Math-3
Lesson 3-1
Unit 1 and 2 Weak Area Review
(Part 1)

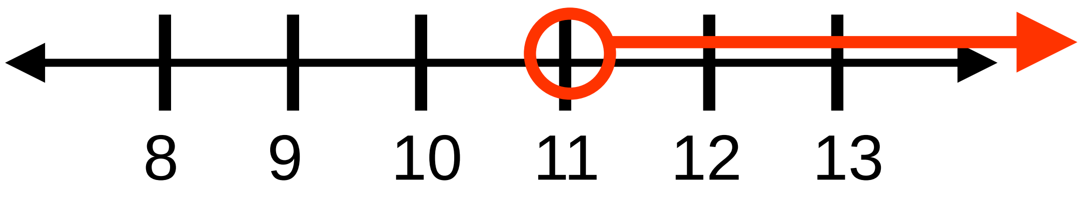
What is *interval notation*?

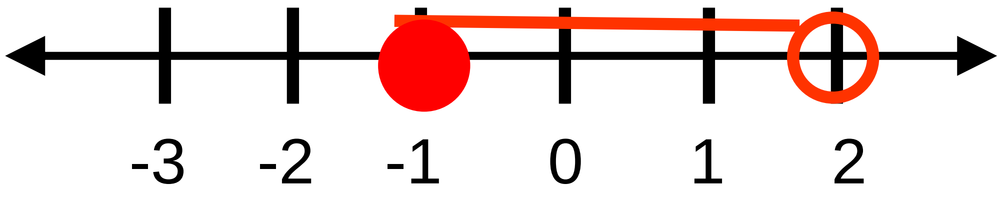
Give an example.

A way to define the shaded region of a number line.

$x = (-\infty, 5]$

Describe the shaded region using interval notation.





$x = (11, \infty)$

$x = [-1, 2)$

What "name" would give the *number '11'* above?

What "name" would give the *numbers '-1'* and *'+2'* above?

Critical value: a number(s) on a number line that separates the *number line* into *separate intervals*.

Critical value: a *boundary number* that separates the number line into separate intervals.

x = (3, 6] x = [-1, ∞)

What is the significance of the “[“ , “]” , “(“ , “)” symbols?

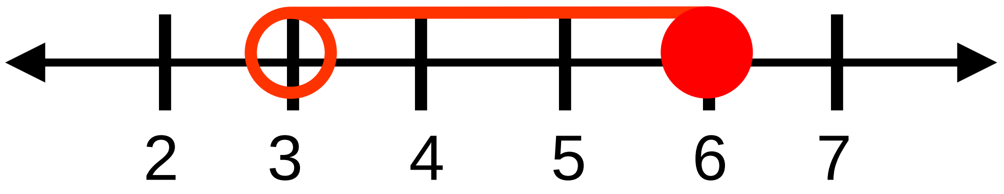
“[“ means the *left-side* “critical value” is *included* in the interval.

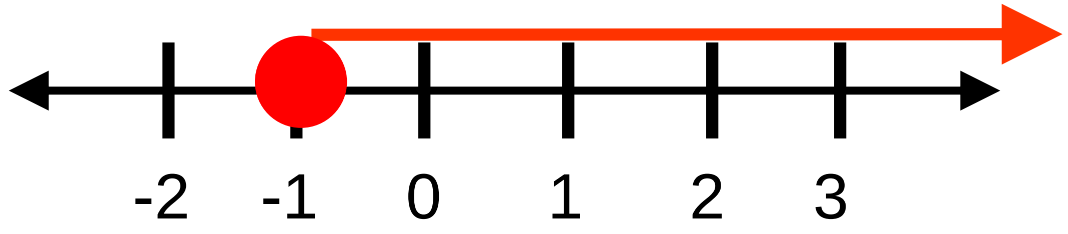
“(“ means the *left-side* “critical value” is *NOT included* in the interval.

“]“ means the *right-side* “critical value” is *included* in the interval.

“)“ means the *right-side* “critical value “is *NOT included* in the interval.

Graph the two intervals shown above.





Which of the following is ***not a correct example*** of Interval Notation?

If not, explain why not.

a) x = (-∞, 5]

b) x = [-3, ∞]

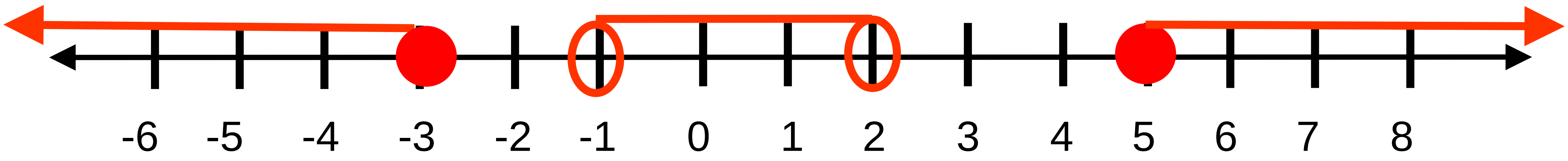
c) x = [-2, 4)

d) x = (5, -∞)

b) "∞" is not a "number" so it can never be "included" in the interval.

d) "-∞" is on the *left end of a number line* so it must be the *left-side* number in interval notation..

Graph the critical values defined by: x = (-∞, -3] U (-1, 2) U [5, ∞)

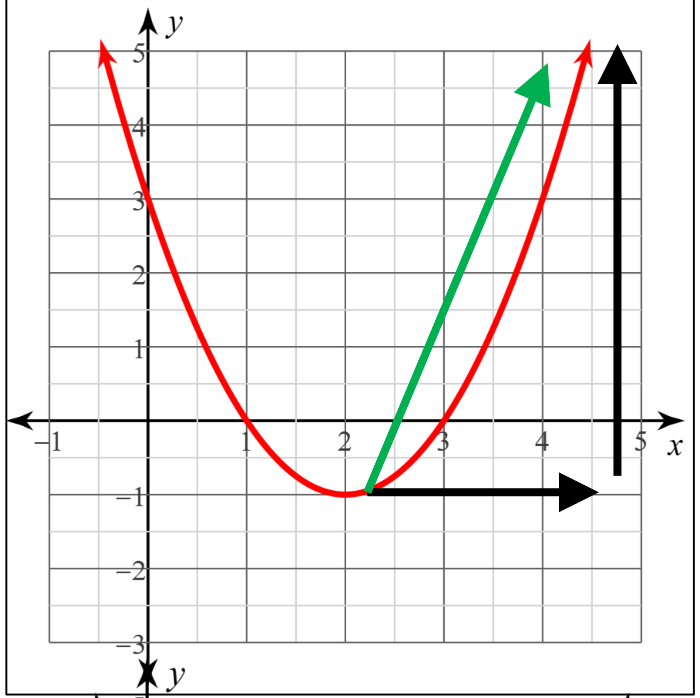


Shade the intervals defined by: x = (-∞, -3] U (-1, 2) U [5, ∞)

Define what it means to say:
"The graph is increasing."

→ From left to right it goes upward.

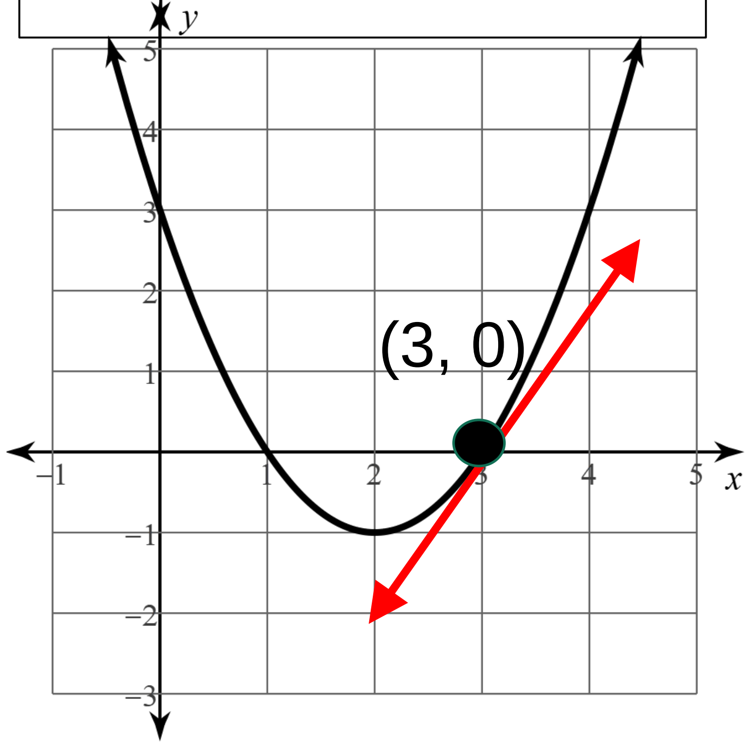
→ Positively sloped.



