Math-3

Lesson 2-8
Divide Polynomials

Long Division

$$\frac{541}{12} = 12\overline{\smash{)}541}$$

Look at the left numbers

$$= \enclose{circle}{12}\overline{\smash{)}\enclose{circle}{54}1}$$

12 divides 54

4 times 12 = 48

$$\begin{array}{r} 45 \\ = 12\overline{\smash{)}541} \end{array}$$

Subtract 48 from 54

Bring the “1” down.

$$-48$$

$$=6$$

5 times 12 = 60;
subtract 60 from 61

$$=61$$

$$-60$$

$$=1$$

$$\frac{541}{12} = 45 + \frac{1}{12}$$

Remainder

Divisor

The Factor Theorem If a polynomial *f(x)* is divided by *(x – k),* and the remainder is “0,” then *(x – k)* is a factor of the original polynomial and the zero of *(x – k)* is a zero of the polynomial.

Polynomial Long division: One method used to divide polynomials similar to long division for numbers.

$$\begin{array}{rl} & x \;\; +1 \\ x+4 \,\big) & \overline{x^2+5x+4} \\ & -(x^2+4x) \\ \hline & \qquad x+4 \\ & \quad -(\,x+4\,) \\ \hline \end{array}$$

The remainder = 0 $\boxed{0}$

Is *(x + 4)* a factor of: $x^2+5x\ +4$?

YES, according to the Factor Theorem.

The zero of *(x + 4)* is a zero of

$$f(x)=x^2+5x\ +4$$

Polynomial Long Division

$$x-1 \overset{x^2}{\overline{\big)\, x^3 + 3x^2 + 14x - 18}}$$

1) Look at left-most numbers

2) What # times “left” = “left”?

$$x^3 / x = ? \quad = x^2$$

3) Multiply

$$x^2\,(x-1) = x^3 - x^2$$

4) Subtract

$$-(x^3 - x^2)$$

Polynomial Long Division

$$
\begin{array}{rl}
 & x^2 \\ \cline{2-2}
x-1 & \big)\, x^3+3x^2+14x-18 \\
 & -(x^3-x^2) \\ \cline{2-2}
 & \qquad 4x^2\ \ +14x-18
\end{array}
$$

4) Subtract

Careful with the negatives!

5) Bring down.

Polynomial Long Division

$$
\begin{array}{r|l}
 & x^2 + 4x \\
\hline
x - 1 & x^3 + 3x^2 + 14x - 18 \\
 & -(x^3 - x^2) \\
\hline
 & 4x^2 + 14x - 18 \\
 & -(4x^2 - 4x) \\
\hline
 & 18x
\end{array}
$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

$$\frac{4x^2}{x} = ? \quad = 4x$$

3) Multiply

$$4x(x-1) \;= 4x^2 - 4x$$

4) Subtract

$$-(4x^2 - 4x)$$

Polynomial Long Division

$$
\begin{array}{r l}
 & x^2 \quad +4x \\
\cline{2-2}
x-1 & \big)\; x^3 + 3x^2 + 14x - 18 \\
 & -(x^3 - x^2) \\
\cline{2-2}
 & \qquad 4x^2 + 14x - 18 \\
 & \quad -(4x^2 - 4x) \\
\cline{2-2}
 & \qquad\qquad 18x \quad -18
\end{array}
$$

4) Subtract

Careful of the negatives

5) Bring down.

Polynomial Long Division

$$\begin{array}{r}
x^2 \; +4x \; +18 \\
x-1 \overline{\big) \; x^3+3x^2+14x-18} \\
-(x^3-x^2) \qquad\qquad\qquad \\
\hline
4x^2+14x-18 \\
-(4x^2-4x) \qquad\; \\
\hline
18x-18 \\
-(18x-18) \\
\hline
0
\end{array}$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times “left” = “left”?

$$\frac{18x}{x} = 18$$

3) Multiply

$$18(x-1)$$

$$18x-18$$

4) Subtract

$$-(18x-18)$$

$$
\begin{array}{r}
x^{2} + 4x - 18 \\
x-1 \,\overline{)\, x^{3} + 3x^{2} + 14x - 18} \\
-(x^{3} - x^{2}) \\
\hline
4x^{2} + 14x - 18 \\
-(4x^{2} - 4x) \\
\hline
18x - 18 \\
-(18x - 18) \\
\hline
0
\end{array}
$$

