Math-3

Unit 2 Weak Areas (Part 2)

“Nice” 3rd Degree Polynomial (with no constant term)

$$y = 3x^3 + 12x^2 - 36x$$

It has no constant term so it can easily be factored into ‘x’ times a quadratic factor.

$$y = 3x(x^2 + 4x - 12)$$

If the quadratic factor is “nice” we can factor that into 2 binomials.

$$y = 3x(x+6)(x-2)$$

x = 0, -6, 2

This is now “intercept form” so we can “read off” the x-intercepts. What are they?

Find the Zeroes

$$y = x^3 - 6x^2 - 16x$$

$$y = x^3 + 4x^2 - 12x$$

$$y = 2x^3 - 6x^2 - 30x$$

"Nice" 3rd Degree Polynomial (with no constant term)

$$y = x^3 + 6x^2 + 4x + 0$$

$$0 = x^3 + 6x^2 + 4x$$

It has no constant term so it can easily be factored into 'x' times a quadratic factor.

$$0 = x(x^2 + 6x + 4)$$

x = 0

What if the quadratic factor is not factorable?

Convert the quadratic factor into vertex form and solve.

$$x = \frac{-b}{2a} = \frac{-(6)}{2(1)}$$

$x = -3$

$$y = f(-3) = (-3)^2 + 6(-3) + 4$$

$y = -5$

$$0 = (x+3)^2 - 5$$

$$x = -3 \pm \sqrt{5}$$

Zeroes:

$x = 0, -3 \pm \sqrt{5}$

Find the zeroes using “box factoring”

$$y = 4x^3 - 5x^2 + 12x - 15$$

$$0 = (x^2 + 3)(4x - 5)$$

$$0 = x^2 + 3 \qquad 4x - 5 = 0$$

$$-3 = x^2 \qquad 4x = 5$$

$$x = \pm i\sqrt{3} \qquad x = {}^5/_4$$

$$x = i\sqrt{3},\ -i\sqrt{3},\ {}^5/_4$$

	$4x$	-5
x^2	$4x^3$	$-5x^2$
3	$12x$	-15

Find the zeroes using "box factoring"

$$y = 2x^3 + 3x^2 + 4x + 6$$

x =

$$y = 4x^3 + 8x^2 + 5x + 10$$

x =

$$y = -6x^3 + 7x^2 - 18x + 21$$

x =

How do we find zeroes of “hard-to-factor” polynomials?

The Factor Theorem   If a polynomial *f(x)* is divided by *(x – k)*, and the remainder is “0,” then *(x – k)* is a factor of the original polynomial and the zero of *(x – k)* is a zero of the polynomial.

$$\begin{array}{r|l} & x \quad +1 \\ \hline x+4 & x^2+5x+4 \\ & -(x^2+4x) \\ \hline & \qquad\quad x+4 \\ & \qquad -(x+4) \\ \hline & \qquad\quad 0 \end{array}$$

The remainder = 0   0

Polynomial Long Division

$$\begin{array}{r|l} & x^2 \\ \hline x-1 & x^3+3x^2+14x-18 \\ & -(x^3-x^2) \end{array}$$

1) Look at left-most numbers

2) What # times "left" = "left"?

3) Multiply

4) Subtract

Polynomial Long Division

$$\begin{array}{r} x^2 \\ x-1 \overline{\big) x^3 + 3x^2 + 14x - 18} \\ -(x^3 - x^2) \\ \hline 4x^2 + 14x - 18 \end{array}$$

4) Subtract

<u>Careful</u> with the negatives!

5) Bring down.

Polynomial Long Division

$$x^2 + 4x$$

$$x - 1 \overline{\big) \, x^3 + 3x^2 + 14x - 18}$$

$$-(x^3 - x^2)$$

$$4x^2 + 14x - 18$$

$$-(4x^2 - 4x)$$

$$18x$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

3) Multiply

4) Subtract

Careful of the negatives!

Polynomial Long Division

$$\begin{array}{r} x^2 \quad +4x \\ x-1 \,\overline{)\, x^3 + 3x^2 + 14x - 18} \\ -(x^3 - x^2) \\ \hline 4x^2 + 14x - 18 \\ -(4x^2 - 4x) \\ \hline 18x \; -18 \end{array}$$

5) Bring down.