Define what it means to say:

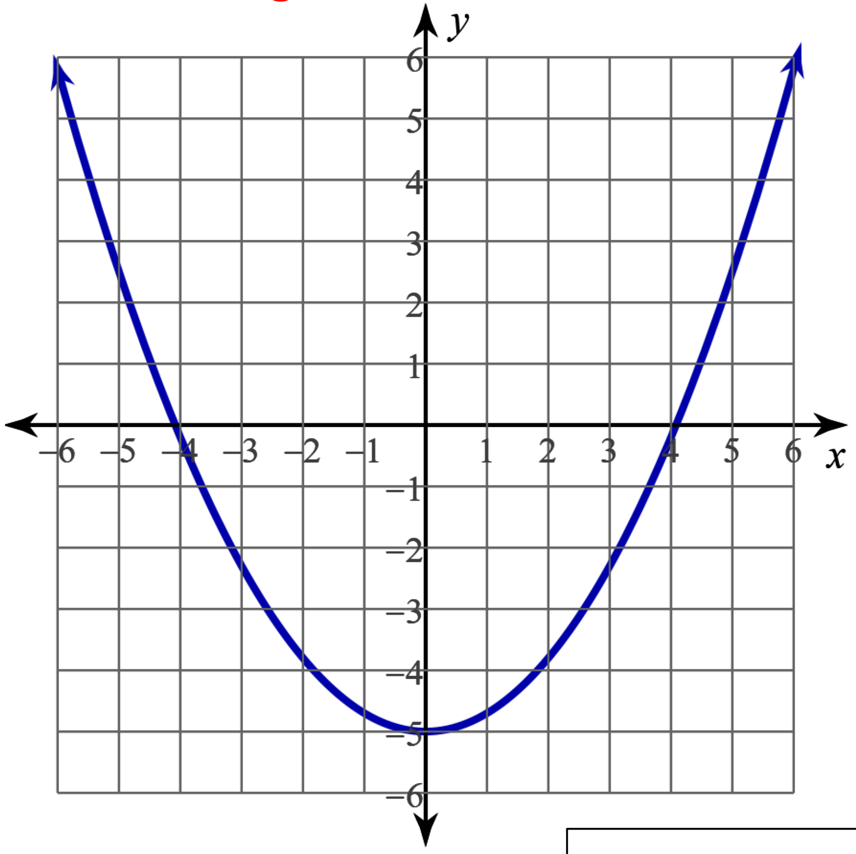
"The graph is increasing at a specific location, for example at (x, y) = (3, 0)."

A line that is tangent to the graph at the point (3, 0) has a positive slope.



Starting from the left side; draw a graph that is:

1) decreasing until x = 0, then increasing

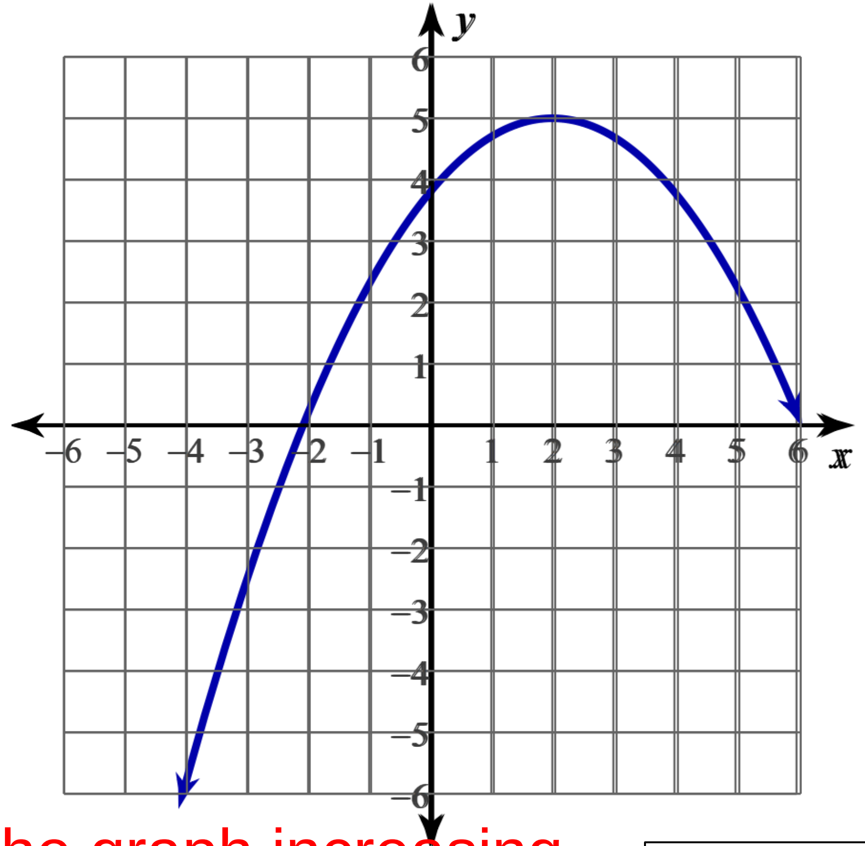


Is the graph increasing or decreasing at x = 0?

Neither increasing nor decreasing

Why not?

2) Increasing until x = 2, then decreasing



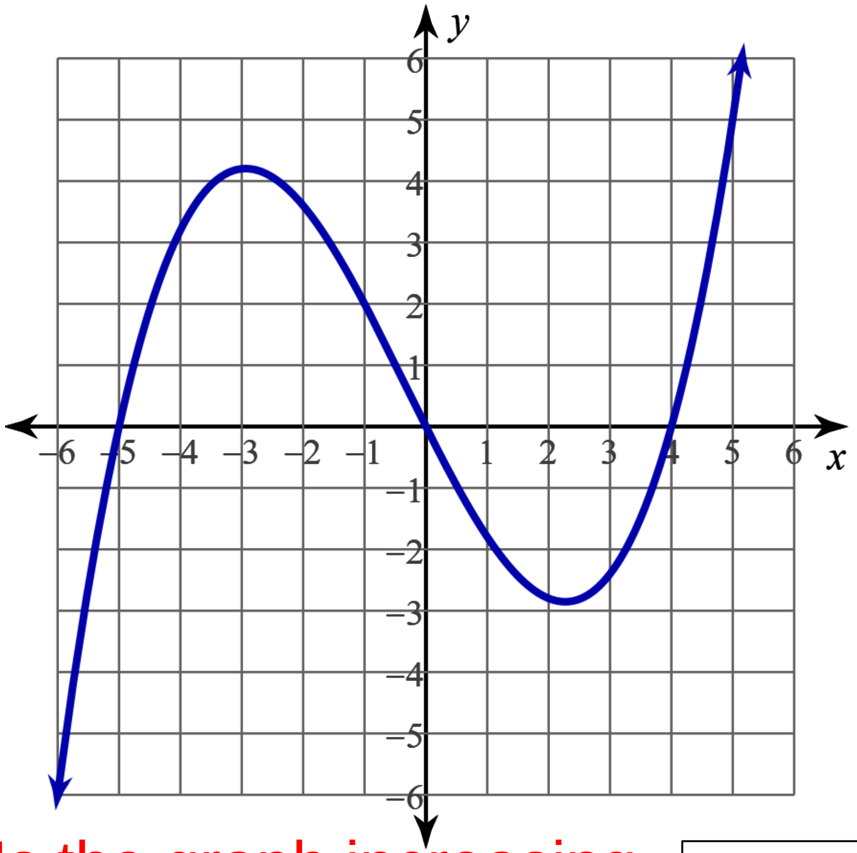
Is the graph increasing or decreasing at x = 2?

Neither

Why not?

Starting from the left side; draw a graph that is:

↑ until x = -3, ↓ until x = 2, then ↑

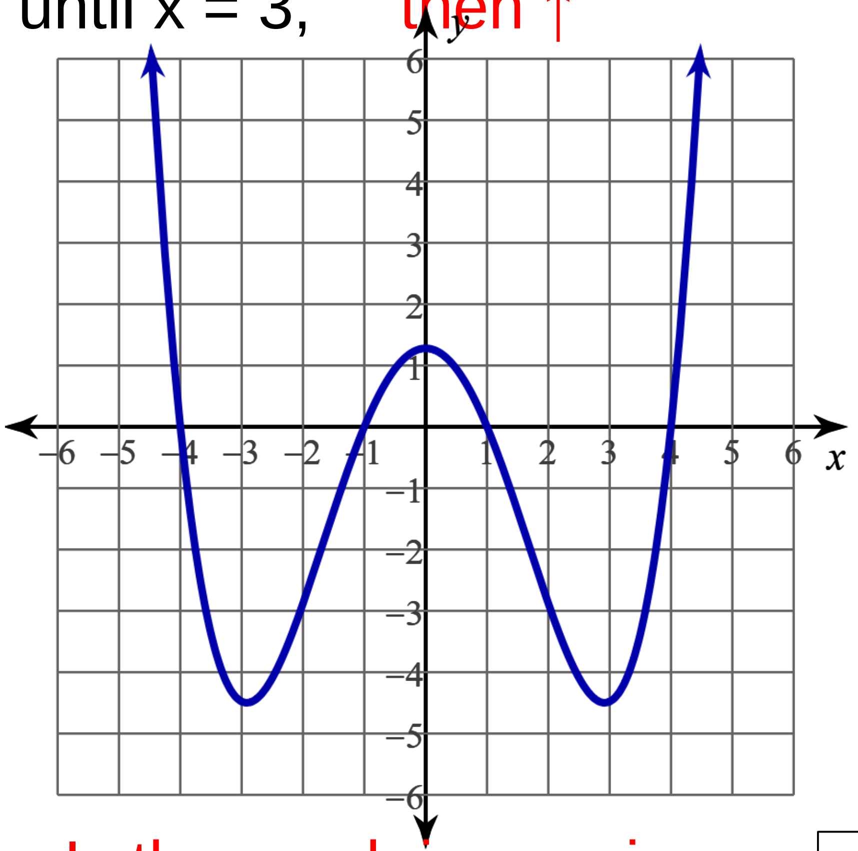


Is the graph increasing or decreasing at x = -3 or x = 2 ?

