Math-3
Lesson 3-9
Applications of Rational Functions

What you should know at the end of the lesson:

1) What a *quantity* is.

2) What a *rate* is.

3) How to use ratios of quantities to come up with new quantities.

4) How slopes of the graphs relating two different quantities can become a new quantities.

5) How to use rates to solve work problems and concentration problems.

Handout:

Pair up with your neighbor and collaborate to answer questions #1 through #4.

Quantities and their Units

Quantity	Unit of Measure
Height	Inches, feet, miles, etc.
Weight	Pounds, ounces, kilograms, grams
Temperature	Degrees F, Degrees C, Degrees K
Volume	Gallons, cubic feet, cups
Length	Meters, kilometers, millimeters
Energy	Joules, watts

New Quantities from Ratios of Quantities

Quantity	Ratio of:	Unit of Measure
"unit price"	Cost/weight	$/lbm (dollars "per" pound)
Speed	Distance/time	Mile/hr (mile "per" hr) Ft/sec (ft "per" sec)
Fuel efficiency	Distance/volume gas used	Miles/gallon (mph) (miles "per" gallon)

When you see the word "per" , what type of quantity is it?

Continue working to complete problems #5 thru #7

What is your ratio of quantities for this situation? What are the units?

A vendor at a state fair is inflating a helium balloon.

"*time rate of change of volume*" $\dfrac{\Delta volume}{\Delta time} \rightarrow \dfrac{in^3}{sec}$ "fill rate"

or

"the change in volume with respect to (the change in) time"

An employee cutting a large patch of grass.

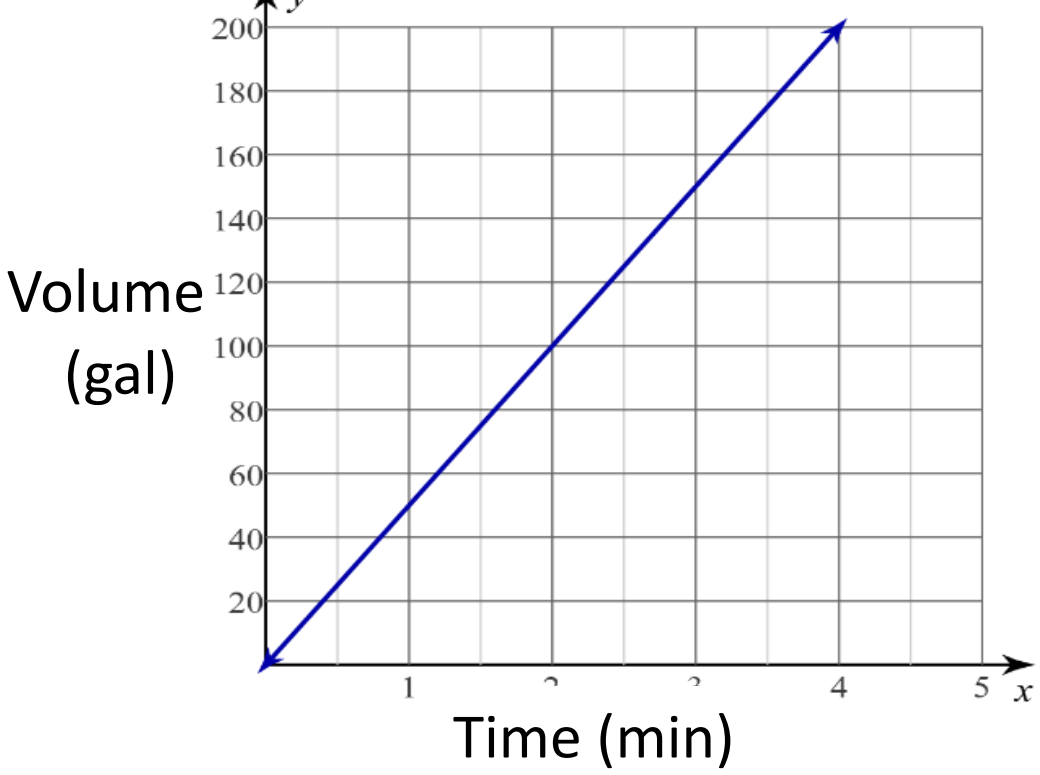
"*time rate of change of area (of cut grass)*" $\frac{\Delta area}{\Delta time} \rightarrow \frac{ft^2}{min}$ "Cutting rate"

or
"the change in cut area with respect to (the change in) time."

