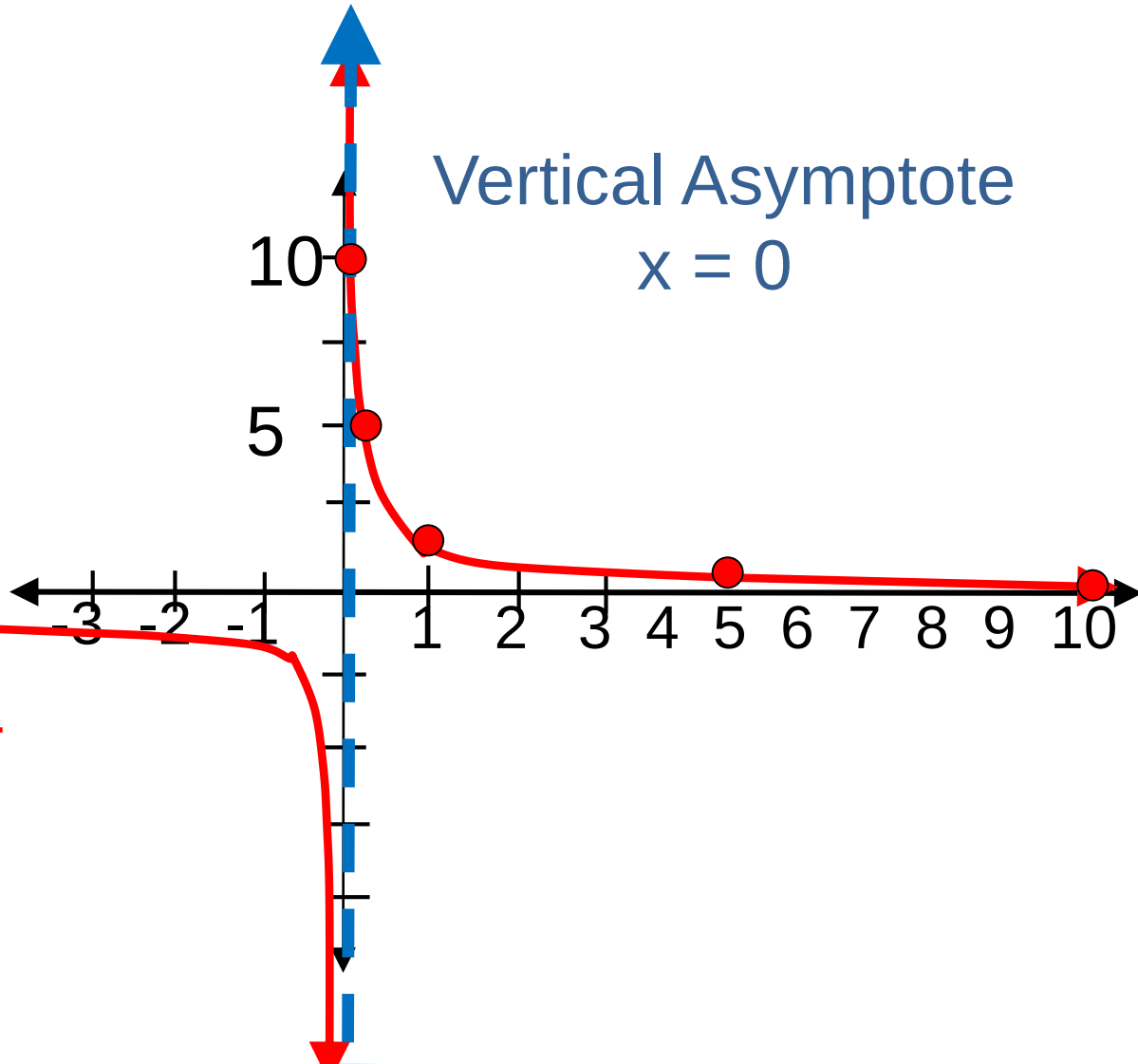
Math-3
Lesson 3-5
The Reciprocal Function

Reciprocal Function

$$f(x) = \frac{1}{x}$$

x	f(x)
1/10 = 0.1	10
1/5 = 0.2	5
1	1
5	1/5 = 0.2
10	1/10 = 0.1
0	1/0 = ??

