Math-3
Lesson 4-2

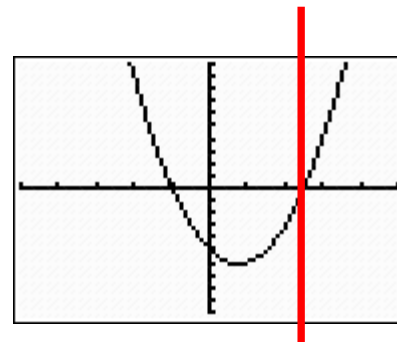
Function Composition

1. Is the following relation a function?

$(-2, 5)$, $(5, 6)$, $(-2, 6)$, $(7, 6)$

No. Input value -2 has two output values.

2. Is the following relation a function?



Does the graph of the relation pass the “vertical line test” ?

Yes. Each input value has exactly one output value.

Function Notation $y = f(x)$ “y is a function of x”

‘y’ equals “math being done to” ‘x’

A function is a rule that matches input values to output values.

$$f(x) = 2x + 1$$

(Input) x	(rule) $2x + 1$	(output) y	
2	$2(2) + 1$	5	$f(2) = 5$
3	$2(3) + 1$	7	$f(3) = 7$

Compositions of Functions $f(x) = 2x \rightarrow f(3) = ?$

Means: “replace ‘x’ in the function with a 3.

1. Replace the ‘x’ with a set of parentheses.

$$f(3) = 2(\quad)$$

2. Put the input value ‘3’ into the parentheses.

$$f(3) = 2(3)$$

3. Find the output value.

$$f(3) = 6$$

Compositions of Functions

$$f(x) = x^2 - 3x + 2 \quad \rightarrow f(2) = ?$$

1. Replace the 'x' with a set of parentheses.

$$f(x) = (\quad)^2 - 3(\quad) + 2$$

2. Put the input value '2' into the parentheses.

$$f(x) = (2)^2 - 3(2) + 2$$

3. Find the output value.

$$f(2) = 0$$

**Cool, we found a
zero of the function.**

$$f(x) = x^3 - 1 \quad f(-2) = ?$$

$$f(-2) = (-2)^3 - 1 \quad f(-2) = -9$$

$$f(x) = 2x^{1/2} \quad f(9) = ?$$

$$f(9) = 2(9)^{1/2} \quad f(9) = 6$$

$$f(x) = \frac{2(x-4)}{x^2 + x - 20} \quad f(-2) = ?$$

$$f(-2) = \frac{2(-2-4)}{((-2)^2 + (-2) - 20)} \quad f(-2) = \frac{2}{3}$$

Function Notation

$$f(x) = 2x + 1$$

(Input) x	(rule) $2x + 1$	(output) f(x)
2	$2(2) + 1$	5 $f(2) = 5$
3	$2(3) + 1$	7 $f(3) = 7$
$x - 1$	$2(x - 1) + 1$	$2x - 1$
$3x$	$2(3x) + 1$	$6x + 1$

$$f(x - 1) = 2x - 1$$

$$f(3x) = 6x + 1$$

If your input is an expression instead of a number:
replace 'x' with parentheses and "plug in" the expression
→ parentheses, substitute, simplify

$$f(x) = 3x - 1$$

(Input) x	(rule) $3x - 1$	(output) f(x)
2	$3(2) - 1$	5 $f(2) = 5$
x^2	$3() - 1$? $f(x^2) = 3x^2 - 1$
$x + 2$	$3() - 1$? $f(x + 2) = 3x + 5$
$3 - 2x$	$3() - 1$? $f(3 - 2x) = 8 - 6x$

Your turn:

$$f(x) = x^2 + 1$$

input the expressions

$$f(2) = ? \quad = 5$$

$$f(x^3) = ? \quad = x^6 + 1$$

$$f(x+2) = ? \quad = (x+2)^2 + 1 \quad = x^2 + 4x + 5$$

$$f(-2x+3) = ? \quad = (-2x+3)^2 + 1$$
$$= 4x^2 - 12x + 10$$

Compositions of Functions

$$f(x) = 2x$$

and

$$g(x) = x^2$$

Let's use $f(x)$ as the input to $g(x)$ $g(f(x)) = ?$

$$g(\dots) = (\dots)^2$$

$$g(2x) = (2x)^2$$

$$g(f(x)) = 4x^2$$

1. Replace the 'x' with a set of parentheses.
2. Put the input value "2x" into the parentheses.
3. Find the output value.