A painter is painting a room.

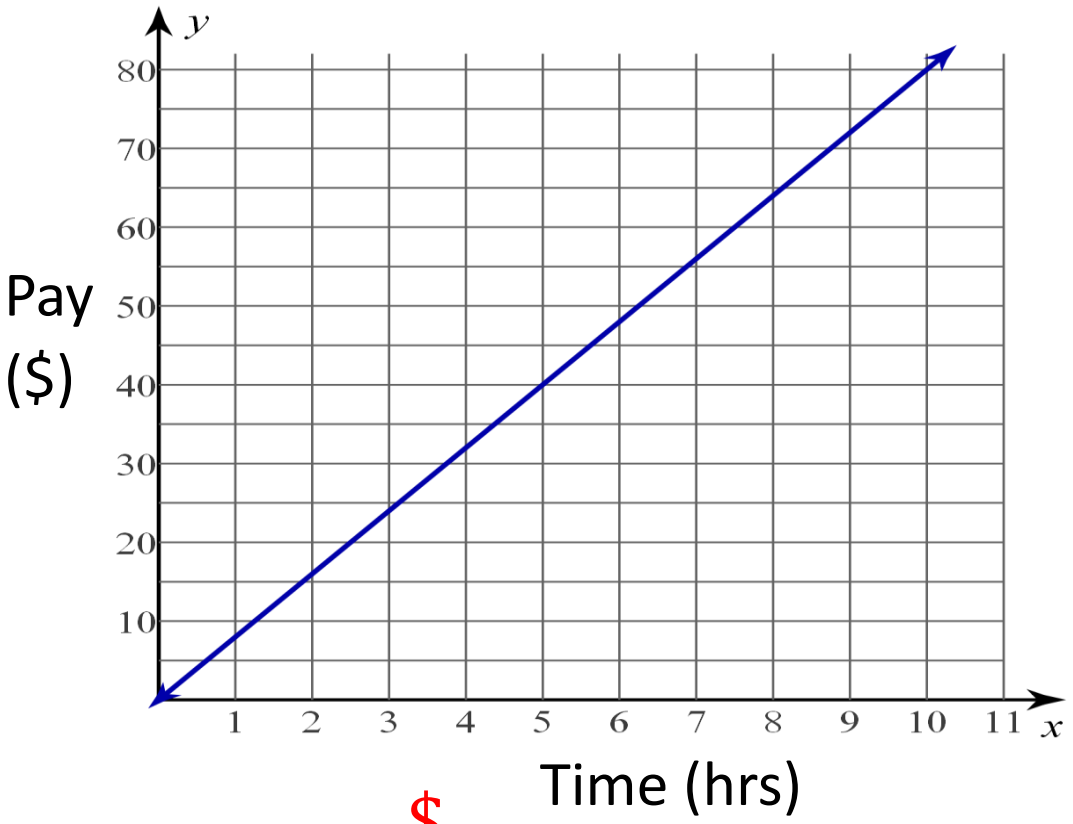
"*time rate of change of painted area* $\frac{\Delta area}{\Delta time} \rightarrow \frac{ft^2}{min}$ "Painting rate"

or
"the change in painted area with respect to (the change in) time."