Polynomial Long Division

$$\begin{array}{r} x^2 + 4x + 18 \\ x - 1 \overline{) \, x^3 + 3x^2 + 14x - 18} \\ -(x^3 - x^2) \\ \hline 4x^2 + 14x - 18 \\ -(4x^2 - 4x) \\ \hline 18x - 18 \\ -(18x - 18) \\ \hline 0 \end{array}$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times “left” = “left”?

3) Multiply

4) Subtract

$$
\begin{array}{r|l}
 & x^2 + 4x - 18 \\
\hline
x-1 & x^3 + 3x^2 + 14x - 18 \\
 & -(x^3 - x^2) \\
\hline
 & 4x^2 + 14x - 18 \\
 & -(4x^2 - 4x) \\
\hline
 & 18x - 18 \\
 & -(18x - 18) \\
\hline
 & 0
\end{array}
$$

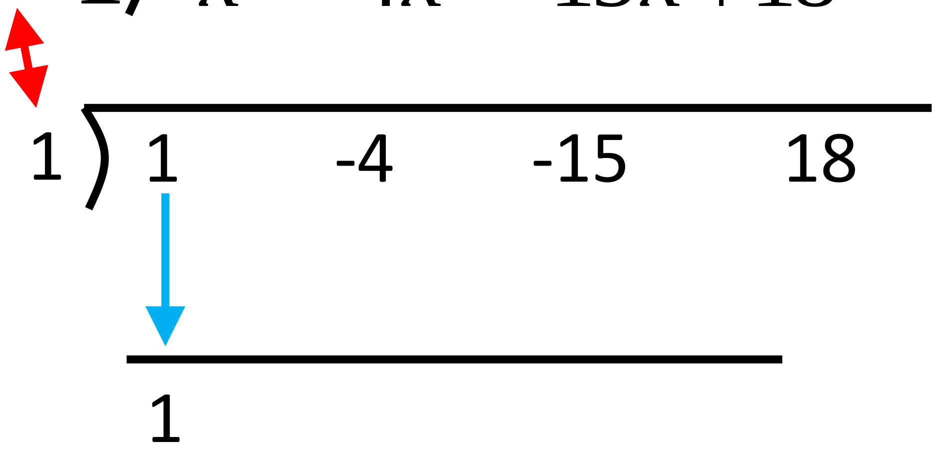
$$x^3 + 3x^2 + 14x - 18 = (x-1)(x^2 + 4x - 18)$$

How do we find the zeroes of the unfactorable quadratic factor?

Convert to vertex form and take square roots.

Is there an easier way to do this? Yes!

$$x-1\overline{)\ x^3-4x^2-15x+18}$$



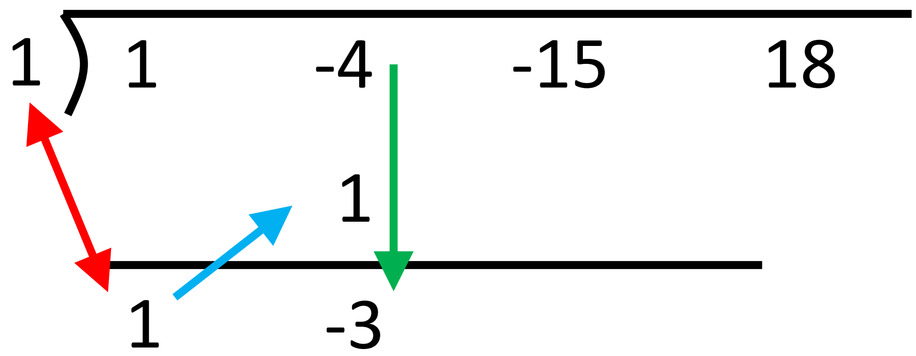
1^{st} step: Write the polynomial with only its coefficients.

2^{nd} step: Write the "zero" of the linear factor.