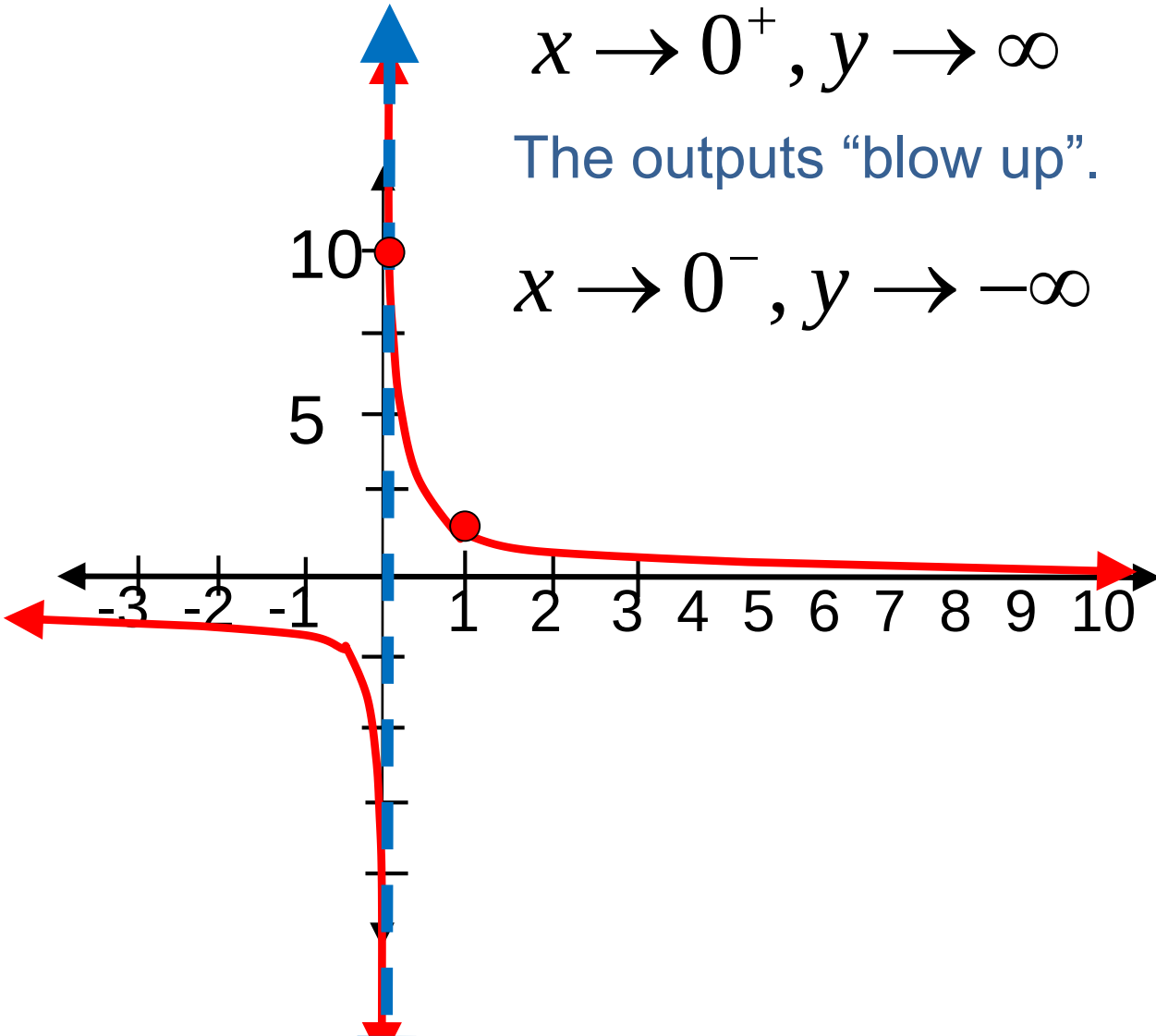


$$f(x) = \frac{1}{x}$$

Why is there a vertical asymptote?

What is the output when we "approach" x = 0 from the "+" side?

x	f(x)
1	1
0.1	10
0.01	100
0.001	1000
10^{-4}	$10^{4} = 10{,}000$
10^{-12}	10^{12}

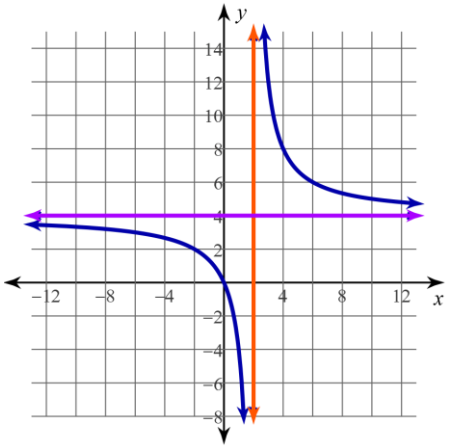


$$x \to 0^{+}, y \to \infty$$

The outputs "blow up".

$$x \to 0^{-}, y \to -\infty$$

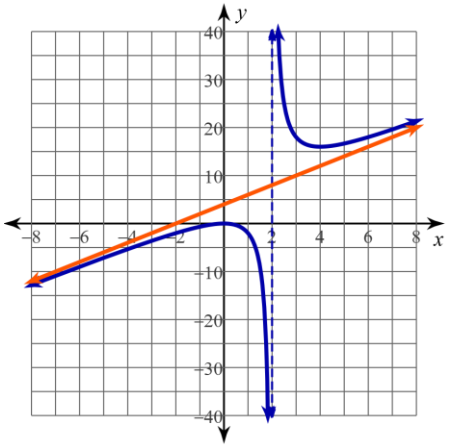
Horizontal Asymptote: An *end-behavior* that is a horizontal line that the graph approaches but NEVER reaches.



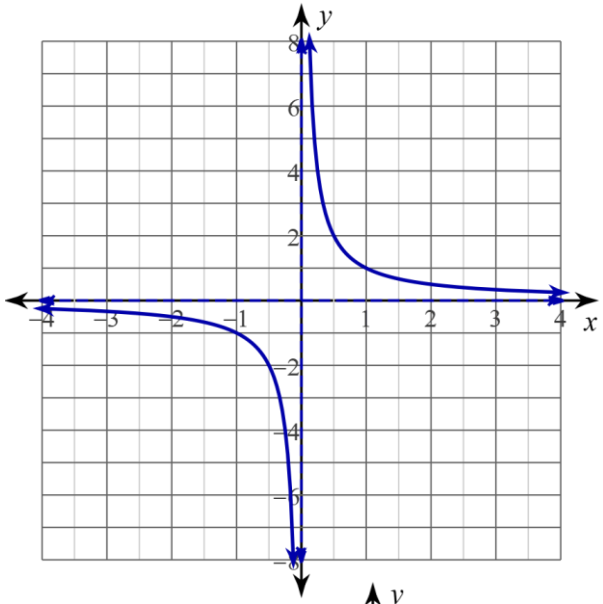
Asymptotes are not part of the graph but you can see them easily. We show them as dotted lines.