Units of slope? $\frac{gal}{min}$

Quantity? Fill rate

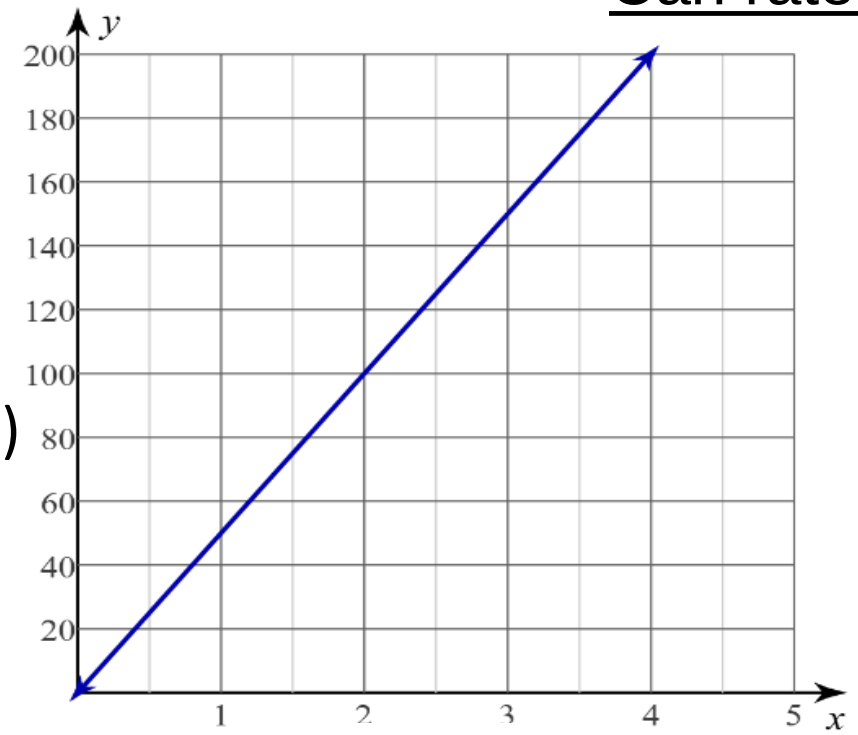


Units of slope? $\frac{\$}{hr}$

Quantity? pay rate cost rate

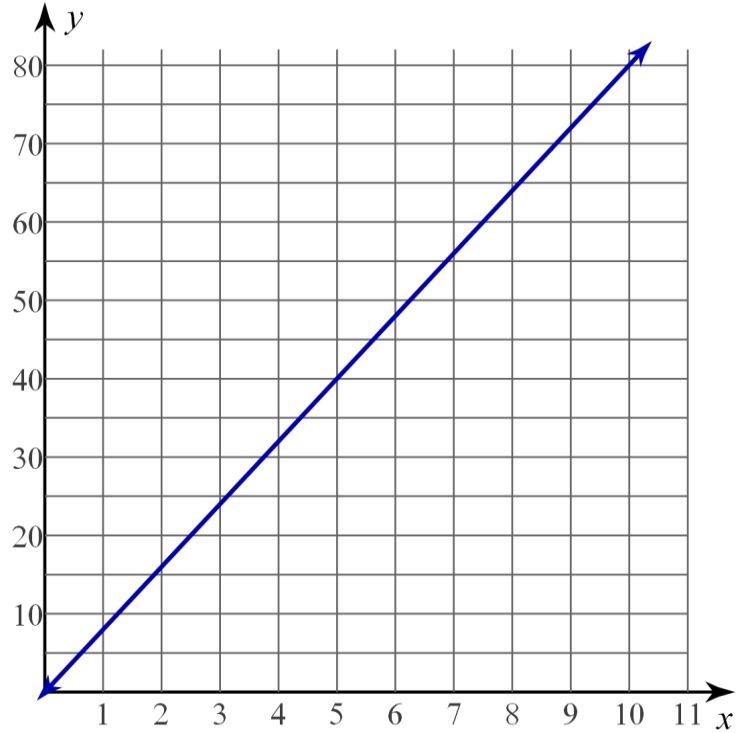
Can rates be added?

