

Math-3

Lesson 3-7

“Holes” and the “Oblique Asymptote”

$$y = \frac{(x + 4)}{(x - 1)} \quad y = \frac{\cancel{(x + 4)}(x - 3)}{\cancel{(x - 1)}(x - 3)}$$

Zeroes of the numerator are x-intercepts (or are imaginary)

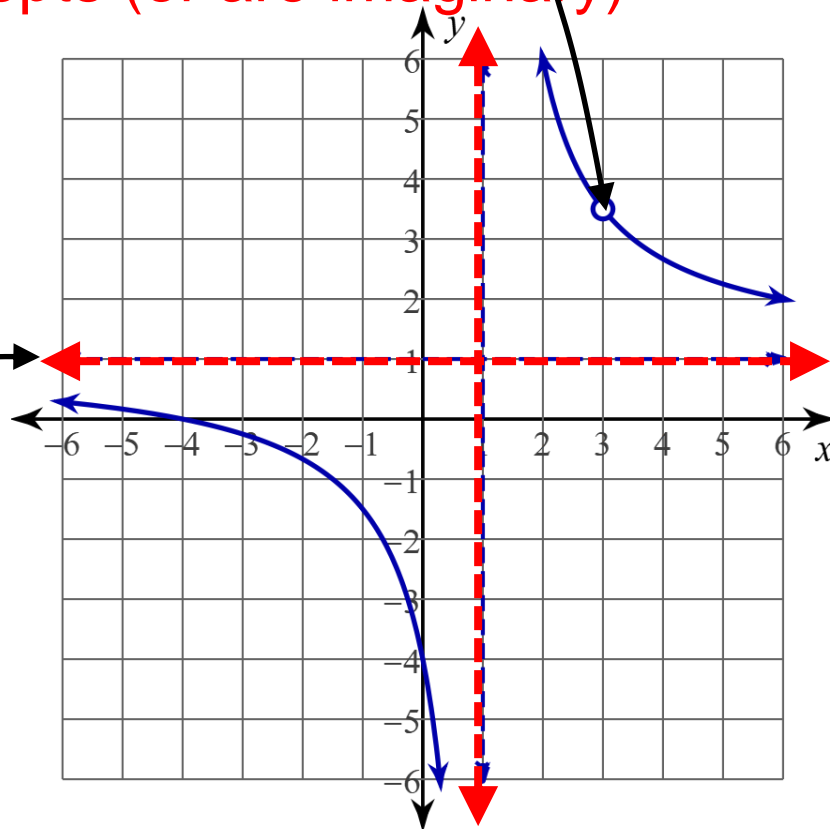
Zeroes of the denominator are either

- (1) Vertical asymptotes or
- (2) Holes

The 'quotient' when you divide a rational equation determines the 'end behavior.'

$$y = \frac{5}{(x - 1)} + 1$$

$$\begin{array}{r} x - 1 \overline{) x + 4} \\ \underline{-(x - 1)} \\ 5 \end{array}$$



The 'quotient' when you divide a rational equation determines the 'end behavior.'

$$y = \frac{x^3}{x^2 - 4}$$

$$\begin{array}{r} x \\ x^2 - 4 \overline{) x^3} \\ \underline{-(x^3 - 4x)} \\ 4x \end{array}$$