Neither increasing nor decreasing

Why not?

↓ until x = -3, ↑ until x = 0,
↓ until x = 3, then ↑



Is the graph increasing or decreasing at x = -3, x = 0, or x = 3 ?

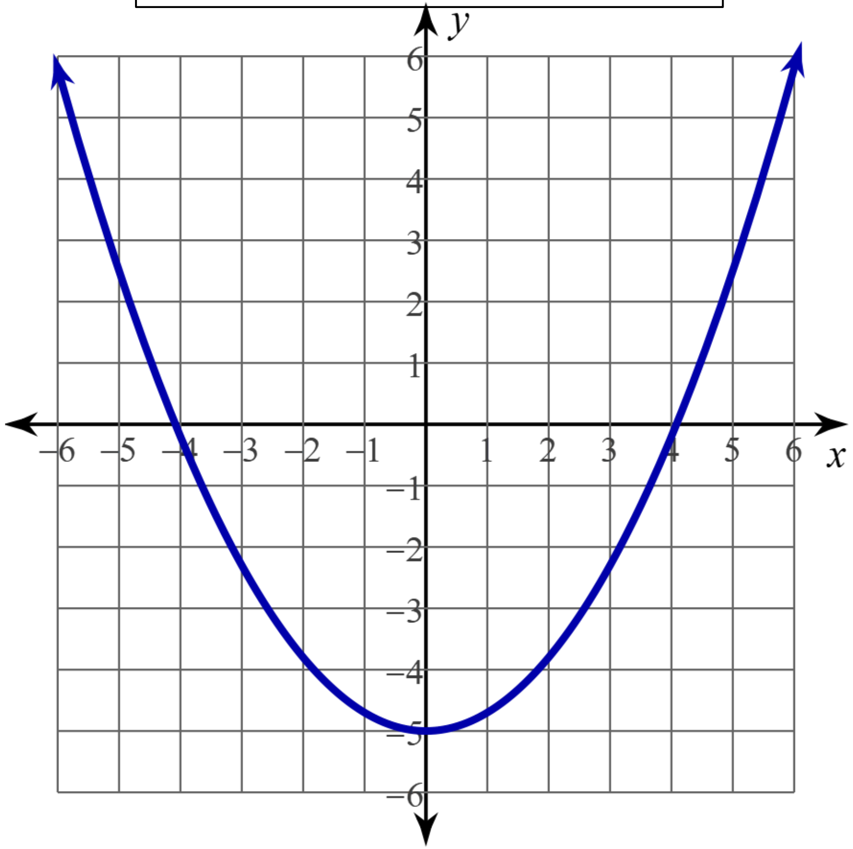
Neither

Why not?

Using Interval Notation, where is the graph:

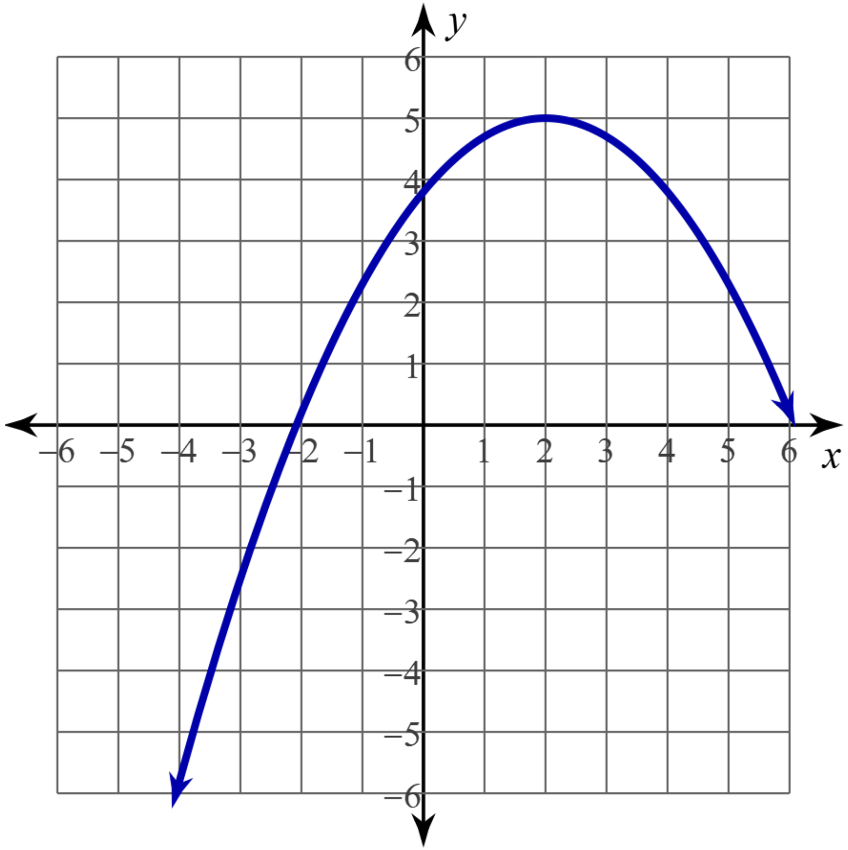
Increasing?

f(x) ↑ on x = (0, ∞)



Increasing?

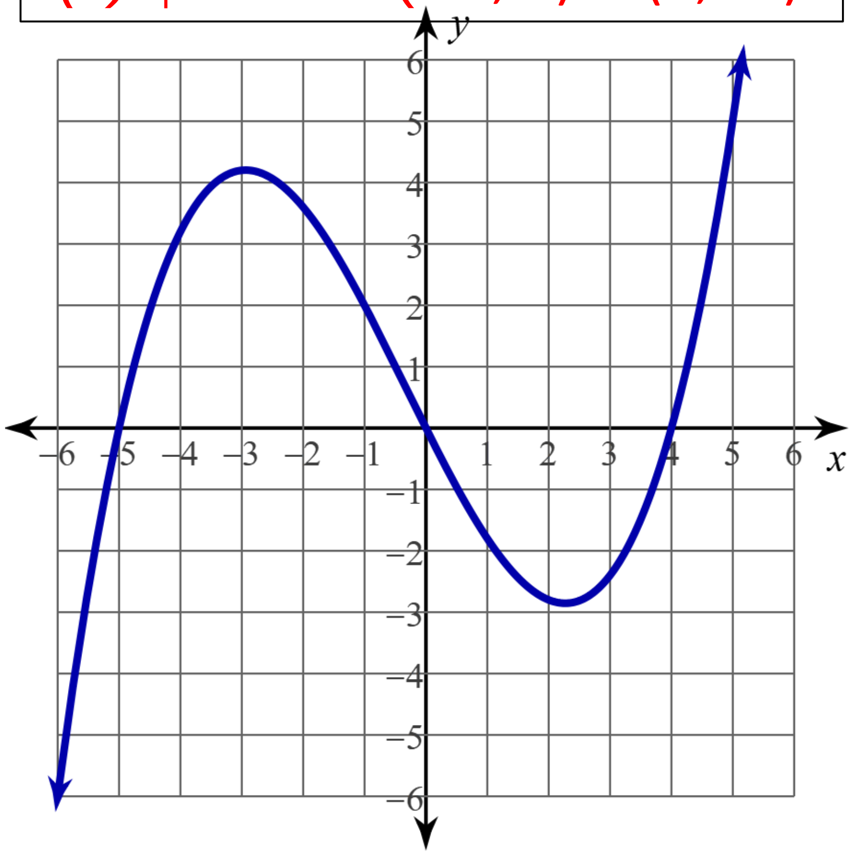
f(x) ↑ on x = (-∞, 2)



Using Interval Notation, where is the graph:

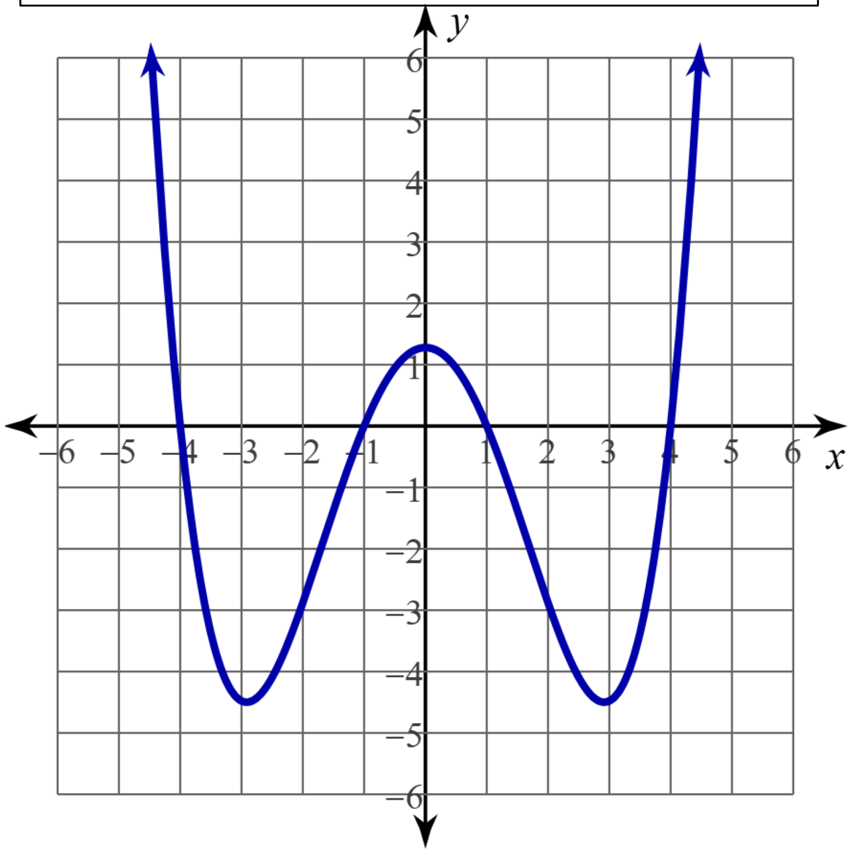
Increasing?

$f(x)$ ↑ on x = (-∞, 3) U (2, ∞)



Increasing?

$f(x)$ ↑ on x = (-3, 0) U (3, ∞)



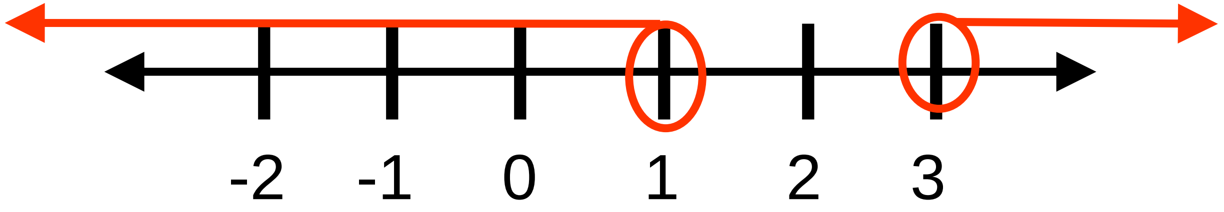
Define what it means to say:
"The portion of the graph that is positive."

→ The part of the graph *above the x-axis.*

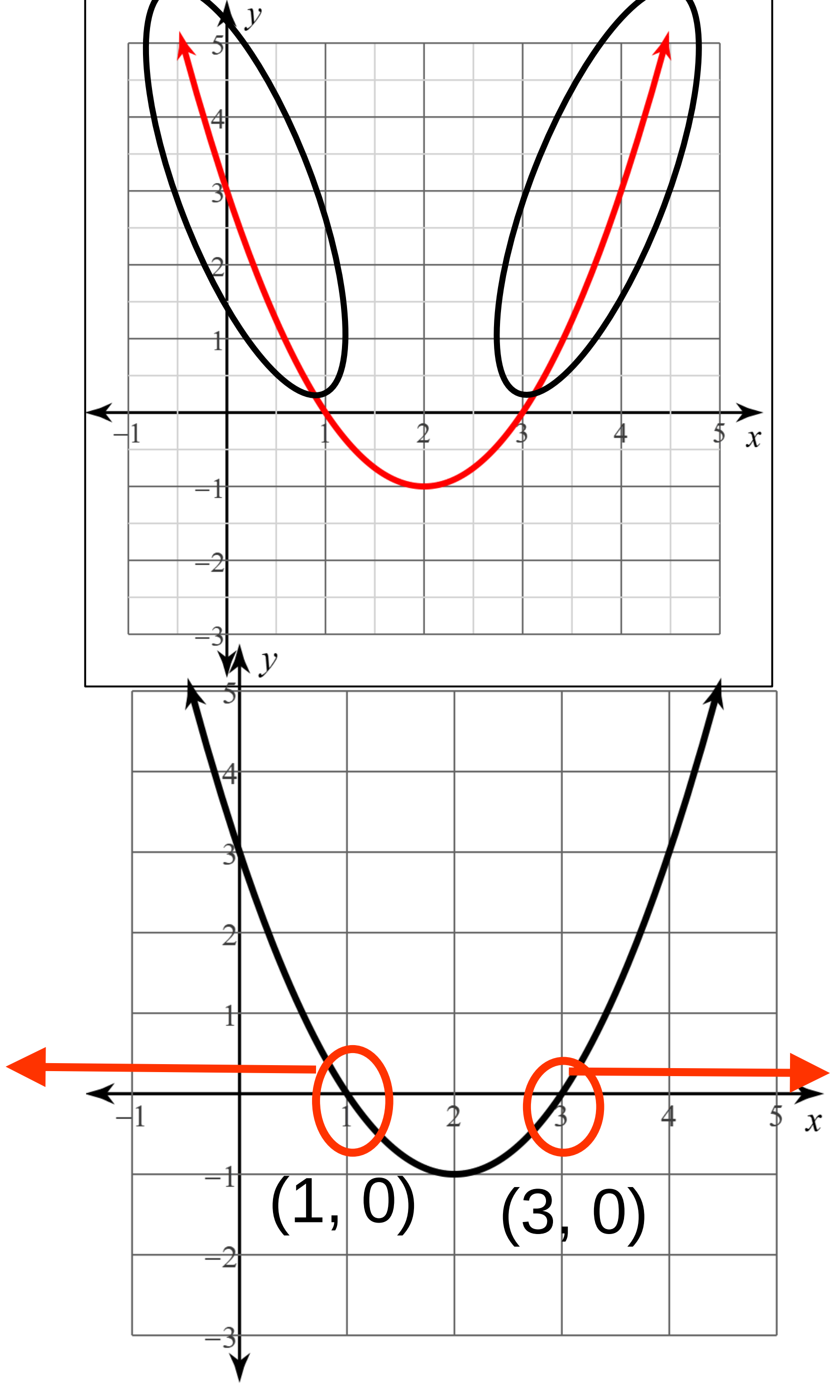
→ All x-y pairs whose y-values are positive.

Shade the *input values* whose corresponding output values are positive.

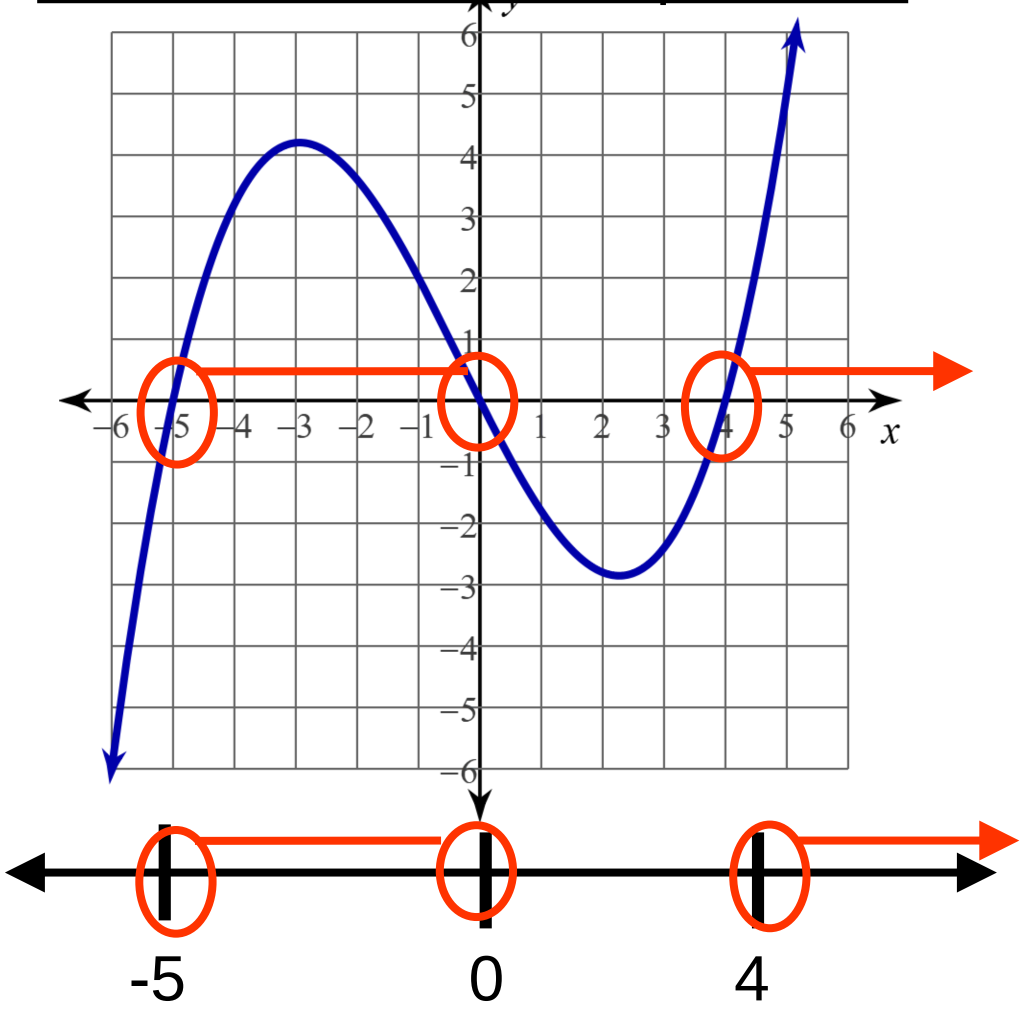
Define the *shaded region* using interval notation.