Volume (from Hose #1) (gal)



Time (min)

Volume (from Hose #2) (gal)



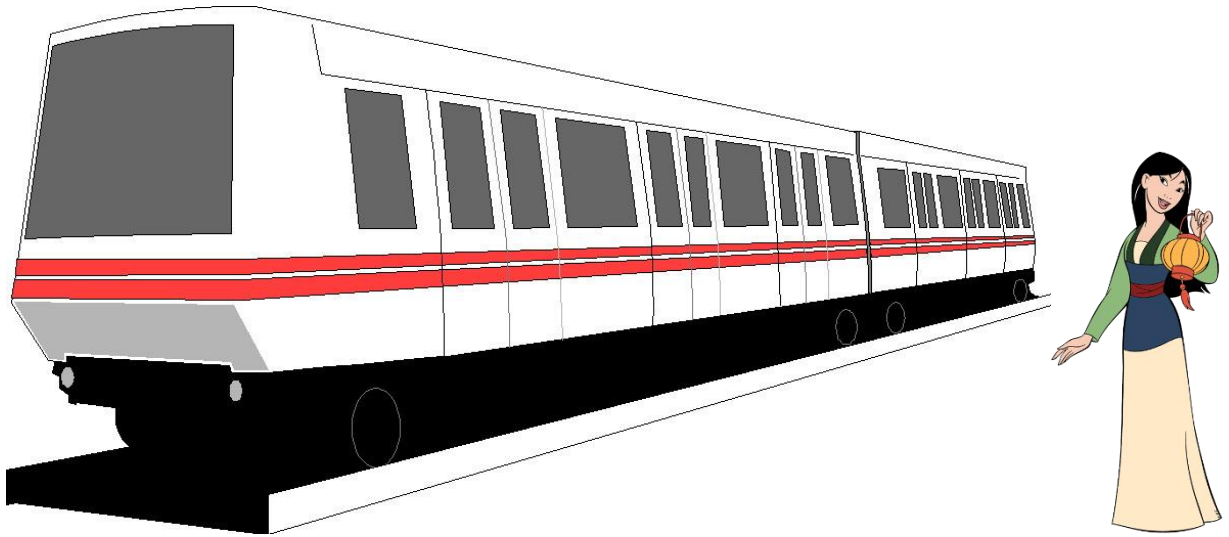
Time (min)

Hose #1 fill rate? $50\ gal/min$

Hose #2 fill rate? $8\ gal/min$

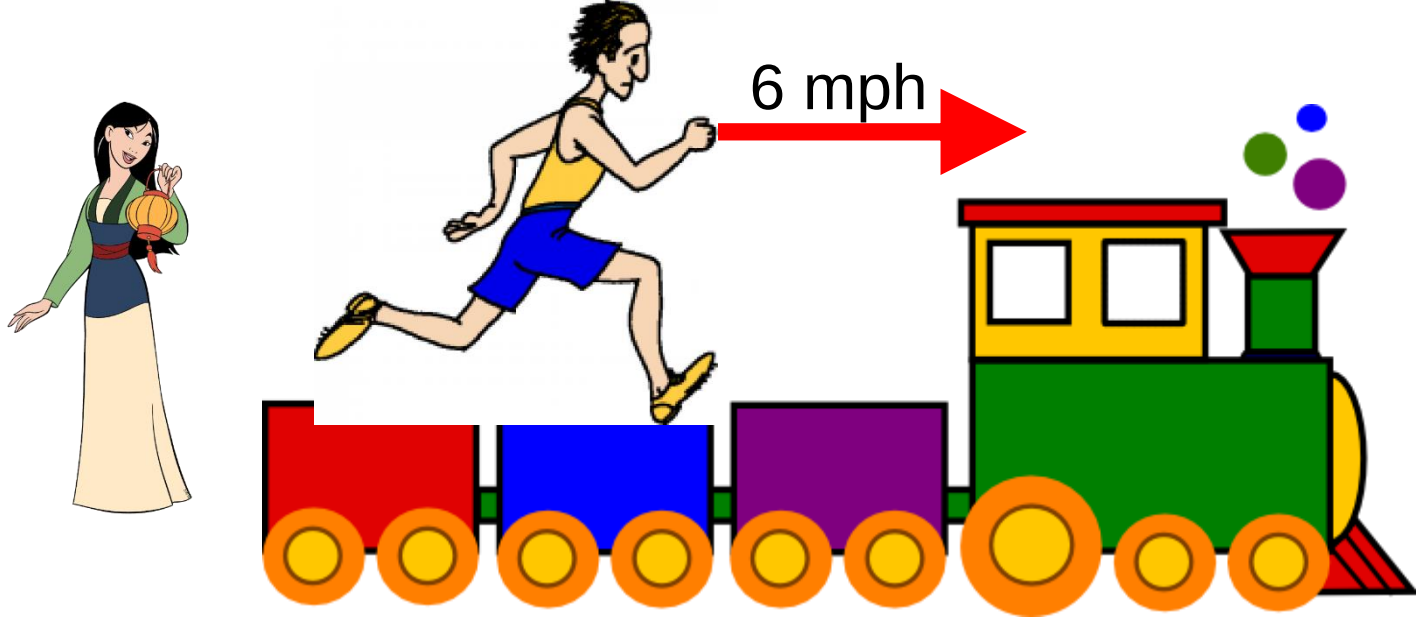
What is the combined fill rate? $58\ gal/min$