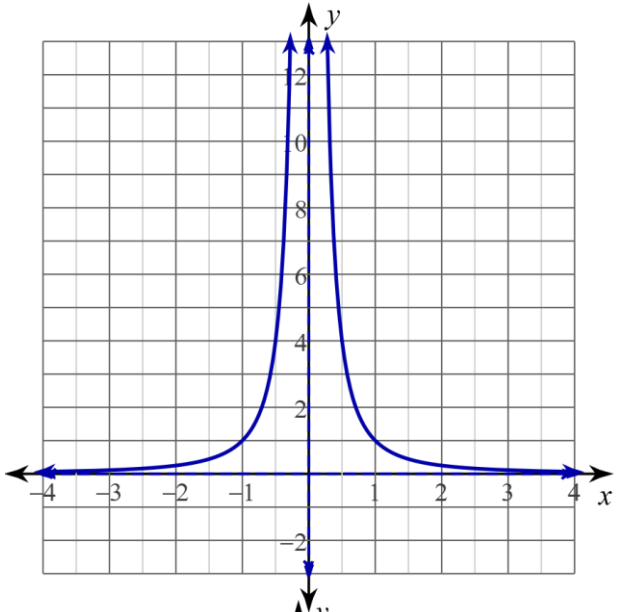
Vertical Asymptote: (a vertical line the graph approaches but never reaches) is caused by a zero of the denominator that does NOT disappear due to simplification.

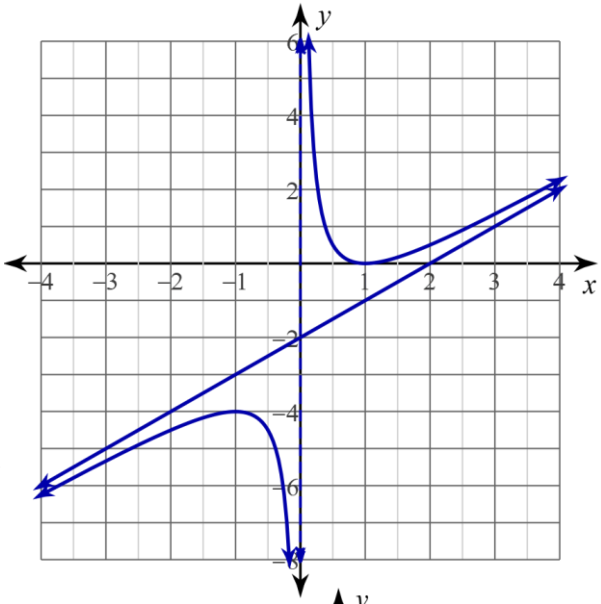
Oblique (Slant) Asymptote: We'll learn about how that happens in our next lesson.

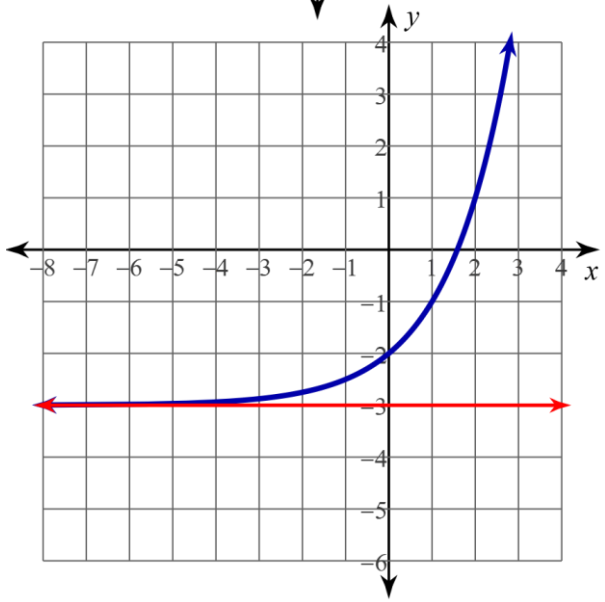


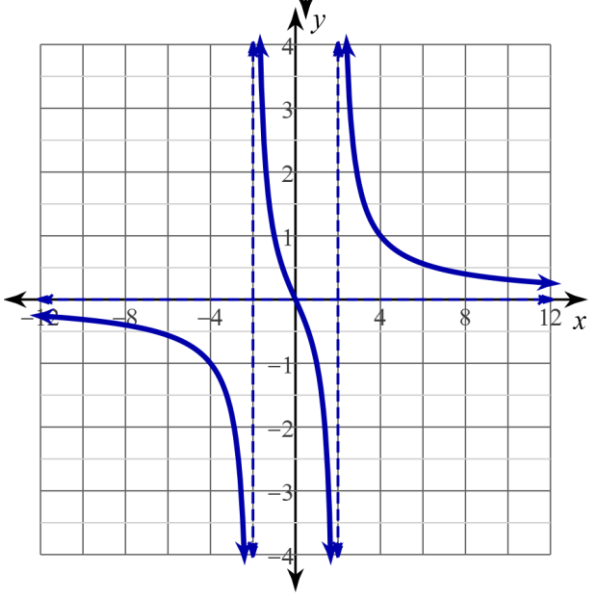
Asymptotic Behavior: the graph must clearly show the graph approaching but never reaching the graph.

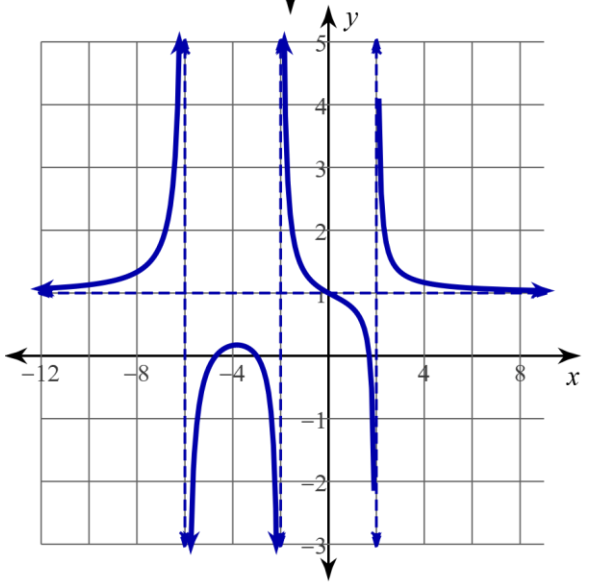












Fractions and the number zero. $y = \frac{1}{x}$

Can the denominator a fraction equal to zero?

$$0 = \frac{1}{x} \rightarrow 0 * x = \frac{1}{x} * x \rightarrow 0 = 1$$

There is no solution to this equation → the denominator can never make a fraction equal to zero.