$$y = x + \frac{4x}{x^2 - 4}$$

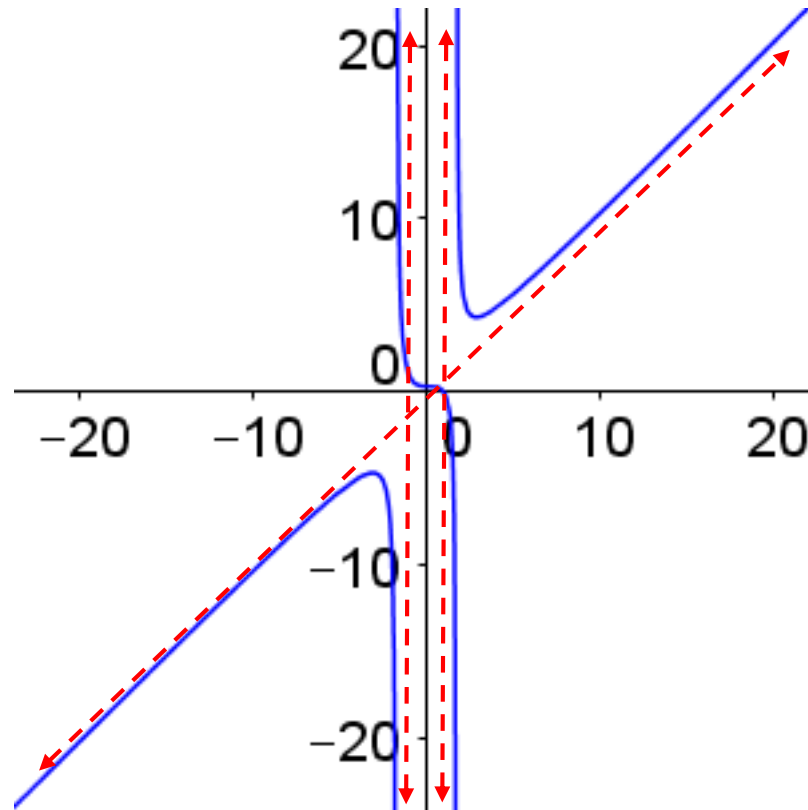
Oblique asymptote:

$$y = x$$

VA:

$$x = -2$$

$$x \dots = 2$$



Will there be a horizontal asymptote?

$$g(x) = \frac{x^2 + 2x + 4}{x - 1}$$

Use long division.

$$g(x) = x + 3 + \frac{7}{x - 1}$$

$$\begin{array}{r} x-1 \overline{) x^2 + 2x + 4} \\ \underline{-(x^2 - x)} \\ 3x + 4 \\ \underline{-(3x - 3)} \\ 7 \end{array}$$

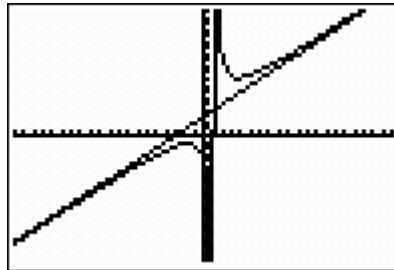
The fraction approaches zero.

$$X \rightarrow \infty \quad g(x) \rightarrow x + 3$$

Oblique asymptote: $y = x + 3$

$$g(x) = x + 3 + \frac{7}{x - 1}$$

$$X \rightarrow \infty \quad y \rightarrow ?$$



```
Plot1 Plot2 Plot3
\Y1=(X^2+2X+4)/(X
-1)
\Y2=X+3
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
Xmin=20
Xmax=20
Xscl=1
Ymin=-20
Ymax=20
Yscl=1
Xres=1
```

find the oblique asymptote

$$g(x) = \frac{6x^2 + 5x - 2}{3x - 2}$$

Use long division.

$$g(x) = 2x + 3 + \frac{4}{3x - 2}$$

The fraction approaches zero.

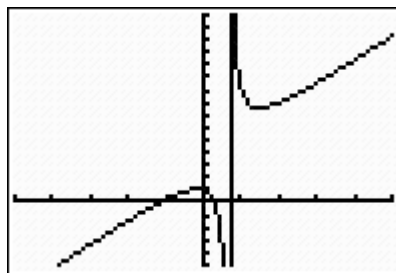
$$X \rightarrow \infty \quad g(x) \rightarrow 2x + 3$$