Compositions of Functions

$$f(x) = 2x + 3 \quad \text{and}$$

$$g(x) = x^2$$

$$f(g(x)) = ?$$

1. The input value to $f(x)$ is $g(x)$.

$$f(..) = 2(..) + 3$$

2. Replace the 'x' in $f(x)$ with a set of parentheses.

$$f(x^2) = 2(x^2) + 3$$

3. Put the input value ($g(x)$) into the parentheses.

$$f(g(x)) = 2x^2 + 3$$

4. Find the output value.

Function “composition”

$$f(x) = x^2 + 1$$

$$g(x) = x^2$$

$$f(2) = ?$$

What does this mean?

“Substitute ‘2’ in for ‘x’ in the function f(x).”

$$f(g(x)) = ?$$

What does this mean?

“Substitute ‘g(x)’ in for ‘x’ in the function f(x).”

$$f(g(x)) = (g(x))^2 + 1$$

“Which means the same as...”

$$f(x^2) = (x^2)^2 + 1 = x^4 + 1$$

Composition of Functions

$$f(x) = 2x + 1 \quad g(x) = 3x + 2 \quad h(x) = x + 5$$

$$f(g(x)) = ? = 2(\quad) + 1 = 2(3x + 2) + 1$$

$$h(g(x)) = ? = (\quad) + 5 = (3x + 2) + 5$$

$$h(f(x)) = ? = (\quad) + 5 = (2x + 1) + 5$$

$$g(h(x)) = ? = 3(\quad) + 2 = 3(x + 5) + 2$$

$$f(f(x)) = ? = 2(\quad) + 1 = 2(2x + 1) + 1$$

New Notation for the Composition of Functions

$$(f \circ g)(x) = f(g(x)) \quad \text{"g" plugged into rule "f"}$$

$$f(x) = 4x - 1 \quad g(x) = -5x + 3$$

$$(f \circ g)(x) = ? \quad = 4(\quad) - 1 \quad = 4(-5x + 3) - 1$$

$$\text{"g" plugged into rule "f"} \quad (f \circ g)(x) = -20x + 11$$

$$(g \circ f)(x) = ? \quad = -5(\quad) + 3 \quad = -5(4x - 1) + 3$$

$$\text{"f" plugged into rule "g"} \quad (g \circ f)(x) = -20x + 8$$

$$(f \circ f)(x) = ? \quad = 4(\quad) - 1 \quad = 4(4x - 1) - 1$$

$$\text{"f" plugged into rule "f"} \quad (f \circ f)(x) = 16x - 5$$

$$(g \circ g)(x) = ? \quad = -5(\quad) + 3 \quad = -5(-5x + 3) + 3$$

$$\text{"g" plugged into rule "g"} \quad (g \circ g)(x) = 25x - 12$$

One more layer!

$$f(x) = 3x \quad g(x) = x^2$$

$$f(g(4))$$

4

$$g(\) = (\)^2$$

$$g(4) = (4)^2$$

16

$$f(\) = 3(\)$$

$$f(16) = 3(16)$$

48

$$f(g(x)) = 3(g(x)) = 3x^2$$

$$f(g(4)) = 3(g(4)) = 3(4)^2 = 48$$

One more layer. $g(x) = x^2$ $f(x) = 3x$

$$(g \circ f)(-1) = ?$$

Rewrite in “old” notation

$$g(f(-1)) = ?$$

The input to $f(x)$ is -1.

$$f(-1) = 3(-1)$$

$$f(-1) = -3$$

The output of $f(-1)$ is -3.

The input to $g(x)$ is -3.

$$g(-3) = 9$$

$$g(f(-1)) = 9$$