The Train Problem



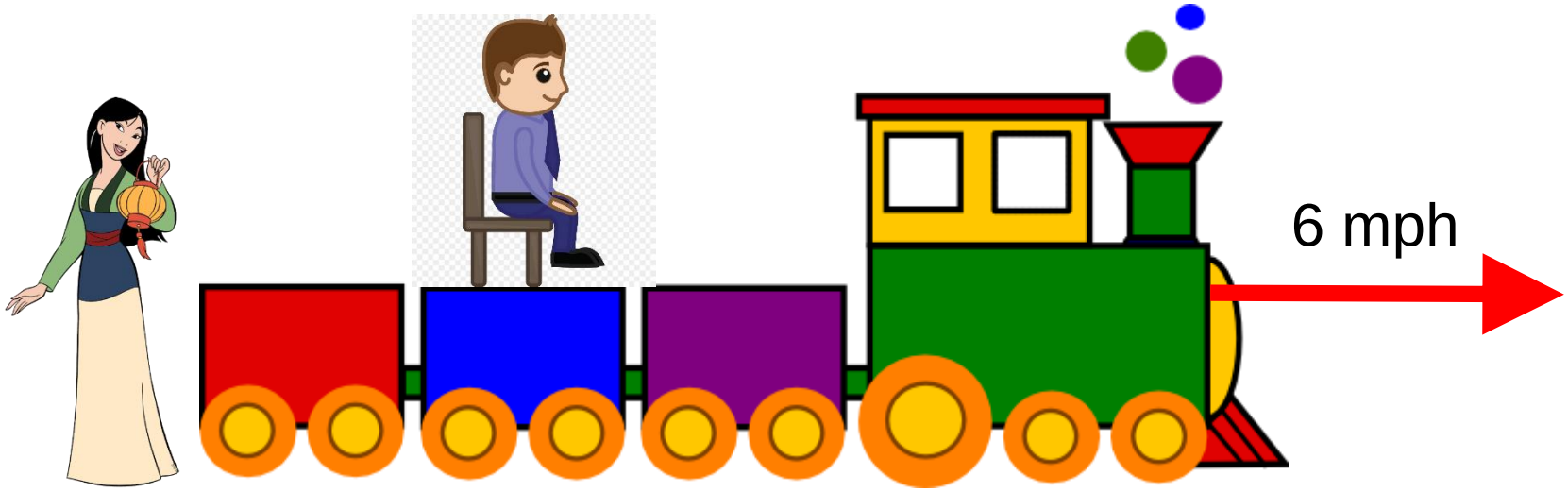
Running away from your mother (at 6 mph) on a stationary train. How fast are you going relative to your mother?