x = (-∞, 1) U (3, ∞)

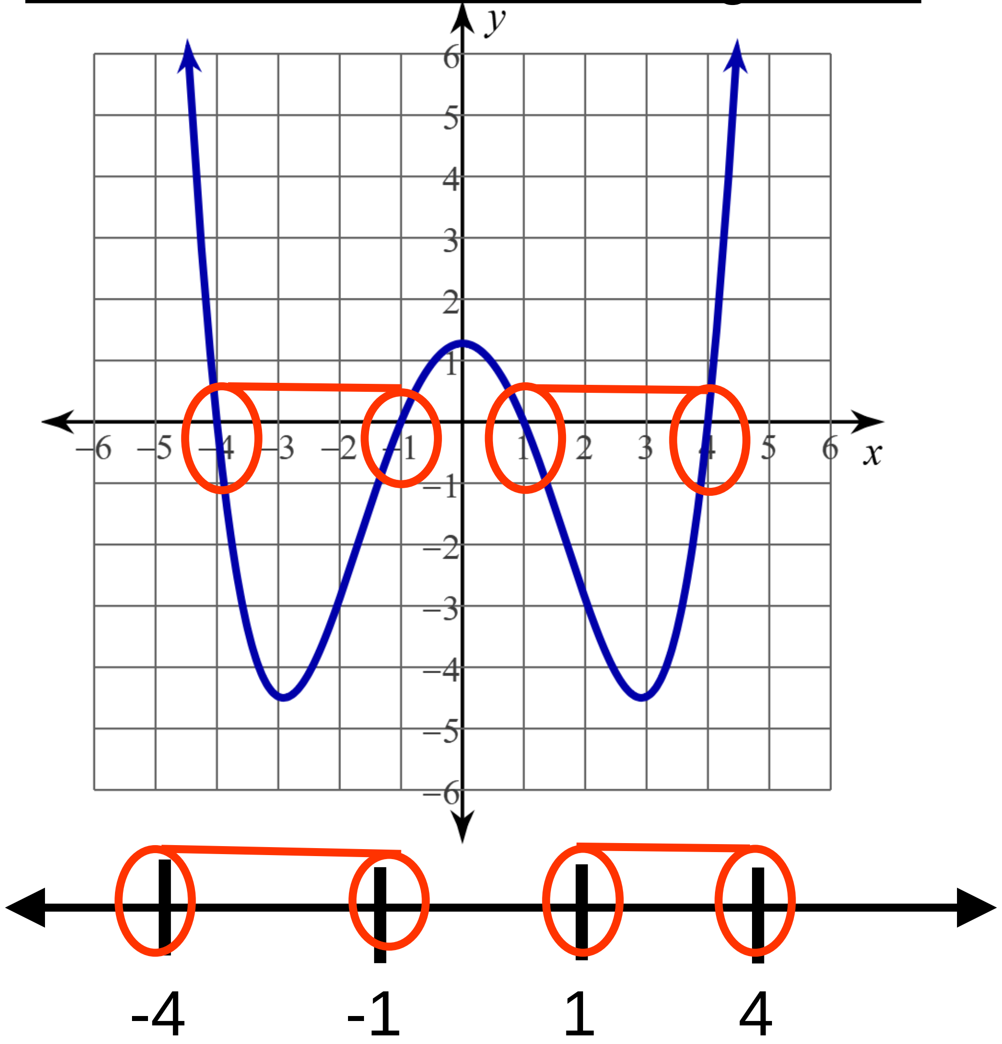


Where is the function positive?



$f(x) > 0$ on $x = (-5, 0) \cup (4, \infty)$

Where is the function negative?



$f(x) < 0$ on $x = (-4, -1) \cup (1, 4)$

What does the "multiplicity" of a "zero" refer to?

The number of times the same zero occurs from separate and distinct linear factors of the polynomial. $y=(x-1)^2 = (x-1)(x-1)$

What are the zeroes? $0 = (x-1)(x-1)$ → x – 1 = 0

x = 1 is a "zero" from 2 separate and distinct linear factors

Zeroes: "x = 1 with multiplicity 2"

State the **Fundamental Theorem of Algebra**: If a polynomial has a degree of "n", then the polynomial has "n" zeroes (provided that repeat zeroes, called "multiplicities" are counted separately).

$y = 6x^4 + 42x^3 + 96x^2 + 28x + 48$ "4th Degree" → 4 zeroes

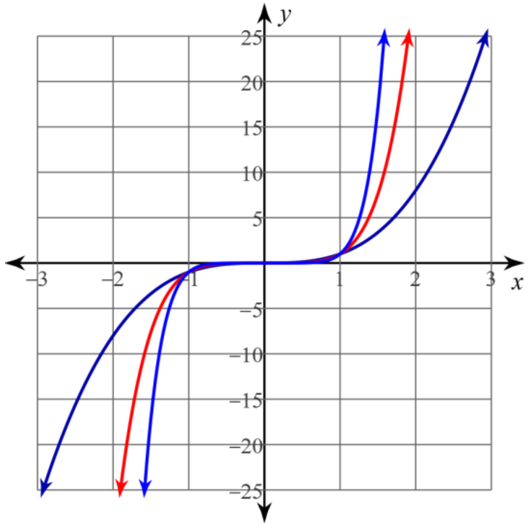
$y = x^4 - 5x^2 + 4$ "4th Degree" → 4 zeroes

What is the relationship between the sign of the lead coefficient and degree of the polynomial and the end behavior of the graph?

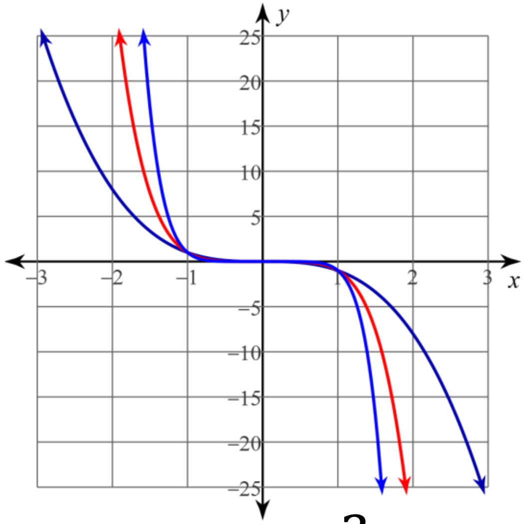
Positive Lead coefficient and Odd degree: ↓ left and ↑ right

Positive Lead coefficient and even degree: ↑ left and ↑ right

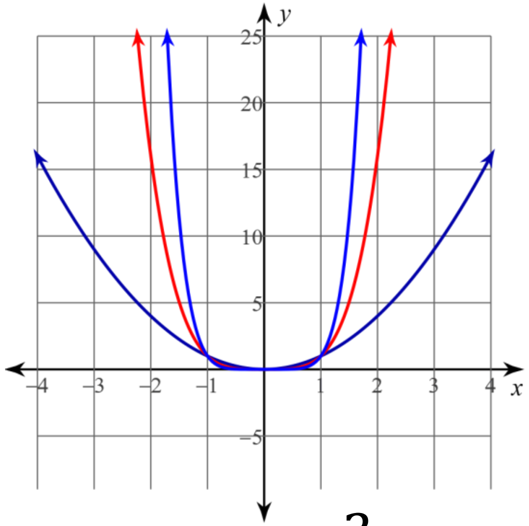
Negative lead coefficient: (reflection across x-axis) switches to opposite end behavior