What part of the fraction makes it equal to zero? $y = \frac{m}{x}$

$$0 = \frac{m}{x} \rightarrow 0 * x = \frac{m}{x} * x \rightarrow 0 = m$$

Only the a zero of the numerator can make a fraction equal zero. $\frac{0}{3}$

Fractions and the number zero. $y = \frac{1}{x}$

Division by zero is not a number.

→ Vertical asymptote: x = 0.

Only the a zero of the numerator can make a fraction equal zero.

Is there any input value for 'x' that will make the numerator = 0?

The output value "y" of this function will never equal zero.

→ Horizontal asymptote: y = 0.

$$f(x) = \frac{1}{x}$$

Domain?

Domain:

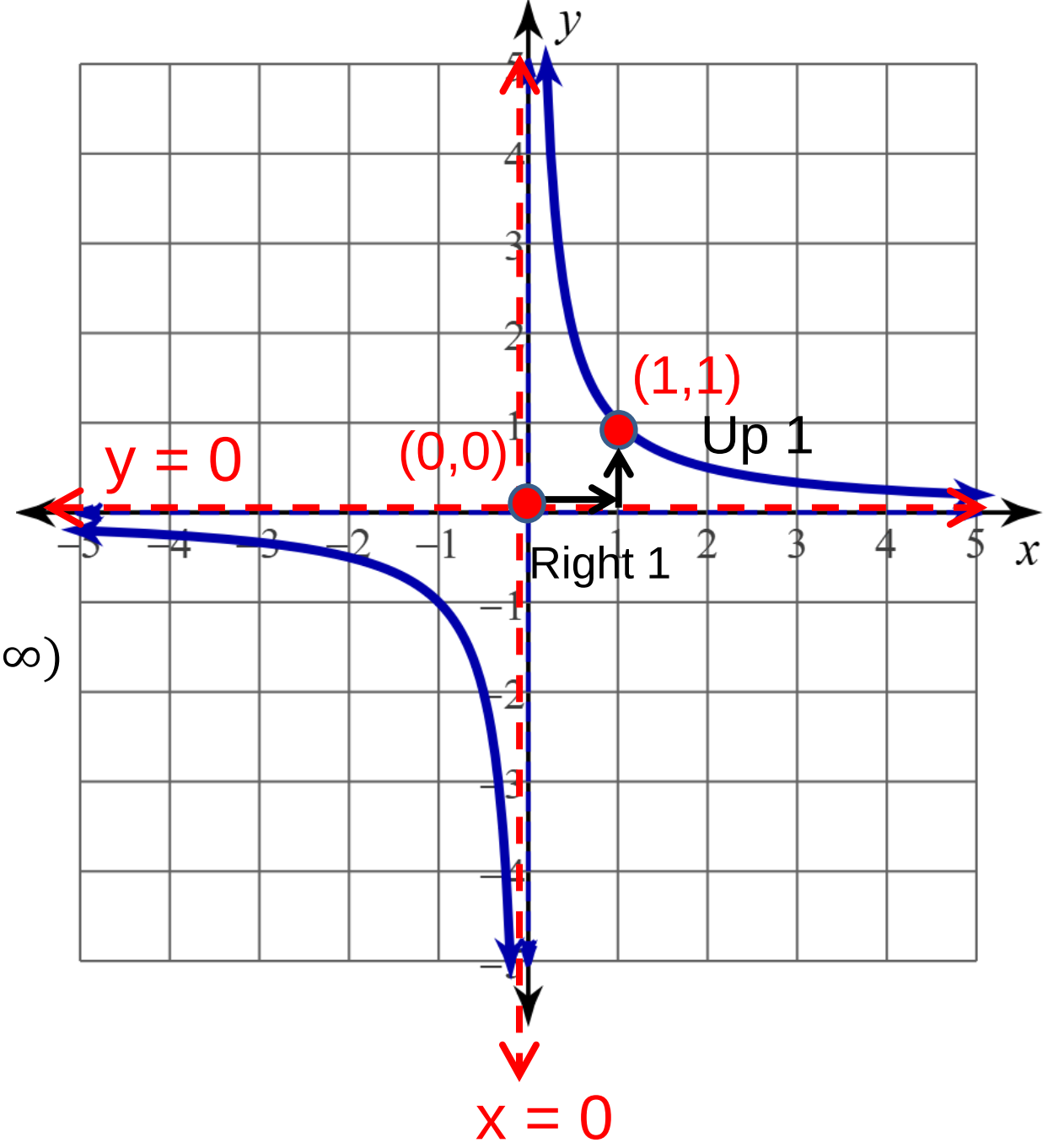
$x = (-\infty, 0) \cup (0, \infty)$

Domain: $x \neq 0$

Range?

Range: $y = (-\infty, 0) \cup (0, \infty)$

Range: $y \neq 0$



Generalized Transformations of the Square Function:

$f(x) = x^2$ $\qquad$ $y = (-1)a(x-h)^2 + k$

Reflection across x-axis

vertical stretch factor

Translates left/right

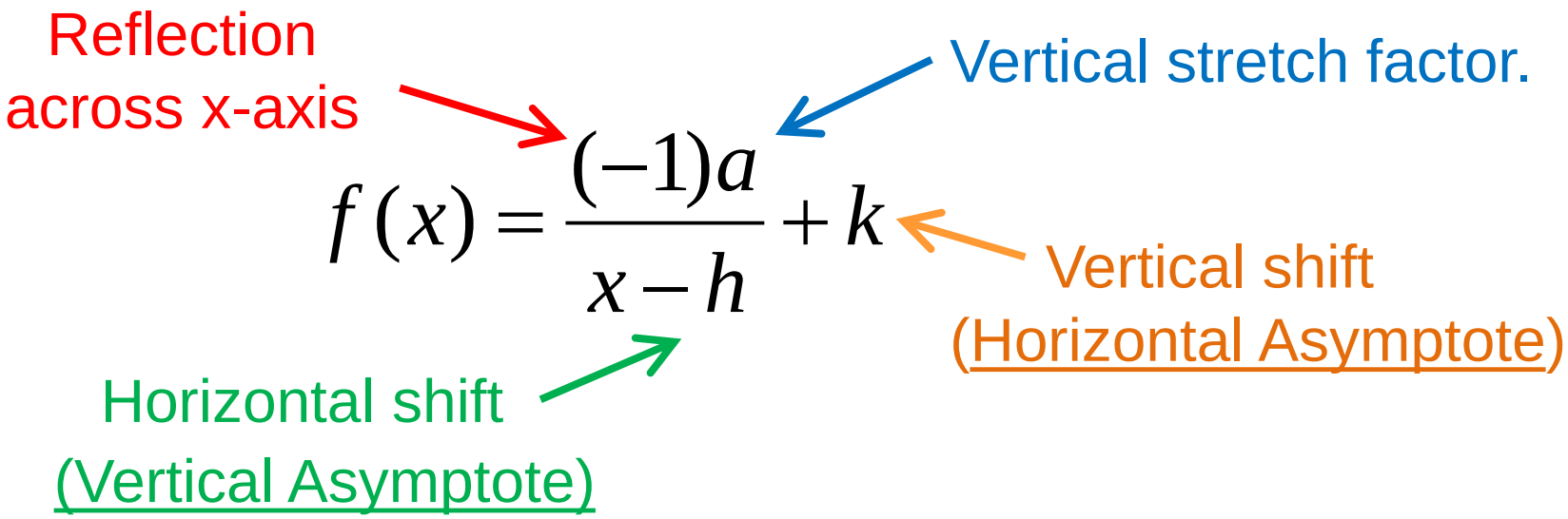
translating up or down

$$y = -2(x-3)^2 + 4$$

Reflected (x-axis), VSF=2, right 3, up 4

Vertex: $(3, 4)$

Reciprocal Function General Transformation Equation



(h, k) The point of intersection of the vertical and horizontal asymptotes.

$Domain: x \neq h$ $Range: y \neq k$

a) Describe the transformations of the reciprocal function.
b) What is the intersection of the asymptotes?
c) What is the horizontal asymptote?
d) What is the vertical asymptote?
e) What is the domain?
f) What is the range?

$$g(x) = \frac{1}{x} + 7$$

(a) Up 7
(b) (0, 7)
(c) x = 0
(d) y = 7
(e) x ≠ 0
(f) y ≠ 7

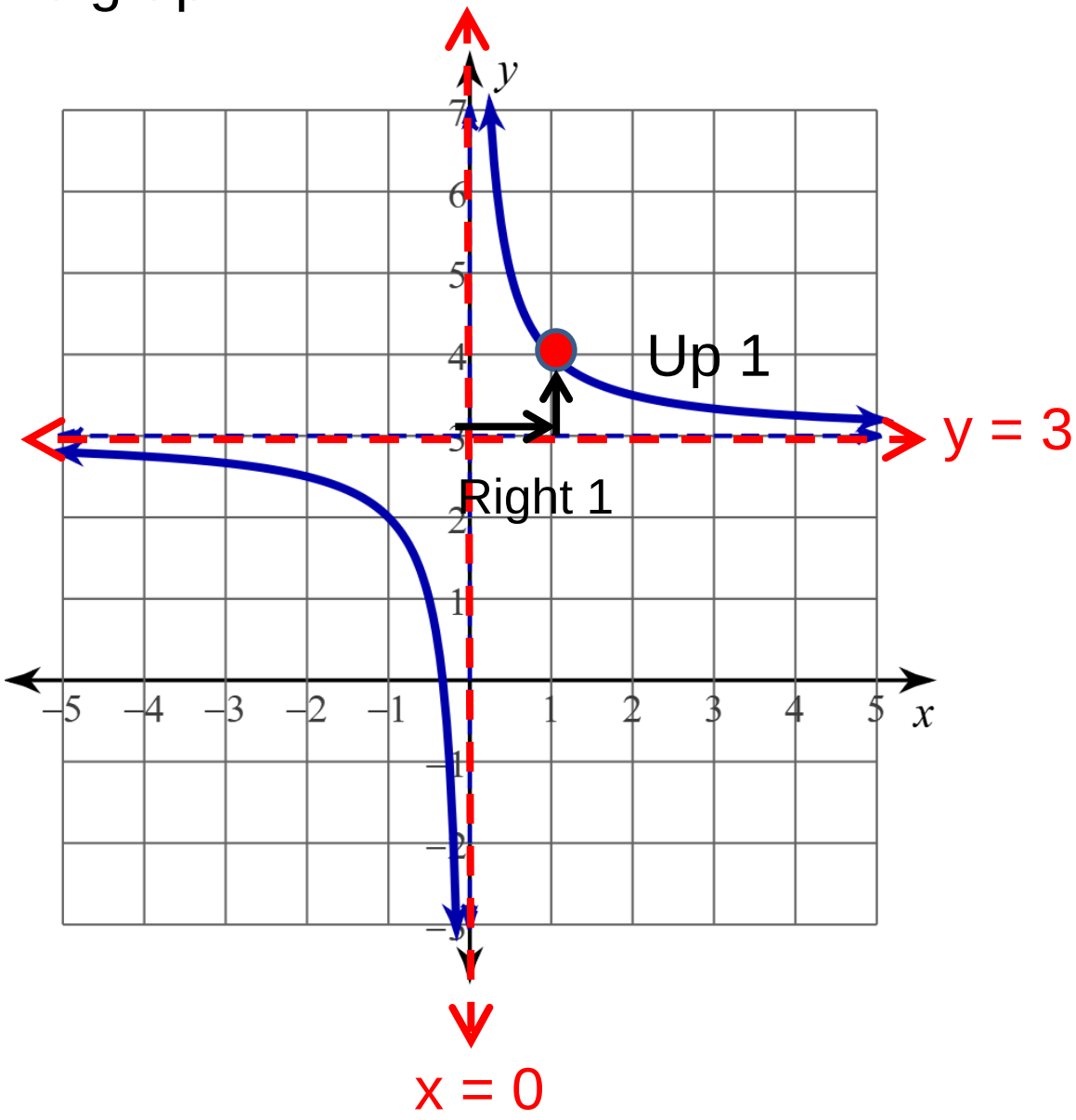
$$f(x) = \frac{-3}{(x+3)} - 5$$

(a) Reflect (x-axis), left 3, down 5
(a) (-3, -5)
(b) x = -3
(c) y = -5
(d) x ≠ -3
(e) y ≠ -5

