

Math-3

Lesson 4-1

Linear Combinations of Functions

Combining Functions Algebraically

Multiplication by a number

$$f(x) = x^2 - 1$$

$$2f(x) = ? \quad \text{Replace "f(x)" with parentheses}$$

$$2(\quad)$$

$$\text{Substitution step} \rightarrow f(x) = x^2 - 1$$

$$2f(x) = 2(x^2 - 1) \quad \text{Simplify} \rightarrow$$

$$f(x) = x^2 - 1 \quad 2f(x) = 2x^2 - 2$$

Perform the indicated operation:

$$f(x) = 4x + 2 \quad g(x) = -5x + 3$$

$$3f(x) = ? = 3(4x + 2) = 12x + 6$$

$$-2g(x) = ? = -2(-5x + 3) = 10x - 6$$

$$5g(-3) = ? \Rightarrow g(-3) = -5(-3) + 3$$

$$g(-3) = 18$$

$$\Rightarrow 5g(-3) = 5(18) = 90$$

“Linear combinations of functions”

$$f(x) = 4x + 2 \quad g(x) = -5x + 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

$$f(x) - g(x) = ? = (\quad) - (\quad)$$

$$= (4x + 2) - (-5x + 3)$$

$$= 4x + 2 + 5x - 3$$

$$f(x) - g(x) = 9x - 1$$

Perform the indicated operation:

$$f(x) = 4x + 2 \quad g(x) = -5x + 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

$$\begin{aligned} g(x) - 2f(x) &= (\quad) - 2(\quad) \\ &= (-5x + 3) - 2(4x + 2) \\ &= -5x + 3 - 8x - 4 \end{aligned}$$

$$g(x) - 2f(x) = -13x - 1$$

Perform the indicated operation:

$$f(x) = 4x + 2 \quad g(x) = -5x + 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

$$-3 f(x) - 2 g(x) = -3(\quad) - 2(\quad)$$

$$= -3(4x + 2) - 2(-5x + 3)$$

$$= -12x - 6 + 10x - 6$$

$$-3 f(x) - 2 g(x) = -2x - 12$$

Perform the indicated operation:

$$f(x) = x^2 + 4x - 21$$

$$g(x) = x - 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

$$f(x) \div g(x) = \frac{(\quad)}{(\quad)} = \frac{(x^2 + 4x - 21)}{(x - 3)}$$

$$= \frac{(x + 7)(\cancel{x - 3})}{(\cancel{x - 3})}$$

$$f(x) \div g(x) = x + 7$$

Perform the indicated operation:

$$f(x) = x^2 - 5x - 14 \quad g(x) = x + 3$$

(1) Replace with parentheses, (2) plug in, (3) Simplify.

$$\begin{aligned} g(x) \div 2 f(x) &= \frac{(\quad)}{(\quad)} \\ &= \frac{(x + 3)}{2(x^2 - 5x - 14)} = \frac{(x + 3)}{2(x - 7)(x + 2)} \end{aligned}$$

New Notation

$$f(x) - g(x) = (f - g)(x)$$

Rewrite the following in the new notation.

$$g(x) - 2f(x) = (g - 2f)(x)$$

$$-3f(x) - 2g(x) = (-3f - 2g)(x)$$

$$f(x) \div g(x) = \left(\frac{f}{g} \right)(x)$$

Perform the indicated operation:

$$j(x) = 15x^2 \quad k(x) = -5x^3$$

$$(2j - 3k)(x) = 2(15x^2) - 3(-5x^3) \\ = 15x^3 + 30x^2$$

$$(2j - 3k)(-1) = 15(-1)^3 + 30(-1)^2$$

$$(2j - 3k)(-1) = 15$$

$$\left(\frac{2k}{j}\right)(x) = ? = \frac{2(-5x^3)}{15x^2}$$

$$= \frac{\cancel{5} * (-2) * \cancel{x} * \cancel{x} * x}{\cancel{5} * 3 * \cancel{x} * \cancel{x}} = \frac{-2x}{3} = -\frac{2}{3}x$$

$$j(x) = 15x^2$$

$$k(x) = -5x^3$$

$(j + k)(-2)$ What does this mean?

$$(j + k)(x) = j(x) + k(x) = 15x^2 - 5x^3$$

$$j(x) + k(x) = 15x^2 - 5x^3$$

$$(j + k)(-2) = 15(\quad)^2 - 5(\quad)^3$$

Another way to do it:

$$= 15(-2)^2 - 5(-2)^3 \quad (j + k)(-2) = j(-2) + k(-2)$$

$$= 60 + 40$$

$$= 100$$

$$j(-2) + k(-2) = 15(-2)^2 - 5(-2)^3$$

$$= 60 + 40$$

$$= 100$$

Perform the indicated operation:

$$f(x) = x^2 + 4x - 21 \quad g(x) = x - 3$$

$$(f - 2g)(-1)$$

$$f(-1) - 2g(-1)$$

$$f(-1) = (-1)^2 + 4(-1) - 21 = -24$$

$$g(-1) = (-1) - 3 = -4$$

$$f(-1) - 2g(-1) = -24 - 2(-4)$$

$$= -24 + 8$$

$$= -16$$

Combining Functions Algebraically

Product: $(fg)(x) = f(x) * g(x)$

$$f(x) = x + 2 \quad g(x) = x - 2$$

$$(fg)(x) = (x + 2)(x - 2)$$

$$(fg)(x) = x^2 - 4$$

$$f(x) = 3x \quad g(x) = x - 4$$

$$(2f + 3g)(-1) = ?$$

Method 1: Write as two separate functions then combine the result

$$(2f + 3g)(-1) = 2f(-1) + 3g(-1)$$

$$= 2[3(-1)] + 3[(-1) - 4]$$

$$= 2(-3) + 3(-5)$$

$$= -6 - 15$$

$$= -21$$

$$f(x) = 2x - 3 \qquad g(x) = 4x - 5$$

$$(3f + 2g)(-1) = ?$$

Method 2: Find linear combination of the functions THEN plug in.

$$\begin{aligned}(3f + 2g)(x) &= 3(2x - 3) + 2(4x - 5) \\ &= 6x - 9 + 8x - 10 \\ &= 14x - 19\end{aligned}$$

$$(3f + 2g)(x) = 14x - 19$$

$$\begin{aligned}(3f + 2g)(-1) &= 14(-1) - 19 \\ &= -14 - 19 = -33\end{aligned}$$