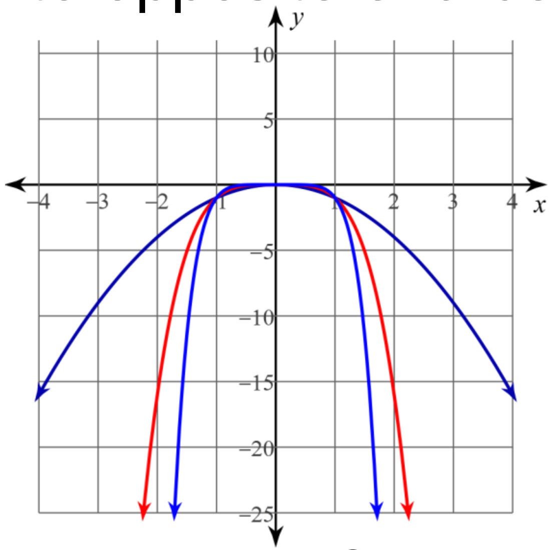
$y = x^3$
$y = 2x^5$
$y = 3x^7$



$y = -x^3$
$y = -2x^5$
$y = -3x^7$



$y = x^2$
$y = 6x^4$
$y = 9x^6$

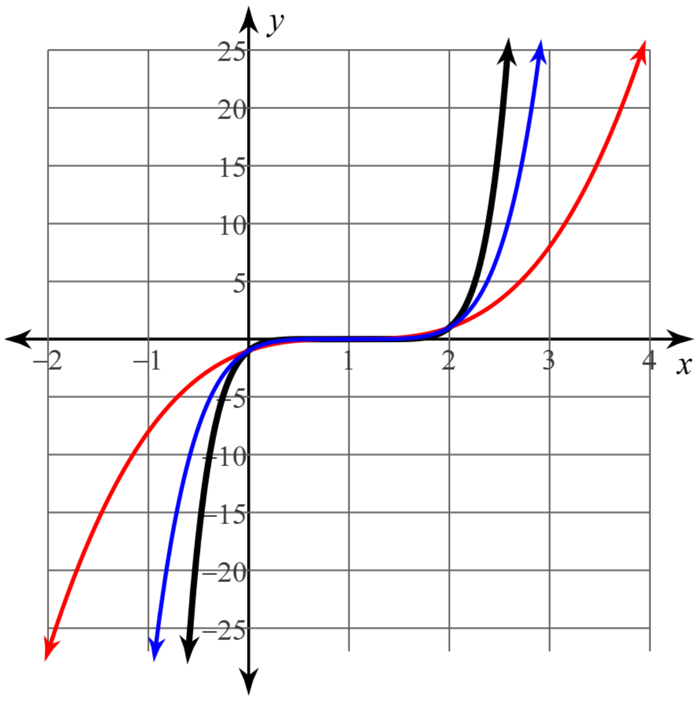


$y = -x^2$
$y = -6x^4$
$y = -9x^6$

What is the relationship between multiplicity and whether the graph crosses or kisses the real-number zero of the polynomial?

Odd multiplicity:

Crosses x-axis at the zero.



$y = (x-1)^3$

$y = 2(x-1)^5$

$y = 3(x-1)^7$

Zeroes:

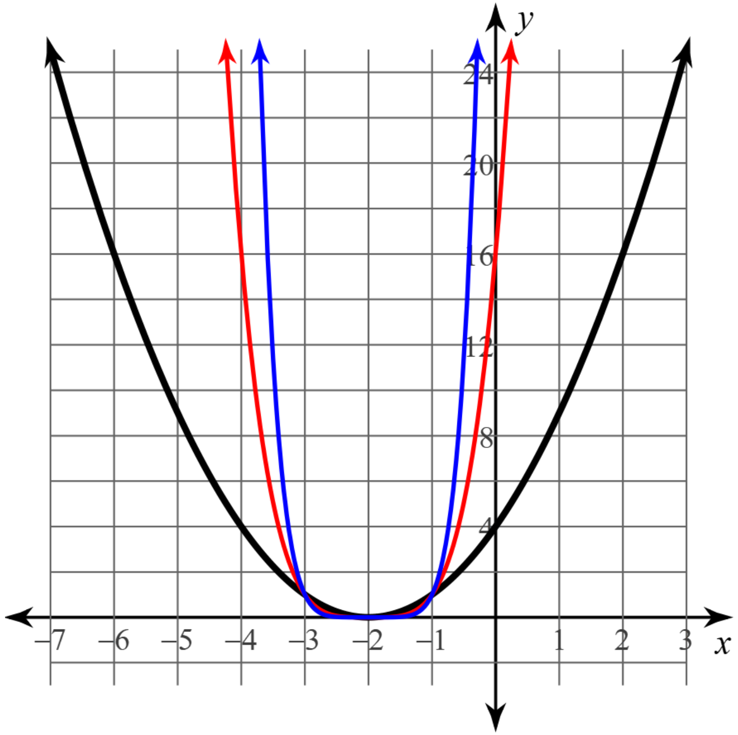
x = 1 mult. 3

x = 1 mult. 5

x = 1 mult. 7

Even multiplicity:

“kisses” x-axis at the zero.



$y = (x+2)^2$

$y = 6(x+2)^4$

$y = 9(x+2)^6$

Zeroes:

x = -2 mult. 2

x = -2 mult. 4

x = -2 mult. 6

Graph the polynomial $f(x) = x(x+1)(x-1)(x-2)$

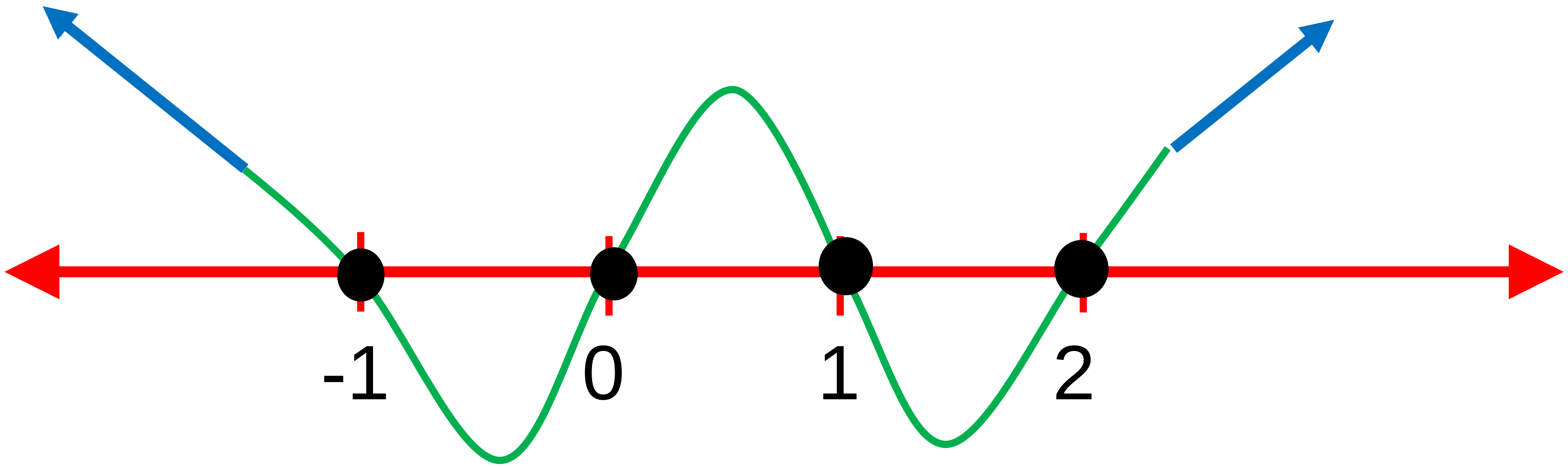
"4^{th} Degree" → 4 zeroes

Zeroes: x = -1, 0, 1, and 2.

Sign (lead coefficient) and degree: (+) even

The end behavior is up right, up left?

All zeroes occur once → Odd multiplicity: Graph crosses x-axis at the zero.



Graph the polynomial $f(x) = 2(x+1)^2(x+3)(x-4)$

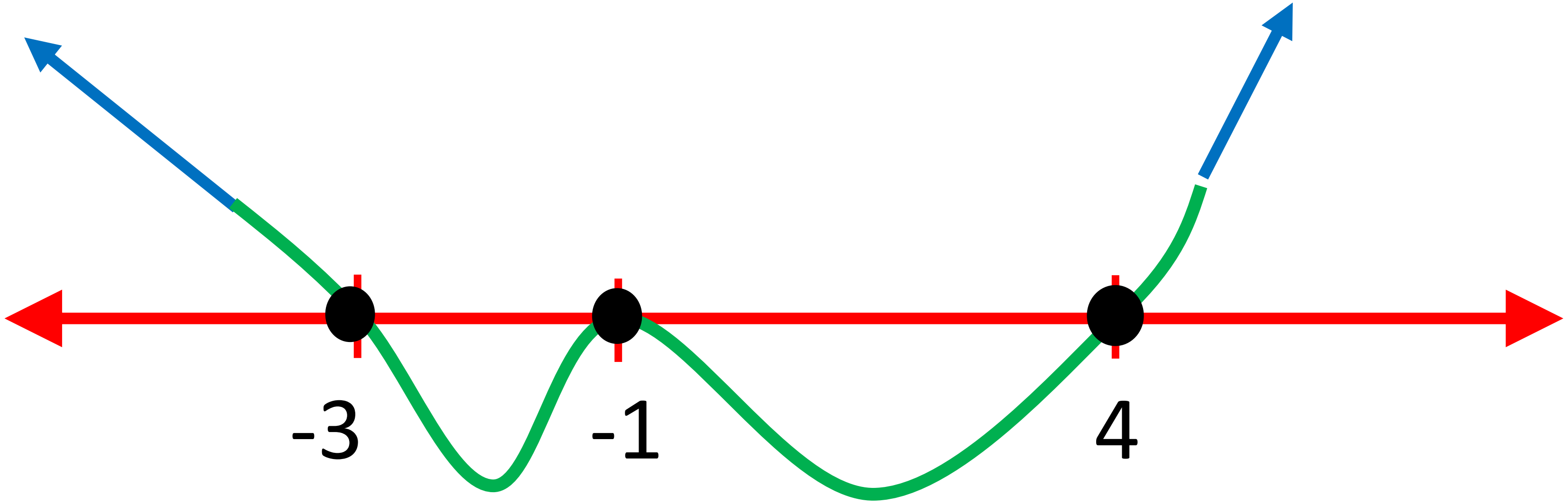
"4th Degree" → 4 zeroes

Zeroes: x = -3, -1 mult. 2, and 4.