What is the equation of the graph?

$$f(x) = \frac{1}{x}$$

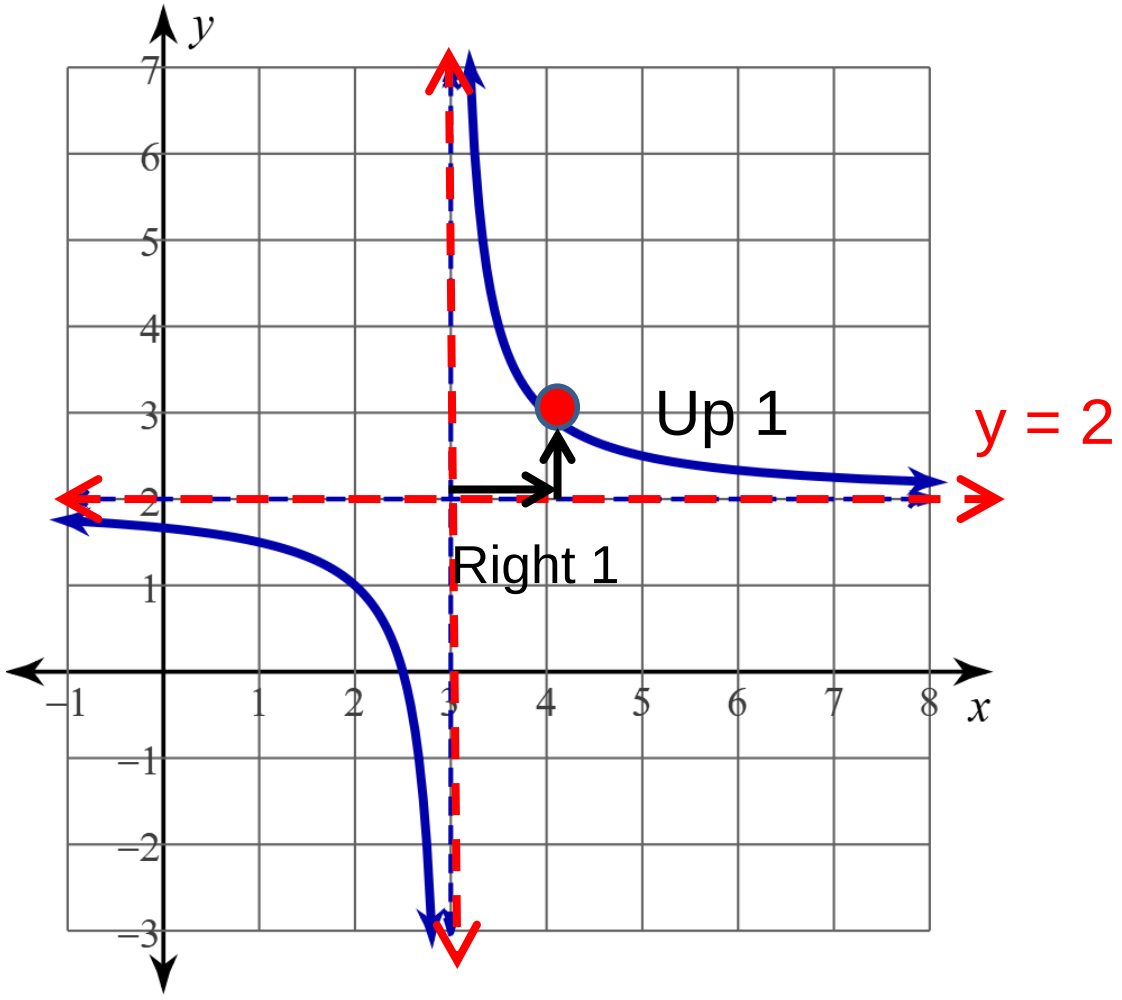
$$g(x) = \frac{1}{x} + 3$$



What is the equation of the graph?