3rd step: add down

Is there an easier way to do this? Yes!

$$x-1\overline{\big)\, x^3-4x^2-15x+18}$$



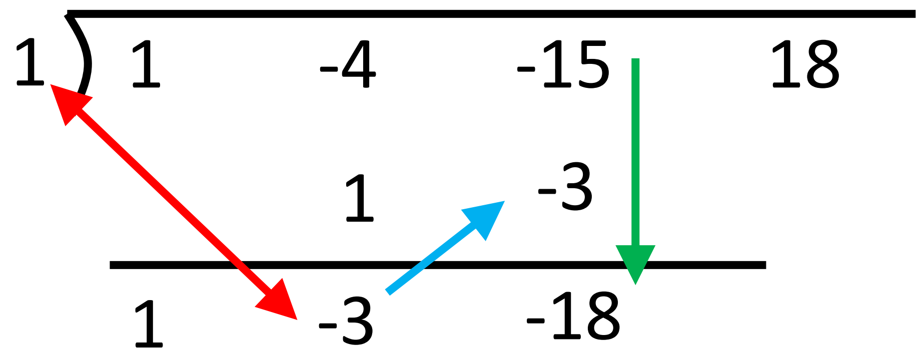
4th step: Multiply the “zero” by the lead coefficient.

5th step: Write the product under the next term to the right.

6th step: add the second column downward

Is there an easier way to do this? Yes!

$$x-1\,\overline{)\,x^3-4x^2-15x+18}$$



7th step: Multiply the "zero" by the second number

8th step: Write the product under the next term to the right.

9th step: add the next column downward

Is there an easier way to do this? Yes!

$$x - 1 \overline{\smash{\big)}\, x^3 - 4x^2 - 15x + 18}$$

1	1	-4	-15	18
		1	-3	-18
	1	-3	-18	0

10th step: Multiply the "zero" by the 3rd number

11th step: Write the product under the next term to the right

12th step: add the next column downward

$$x - 1 \overline{\smash{)}\, x^3 - 4x^2 - 15x + 18}$$

Quotient: $x^2 - 3x - 18$

1	1	-4	-15	18
		1	-3	-18
	1	-3	-18	0

0 ← remainder

$$x^3 - 4x^2 - 15x + 18 \div (x - 1) \quad \boxed{= x^2 - 3x - 18}$$

Because the remainder = 0, then (x – 1) is a factor AND x = 1 is a zero of the original polynomial!

$$y = x^3 + 8$$

$$y = x^3 + 0x^2 + 0x + 8$$

<u>Use Division to Factor</u>:

Find the 1st zero.

$$0 = x^3 + 8$$

$$-8 = x^3$$

$$-2 = x$$

→ Comes from the factor (x + 2)

-2	1	0	0	8
		-2	4	-8
	1	-2	4	0

$$y = (x + 2)(x^2 - 2x + 4)$$

Find the Zeroes of the Difference of cubes:

$$y = x^3 - 64$$

$$y = x^3 + 125$$

Difference of Squares (of higher degree):

$$y = x^4 - 81$$

$$y = x^4 + 0x^3\ 0x^2 + 0x - 81$$

3	1	0	0	0	-81
		3	9	27	81
	1	3	9	27	0

Find the 1st zero.

$$0 = x^4 - 81$$

$$81 = x^4$$

$$3 = x$$

→ Comes from the factor (x – 3)

$$y = (x - 3)(x^3 + 3x^2 + 9x + 27)$$

→ Try to box-factor the 3rd degree polynomial.

"Quadratic Form": a trinomial that looks like a quadratic but has a larger degree.

$y = x^4 + 3x^2 + 2$ Looks like → $y = m^2 + 3m^1 + 2$

Factors similarly

$y = (x^2 + 2)(x^2 + 1)$ $y = (m + 2)(m + 1)$

BUT, according to the Linear Factorization Theorem, it factors into 4 linear factors.

$$y = (x + i\sqrt{2})(x - i\sqrt{2})(x + i)(x - i)$$

Zeroes: $x = i\sqrt{2}, -i\sqrt{2}, i, -i$

Find the zeroes

$$y = x^4 - 16x^2 + 28$$

$$y = (x^2 - 14)(x^2 - 2)$$

$$y = (x + \sqrt{14})(x - \sqrt{14})(x + \sqrt{2})(x - \sqrt{2})$$

Zeroes: $x = \sqrt{14}, -\sqrt{14}, \sqrt{2}, -\sqrt{2}$

$$y = 16x^4 - 81$$

$$y = (4x^2 - 9)(4x^2 + 9)$$

$$y = (2x - 3)(2x + 3)(2x - 3i)(2x + 3i)$$

$$x = \frac{3}{2}, -\frac{3}{2}, \frac{3i}{2}, -\frac{3i}{2}$$