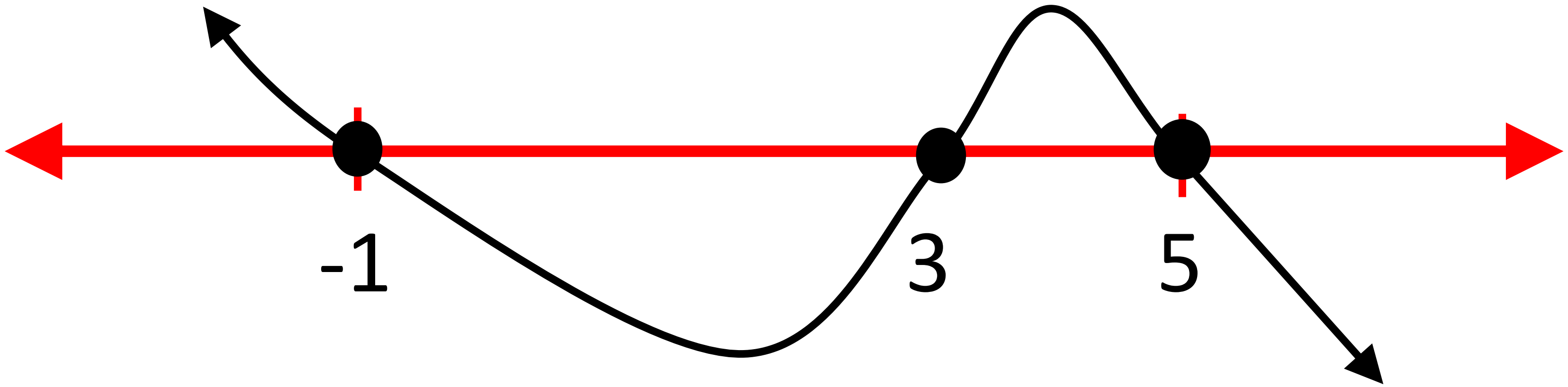
Sign (lead coefficient) and degree: (+) even

The end behavior is up right, up left?

x = -1 has an even multiplicity: Graph kisses x-axis at the zero.



Find the errors in the graph of: $f(x) = -2x^2(x-3)^2(x+1)(x-5)$



"6^{th} Degree" → 6 zeroes

Zeroes: x = -1, **0 (mult. 2),** 3 (mult. 2), and 5.

Error #1: Did not graph x = 0 (mult. 2)

Error #2: Did not "kiss" at x = 3 (mult. 2)

Sign (lead coefficient) and degree: (-) even ↓ left and ↓ right

Error #3: Wrong end behavior.

Write the equation of the line that passes through the data the table.

$y = mx + b$

x	0	2	4
y	-2	5	7

$y = m(0) - 2$

$y = mx - 2$

$5 = m(2) - 2$

$m = \frac{5-2}{2} = \frac{3}{2}$

$y = \frac{3}{2}x + 4$

Write the equation of the line that passes through the data for each table. $y = mx + b$

$\Delta x = -2$

x	-3	-5
y	5	-6

$\Delta y = -11$

$$m = \frac{\Delta y}{\Delta x} = \frac{-11}{-2} = \frac{11}{2}$$

$$y = \frac{11}{2}x + b$$

Substitution: (x, y) = (-5, -6)

$$-6 = \frac{11}{2}(-5) + b$$

Simplify right side

$$-6 = \frac{-55}{2} + b$$

Add $\frac{55}{2}$ left/right of “=“

$$-6 + \frac{55}{2} = b$$

Simplify left side

$$\frac{43}{2} = b$$

$$y = \frac{11}{2}x + \frac{43}{2}$$

Factoring Quadratic trinomial whose lead coefficient ≠ 1

$$3 * 8 = 24$$

$$3x^2 + 14x + 8$$

$$24 = \underline{12} * \underline{2}$$

$$14x = \underline{12x} + \underline{2x}$$

This tells us to break 14x into 2x + 12x

$$3x^2 + 14x + 8$$

$$3x^2 + 2x + 12x + 8$$

These are all of the terms in “the box”

	3x²	12x
	2x	8

Factor out common factor of the 1st row.

3x	3x²	12x
	2x	8

Fill in the rest of the table.

	x	4
3x	3x²	12x
2	2x	8

$$3x^2 + 14x + 8$$

$$\to (3x + 2)(x + 4)$$

$5x^2 + 10x - 4$

-20 = 10 * (-2)

10x = 10x + (-2x)

	x	2
5x	$5x^2$	10x
-2	-2x	-4

$5x^2 + 10x - 4$

$\rightarrow (5x-2)(x+2)$

$6x^2 - x - 2$

-12 = -4 * 3

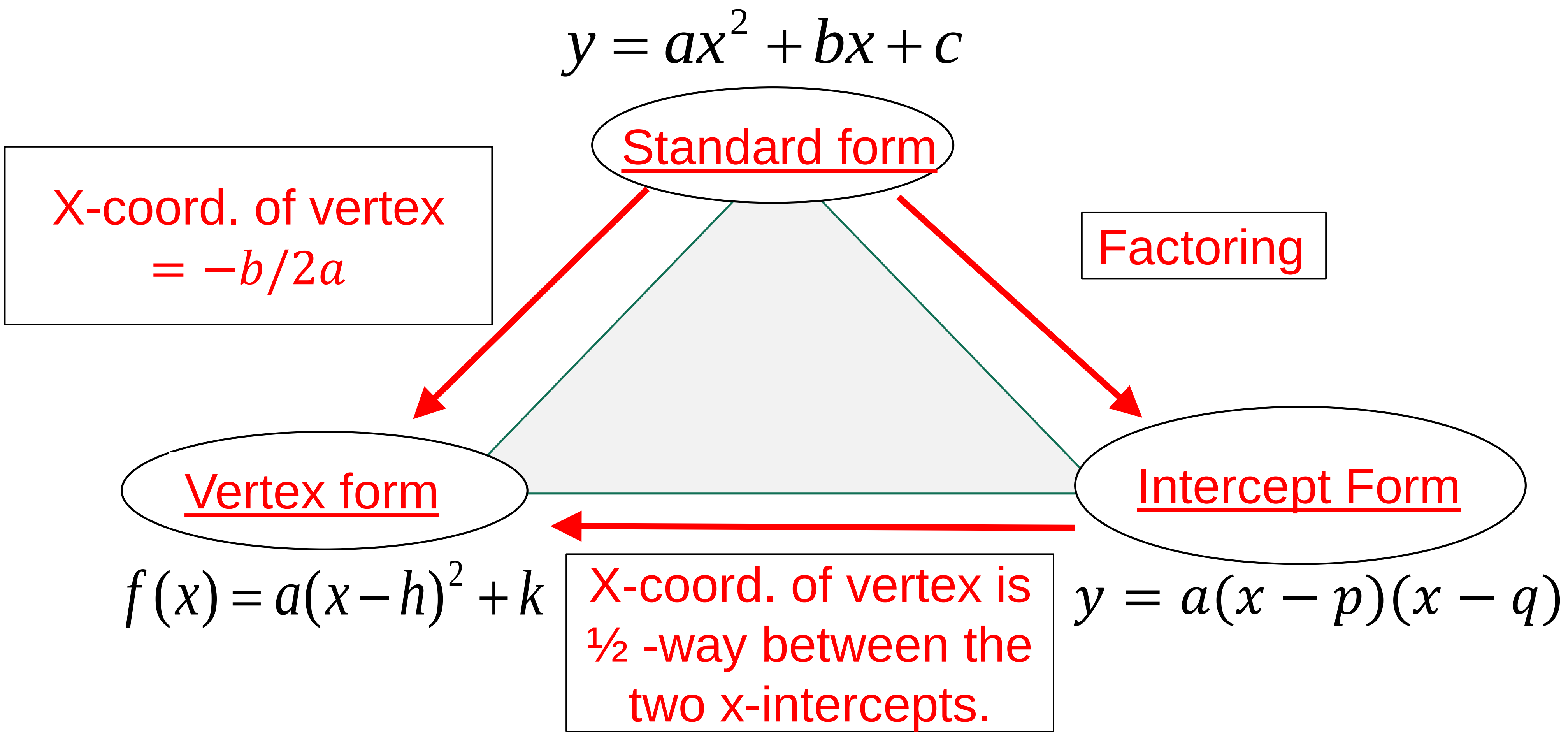
-x = -4x + 3x

	3x	-2
2x	$6x^2$	-4x
1	3x	-2

$6x^2 - x - 2$

$\rightarrow (2x+1)(x-2)$

Forms of the Quadratic Equation



Standard form → Vertex Form

$$y = 3x^2 + 6x - 12$$

a = 3   b = -4

Vertex: (x, y) = (____, ____)