Oblique asymptote: $y = 2x + 3$

$$\begin{array}{r} 2x+3 \\ 3x-2 \overline{) 6x^2 + 5x - 2} \\ \underline{-(6x^2 - 4x)} \\ 9x - 2 \\ \underline{-(9x - 6)} \\ 4 \end{array}$$

$$g(x) = 2x + 3 + \frac{4}{3x - 2}$$

$$X \rightarrow \infty \quad y \rightarrow ?$$

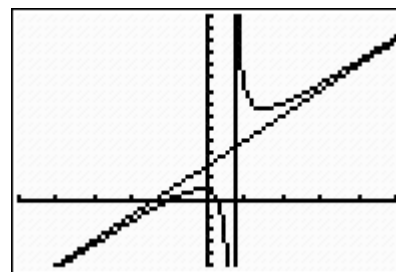


```

Plot1 Plot2 Plot3
Y1=(6X^2+5X-2)/(3X-2)
Y2=2X+3
Y3=
Y4=
Y5=
Y6=
    
```

```

WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-1
Ymax=1
Yscl=1
Xres=1
    
```



$$y = \frac{6(x+4)(x-1)(x+2)}{2(x-1)(x+3)}$$

x-intercept? $(-4, 0), (-2, 0)$

y-intercept? $(0, 8)$

Vertical asymptote? $x = -3$

holes? $f(1) = 45/4$

Oblique asymptote? $y = 3x + 9$

$$y = \frac{6(x+4)(x+2)}{2(x+3)}$$

$$\begin{array}{r} 3x + 9 \\ \hline 2x + 6 \overline{) 6x^2 + 36x + 48} \\ \underline{-(6x^2 + 18x)} \\ 18x + 48 \\ \underline{-(18x + 54)} \\ -6 \end{array}$$

$$g(x) = 2x + 3 + \frac{4}{3x-2}$$

$$y = 3x + 9 + \frac{-6}{2x+6}$$

$x \rightarrow \infty \quad y \rightarrow ?$

$$y = 3x + 9 + \frac{-3}{x+3}$$

Your turn: find the asymptote

$$g(x) = \frac{x^3 + 2x^2 + 3x - 4}{x + 2}$$

Use long division.

$$g(x) = x^2 + 3 - \frac{10}{x + 2}$$

$$\begin{array}{r} x^2 + 3 \\ x + 2 \overline{) x^3 + 2x^2 + 3x - 4} \\ \underline{-(x^3 + 2x^2)} \\ 3x - 4 \end{array}$$

The fraction approaches zero.

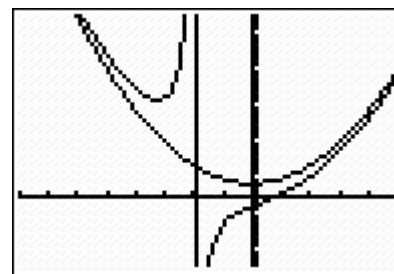
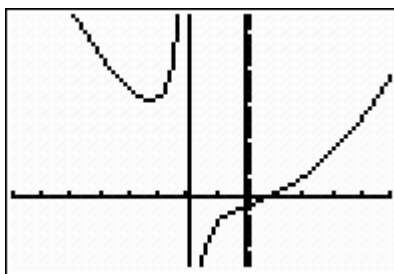
$$X \rightarrow \infty \quad g(x) \rightarrow x^2 + 3$$

Oblique asymptote: $y = x^2 + 3$

$$\begin{array}{r} 3x - 4 \\ \underline{-(3x + 6)} \\ -10 \end{array}$$

$$g(x) = x^2 + 3 - \frac{10}{x + 2}$$

$$X \rightarrow \infty \quad y \rightarrow ?$$



```

WINDOW
Xmin=-8
Xmax=5
Xscl=1
Ymin=-15
Ymax=40
Yscl=1
Xres=1
    
```

```

Plot1 Plot2 Plot3
Y1=(X^3+2X^2+3X-4)/(X+2)
Y2=X^2+3
Y3=
Y4=
Y5=
Y6=
    
```