6 mph



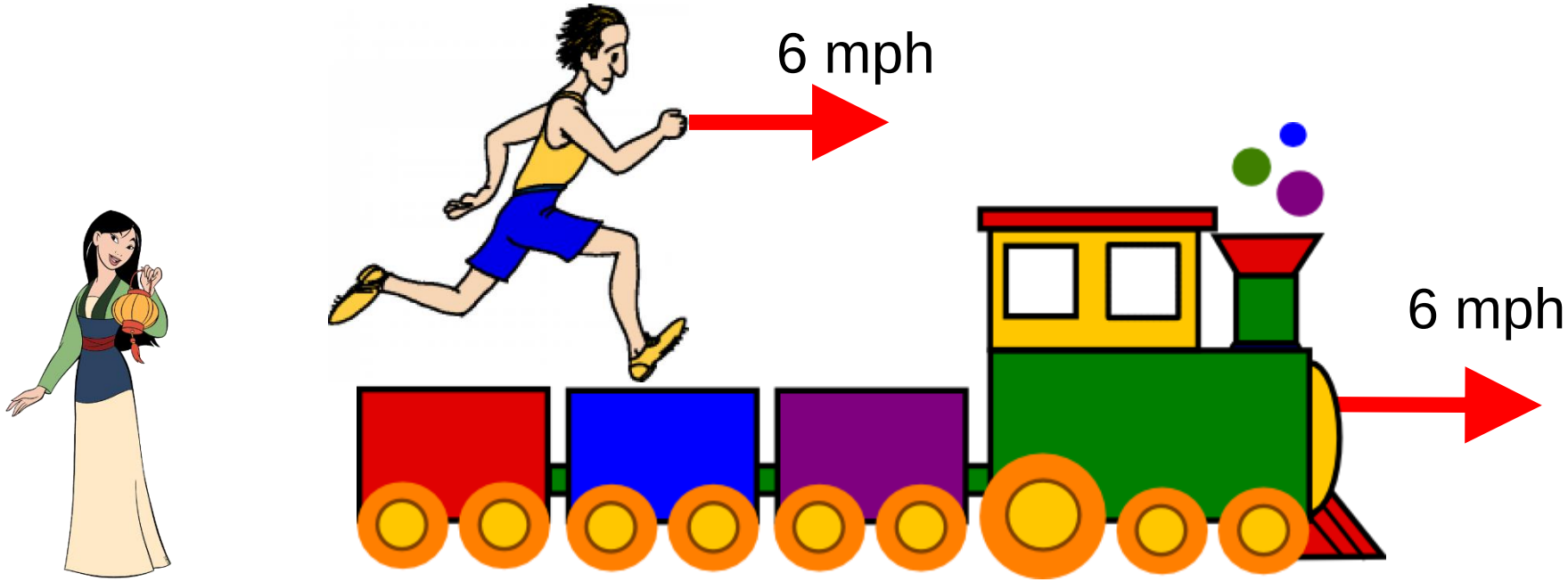
Sitting on a train going 6 mph. How fast are you going relative to your mother?

6 mph



Running away from your mother (at 6 mph) on a train that is also going 6 mph. How fast are you going relative to your mother?

12 mph



Can rates be added? ***YES!***

$$Rate_{you} + Rate_{train} = Rate_{you\ \&\ train}$$

Jose takes 3 hours to clean a house
(time rate of cleaning: one house per 3 hours → $Rate_{Jose} = \frac{1\ house}{3\ hours}$

George takes 4 hours to clean a house
(time rate of cleaning: one Job per 4 hours → $\boxed{Rate_{George} = \frac{1\ house}{4\ hours}}$

How Long for both to clean one house by working together?