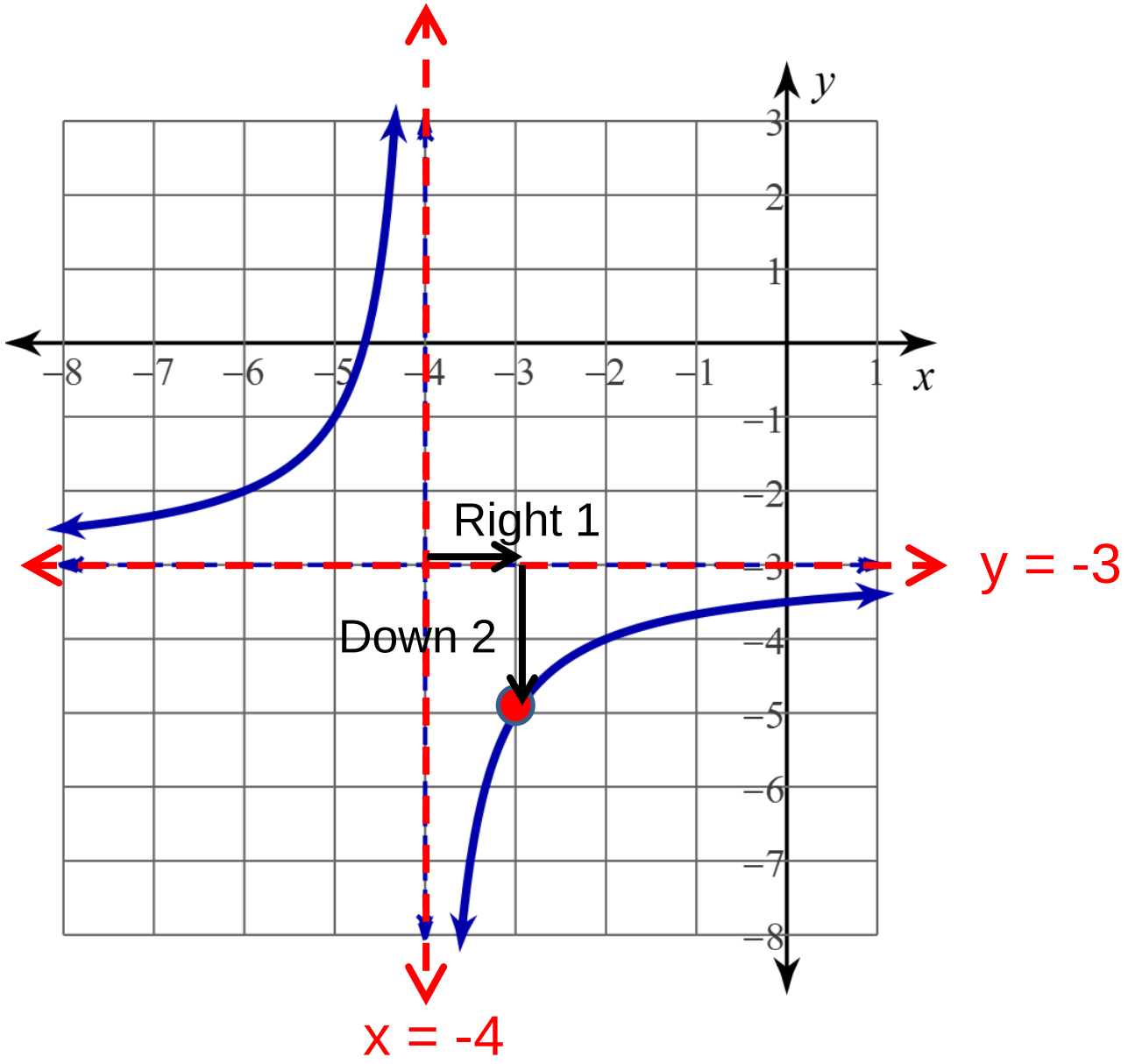
$$f(x) = \frac{1}{x}$$

$$g(x) = \frac{1}{(x-3)} + 2$$



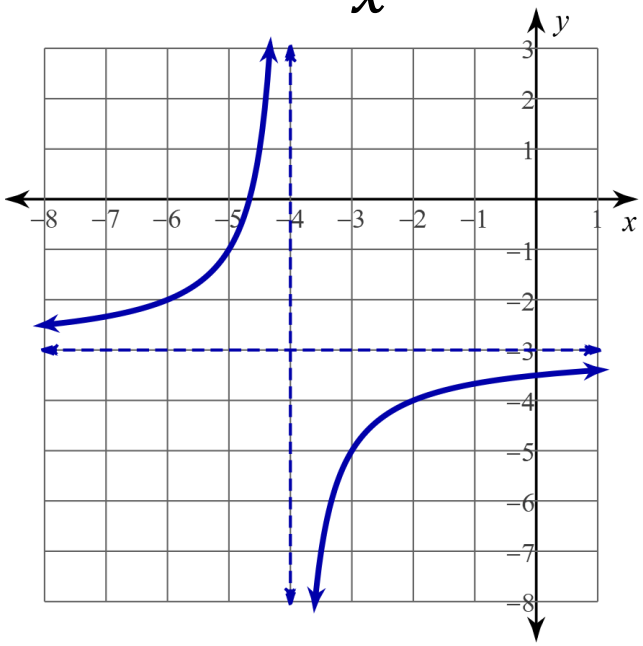
$$f(x) = \frac{1}{x}$$

$$g(x) = \frac{-2}{(x+4)} - 3$$



Another way to understand the horizontal asymptote:

$$g(x) = \frac{1}{x} + 3$$



End behavior!

On right end of the graph, y-value approaches the horizontal asymptote.

If the x-value is ***very*** large, what does y-value approach?

$$g(x) = \frac{1}{x} + 3$$

$$\frac{1}{10} + 3 = 3.1 \qquad \frac{1}{100} + 3 = 3.01 \qquad \frac{1}{1000} + 3 = 3.001$$

(Remember from Math-2), right end behavior is given by:

$$g(x) = \frac{1}{x} + 3 \qquad x \to \infty, \;\; g(x) \to ? \qquad g(x) = \frac{1}{x} + 3 \to 0$$

$$x \to \infty, \;\; g(x) \to 3$$

Horizontal/Oblique Asymptote: the quotient when you divide.

$$g(x) = \frac{2x}{x-3}$$

$$x-3\overline{)2x}$$

$$\begin{array}{r} 2 \\ x-3\overline{)2x} \\ -(2x-6) \\ \hline 6 \end{array}$$

$$g(x) = 2 + \frac{6}{x-3}$$

$x \to \infty, \ g(x) \to ?$

$$g(x) = 2 + \frac{6}{x-3}$$

0

$$x \to \infty, \quad g(x) \to 2$$

Horizontal/Oblique Asymptote: y = 2

Horizontal/Oblique Asymptote: the quotient when you divide.

$$g(x) = \frac{2x}{x-3} \qquad g(0) = \frac{2(0)}{(0)-3} \qquad g(0) = \frac{0}{-3} = 0$$

x-intercept (zero of the numerator) (x, y) = (0, 0)