What is the x-coordinate of the vertex?

x-coord. of vertex = $\frac{-b}{2a}$

$$\frac{-b}{2a} = \frac{-(6)}{2(3)} = -1$$

Vertex: (x, y) = (-1, ____)

What is the y-coordinate of the vertex?

$$f(-1) = 3(-1)^2 + 6(-1) - 12$$

$$f(-1) = -15$$

Vertex: $(-1,$ -15)

What is the Vertex form equation?

$$\text{VSF} = 3, \text{vertex} = (-1, -15)$$

$$y = 3(x+1)^2 - 15$$

Standard form → Vertex Form

$$y = 2x^2 + 16x + 24$$

a = 2   b = 16

Vertex: (x, y) = (____, ____)

What is the x-coordinate of the vertex?

x-coord. of vertex = $\frac{-b}{2a}$

$$\frac{-b}{2a} = \frac{-16}{2(2)} = -4$$

Vertex: (x, y) = (-4, ____)

What is the y-coordinate of the vertex?

$$f(-4) = 2(-4)^2 + 16(-4) + 24$$

$$f(-4) = -8$$

Vertex: $(-4,$ -8)

What is the Vertex form equation?

$$\text{VSF} = 2, \text{vertex} = (-4, -8)$$

$$y = 2(x + 4)^2 - 8$$

Intercept Form → Vertex form: The x-coordinate of the vertex is exactly half-way between the two x-intercepts.

$$f(x) = (x+5)(x-1) \qquad x = \frac{-5+1}{2} = \frac{-4}{2} = -2$$

x = -5 | x = 1

What are the x-coordinates of the x-intercepts?

What is the x-coordinate of the vertex?

Vertex: (x, y) = (-2 , ___)

What is the y-coordinate of the vertex?

Vertex: (x, y) = (-2, -9)

$$f(-2) = (-2+5)(-2-1) = (3)(-3) \qquad f(-2) = -9$$

What is the VSF? a = 1

$$y = a(x-p)(x-q)$$

What is the vertex form equation?

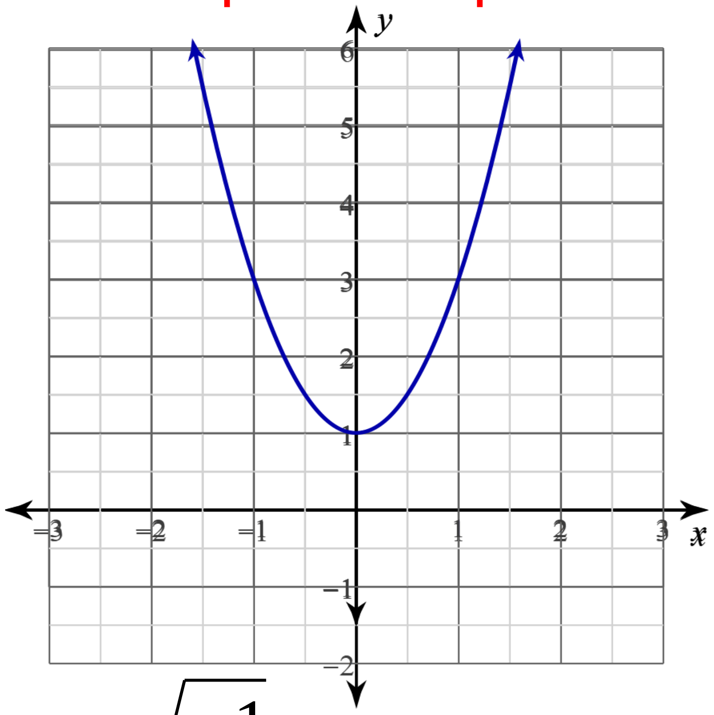
$$y = ax^2 + bx + c$$

$$y = (x+2)^2 - 9$$

$$y = a(x-h)^2 + k$$

$$y = 2x^2 + 1$$

Graph the equation



What are the "zeroes" of the function?

1. Set y = 0

$$0 = 2x^2 + 1$$

2. Isolate the square, undo the square

$$-1 = 2x^2$$

Subtract 1 (left/right)

$$\frac{-1}{2} = x^2$$

Divide by 2 (left/right)

Square root (left/right)

$$\pm\frac{\sqrt{-1}}{\sqrt{2}} = x$$

Convert to '*i*'

Multiply by "one" (in the form of…)

$$x = \pm\frac{i}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = \pm\frac{i\sqrt{2}}{2}$$

Vertex form $y = a(x-h)^2 + k$ $y = (x+3)^2 - 8$

1. Find the zeroes of the function.

Let y = 0 $0 = (x+3)^2 - 8$

Isolate the squared term

$8 = (x+3)^2$ "take square roots"

$\sqrt{8} = \sqrt{(x+3)^2}$

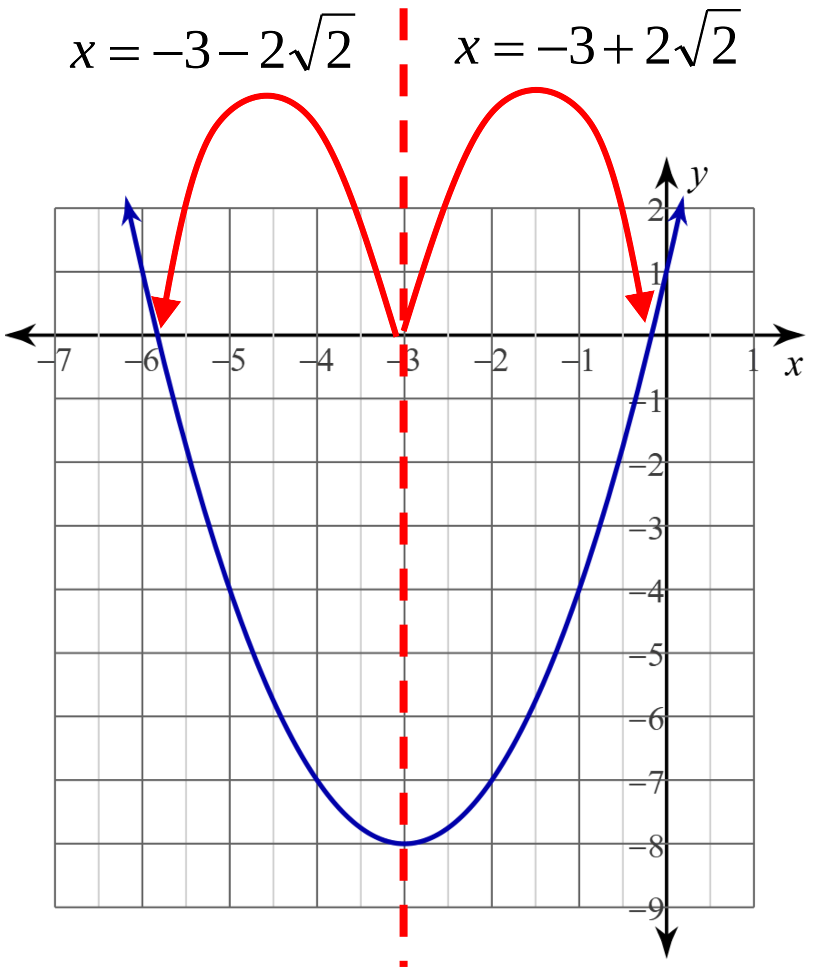
$\pm\sqrt{8} = x+3$ Simplify the radical

$\pm\sqrt{2*2*2} = x+3$

$\pm 2\sqrt{2} = x+3$ Solve for 'x'

$x = -3 \pm 2\sqrt{2}$

x-coord of vertex



Find the zeroes of the function. $y = 2(x+7)^2 - 10$

$0 = 2(x+7)^2 - 10$ set y = 0

$10 = 2(x+7)^2$ Add 10 (left/right)

$5 = (x+7)^2$ Divide by 2 (left/right)

$\pm\sqrt{5} = \sqrt{(x+7)^2}$ “take square roots”

$\pm\sqrt{5} = x + 7$ subtract 7 from both sides

$-7 \pm \sqrt{5} = x$

$x = -7 + \sqrt{5}$ $x = -7 - \sqrt{5}$