Rate George + Rate Jose = Combined Rate (George & Jose) $\boxed{Rate_G + Rate_J = Rate_{G+J}}$

$$\frac{1\,\text{house}}{4\,\text{hrs}} + \frac{1\,\text{house}}{3\,\text{hours}} = \frac{1\,\text{house}}{t\,\text{hours}} \qquad \frac{1}{4} + \frac{1}{3} = \frac{1}{t}$$

Multiply by the common denominator

$$\frac{12t}{4} + \frac{12t}{3} = \frac{12t}{t} \quad \text{simplify} \quad 3t + 4t = 12 \qquad 7t = 12 \qquad t = 12/7$$

$$t = 1.7\ \text{hrs}$$

James, Adam and Paul can paint a room together in 2 hours. If Adam does the job alone he can paint the room in 5 hours. If Paul works alone, he can paint the room in 6 hours. If James works alone, how long would it take him to paint the room?

$$\text{rate}_{\text{Pria, Jamie, Paul}} = \frac{room}{2\,\text{hrs}} = \frac{room}{5\,\text{hrs}} + \frac{room}{6\,\text{hrs}} + \frac{room}{\text{"t" hrs}}$$

$$\text{rate}_{\text{Pria}} = \frac{room}{5\,\text{hrs}} \qquad \frac{room}{2\,\text{hrs}} = \frac{room}{5\,\text{hrs}} + \frac{room}{6\,\text{hrs}} + \frac{room}{\text{"t" hrs}}$$

$$\text{rate}_{\text{Paul}} = \frac{room}{6\,\text{hrs}} \qquad \frac{1}{2} = \frac{1}{5} + \frac{1}{6} + \frac{1}{\text{t}} \qquad 5t = 2t + \frac{10t}{6} + \frac{10}{1}$$

$$\frac{t}{2} = \frac{t}{5} + \frac{t}{6} + \frac{1}{1} \qquad 30t = 12t + 10t + 60$$

$$t = \frac{2t}{5} + \frac{2t}{6} + \frac{2}{1} \qquad 8t = 60 \qquad t = 7.5\,\text{hrs}$$

How would you describe “percent”?

$$grade = \frac{points\ earned}{total\ points}$$

$$\boxed{\% = \frac{partial\ amount}{total\ amount}}$$

In Chemistry, what does the “*concentration*” mean?

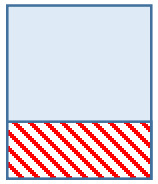
$$\boxed{concentration\ (\%) = \frac{volume\ of\ a\ specific\ chemical}{total\ volume}}$$

Mixture Problem: mixtures of various concentrations of solutions.

30 ml. of a 20% saline (salt/water) solution is mixed with 50 ml. of a 60% saline solution
What is the concentration of the mixture?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{volume_{\text{pure}}}{volume_{\text{total}}}$$

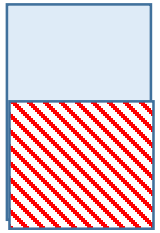
30 ml



20%

30(0.2) = 6 ml

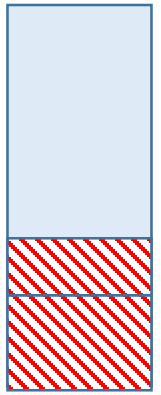
50 ml



60%

50(0.6) = 30 ml

30 ml + 50 ml
= 80 ml



6 ml

30 ml

= 36 ml

$$x\% = \frac{36\ ml}{80\ ml} = 0.45 = 45\%$$

Mixture Problem: mixtures of various concentrations of solutions, allows, items, etc.

30 ml. of a 20% saline (salt/water) solution is mixed with 50 ml. of a 60% saline solution. What is the concentration of the mixture?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{weight_{\text{pure}}}{weight_{\text{total}}}$$

	A	B	A & B
Part	**6 ml**	**30 ml**	**36 ml**
Whole	30 ml	50 ml	80 ml
%	0.2	0.6	0.45

$$20\% = 0.2 = \frac{x}{30ml}$$

$$x = 6\ \text{ml (salt)}$$

$$60\% = 0.60 = \frac{x}{50ml}$$

$$x = 30\ \text{ml (salt)}$$

Add the column values to get the total

$$\%\text{pure} = \frac{36}{80} = 0.45 = 45\%$$

You *cannot add percents!*

You must *calculate the percent concentration.*

5 gallons of a 20% acid mixture was added to 10 gallons of an unknown mixture. The resulting mixture concentration was 26.7%. What was the concentration of the 10 gallon mixture?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{Vol_{\text{pure}}}{Vol_{\text{mixture}}}$$