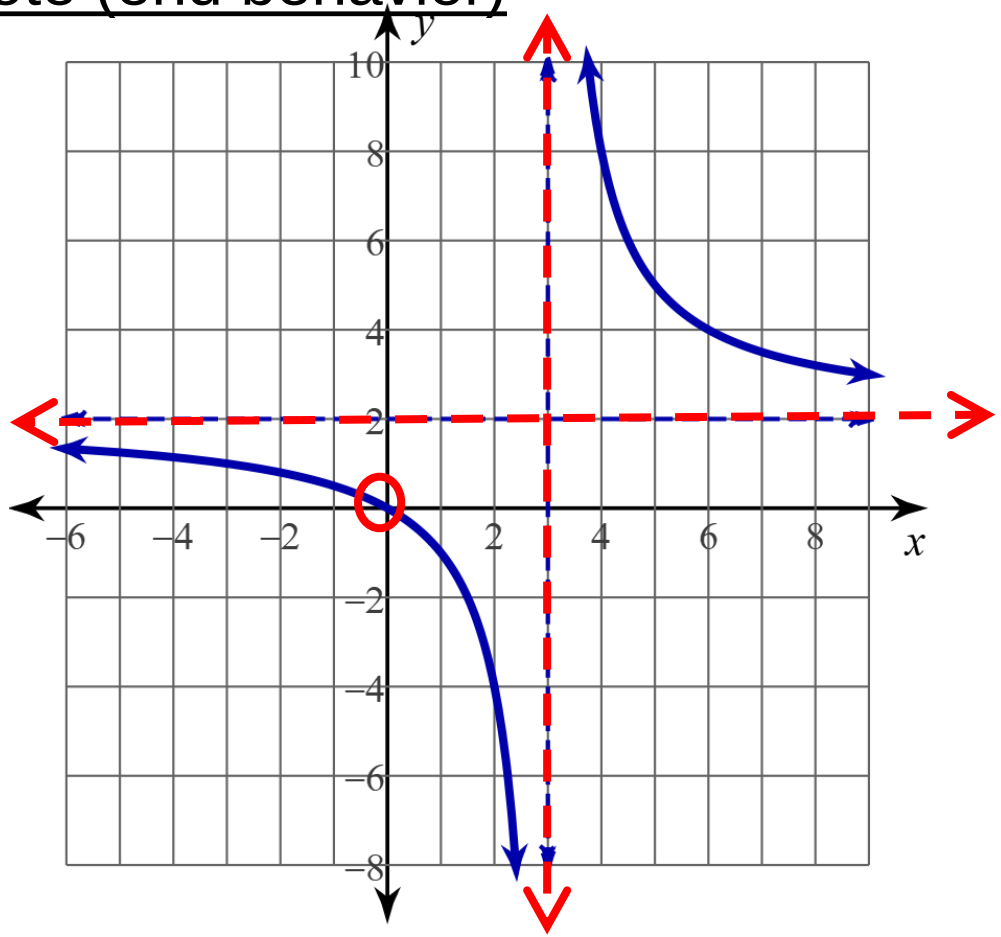
Horizontal/Oblique Asymptote (end behavior)

$x \to \infty,\ g(x) \to 2$ y = 2

Vertical Asymptote (excluded value):

x = 3

$$g(x) = \frac{6}{x-3} + 2$$



Asymptotes cross (3, 2)

VSF = 6

$$g(x) = \frac{3x - 6}{x - 4}$$

x-intercept = ?

$$0 = \frac{3x - 6}{x - 4}$$

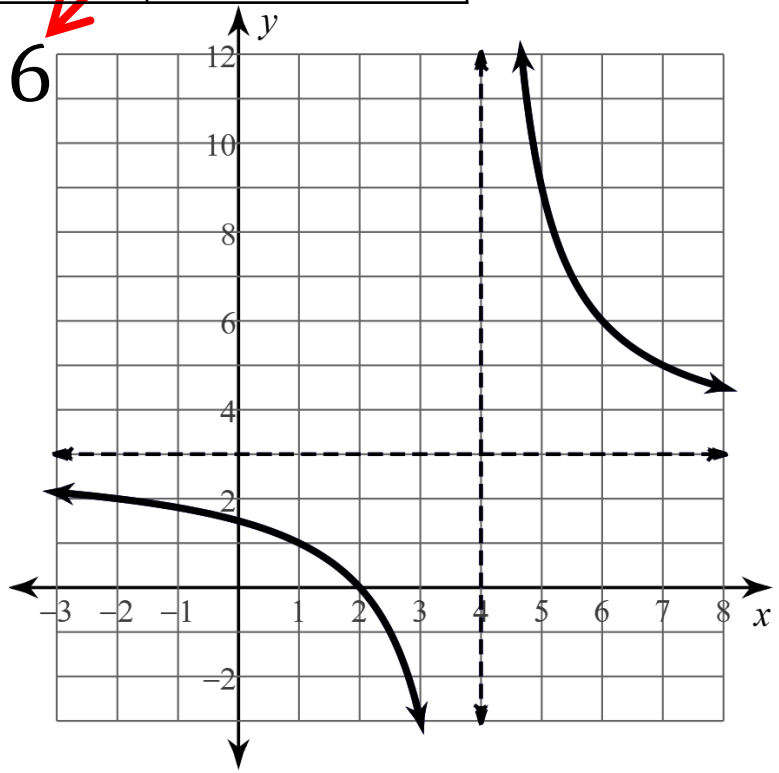
$$0 = 3x - 6$$

$$2 = x$$

Horizontal Asymptote?

	3	remainder
x	$3x$	6
-4	-12	

$3x - 6$



Vertical Asymptote ?

$$0 \neq x - 4 \qquad 4 \neq x$$

$$g(x) = 3 + \frac{6}{x - 4}$$

$$g(x) = \frac{6}{x - 4} + 3$$

$$x \to \infty,\ g(x) \to 3$$

$$y = 3$$

Asymptotes cross (4, 3)

VSF = 6