	A	B	A & B
Part	**1 gal**	**10x gal**	**1+10x**
Whole	5 gal	10 gal	15 gal
%	0.2	x	0.267

Add the column values to get the total

You *cannot add percents!*

You must *calculate the percent concentration.*

5(0.2) = 1 gal

$$26.7\% = 0.267 = \frac{1 + 10x}{15}$$

$$0.267(15) = 1 + 10x$$

$$4.005 = 1 + 10x$$

$$3.005 = 10x$$

$$0.3005 = x \quad = 30.05\%$$

Mixture Problem: mixtures of various concentrations of solutions, allows, items, etc.

75 ml. of a 30% saline (salt/water) solution is mixed with 65 ml. of a 45% saline solution. What is the concentration of the mixture?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{weight_{\text{pure}}}{weight_{\text{total}}}$$

	A	B	A & B
Part	**22.5 ml**	**29.25 ml**	**51.75 ml**
Whole	75 ml	65 ml	140 ml
%	0.3	0.45	0.3693

Mixture Problem: mixtures of various concentrations of solutions, allows, items, etc.

How much of a 40% saline (salt/water) solution must be mixed with 35 ml. of a 25% saline solution to get a solution with 30% concentration?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{weight_{\text{pure}}}{weight_{\text{total}}}$$

	A	B	A & B
Part	**0.4x ml**	**8.75 ml**	**0.4x + 8.75 ml**
Whole	x ml	35 ml	x + 35 ml
%	0.4	0.25	0.3

$$0.3 = \frac{0.4x + 8.75}{x + 35}$$

$$0.3(x + 35) = 0.4x + 8.75$$

$$0.3x + 10.5 = 0.4x + 8.75$$

$$1.75 = 0.1x$$

$$17.5 = x$$

Mixture Problem: mixtures of various concentrations of solutions, allows, items, etc.

How much of a ***pure*** (100%) grape juice must be added to 2 quarts of 35% grape juice mixture to yield 65% grape juice mixture?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{weight_{\text{pure}}}{weight_{\text{total}}}$$

	A	B	A & B
Part	**x qt**	**0.7 qt**	**x + 0.7 qt**
Whole	x qt	2 qt	x + 2 qt
%	1	0.35	0.65

$$0.65 = \frac{x + 0.7}{x + 2}$$

$$0.65(x + 2) = x + 0.7$$

$$0.65x + 1.3 = x + 0.7$$

$$0.6 = 0.35x$$

$$1.71 = x$$

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{Vol_{\text{pure}}}{Vol_{\text{mixture}}}$$

3 gallons of an unknown mixture concentration was added to 4 gallons of a 15% acid mixture. The resulting mixture concentration was 20.5%. What was the concentration of the 3 gallon mixture?

	A	B	A & B
Part	**3x gal**	**0.6 gal**	**3x + 0.6 gal**
Whole	3 gal	4 gal	7 gal
%	x	0.15	0.205

$$0.205 = \frac{3x + 0.6}{7}$$

$$0.8353 = 3x$$

$$1.4353 = 3x + 0.6$$

$$0.2783 = x$$

Metal Alloy: a mixture of different metals. For example “rose gold” is a mixture of copper (reddish color) with gold (yellow color). “Yellow gold” is a mixture of silver and gold. The purity of gold alloy is measured in “carats”.

The a pure substance is mixed with a “filler” we call the ratio of the pure substance to the total amount the concentration.

carats	% Gold
24	100
18	75
12	50
6	25

$$\%\,\text{concentration}_{\text{gold}} = \frac{part_{gold}}{whole_{mixture}} = \frac{weight_{\text{gold}}}{weight_{\text{total}}}$$