$$x^{3} + 3x^{2} + 14x - 18 = (x-1)(x^{2} + 4x - 18)$$

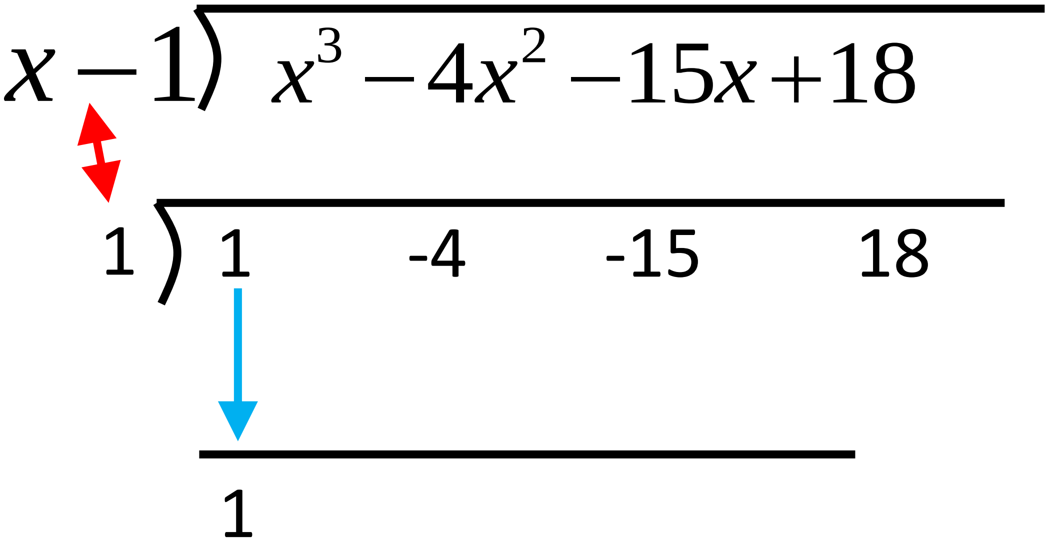
How do we find the zeroes of the unfactorable quadratic factor?

Convert to vertex form and take square roots.

Division with remainders

$$x - 2 \overline{\big)\, x^4 + x^3 - 4x^2 + 2x - 12}$$

Is there an easier way to do this? Yes!



$$x-1\overline{)\,x^3-4x^2-15x+18}$$

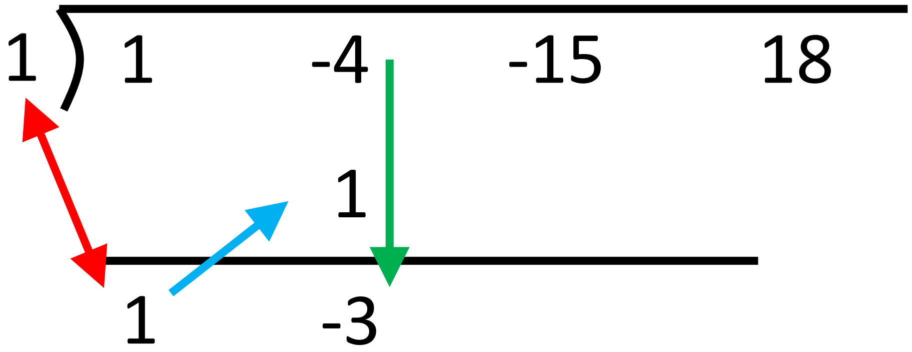
1st step: Write the polynomial with only its coefficients.

2nd step: Write the “zero” of the linear factor.

3rd step: add down

Is there an easier way to do this? Yes!

$$x-1\overline{\big)\,x^3-4x^2-15x+18}$$



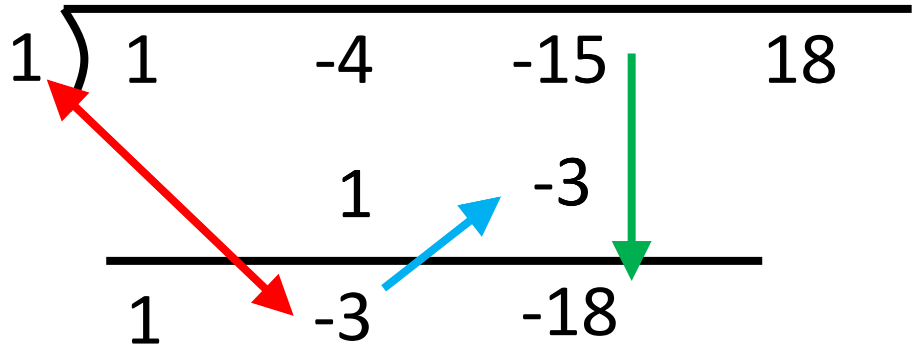
4th step: Multiply the “zero” by the lead coefficient.

5th step: Write the product under the next term to the right.

6th step: add the second column downward

Is there an easier way to do this? Yes!

$$x-1\overline{\smash{)}\,x^3-4x^2-15x+18}$$



1	1	-4	-15	18
		1	-3	
	1	-3	-18	

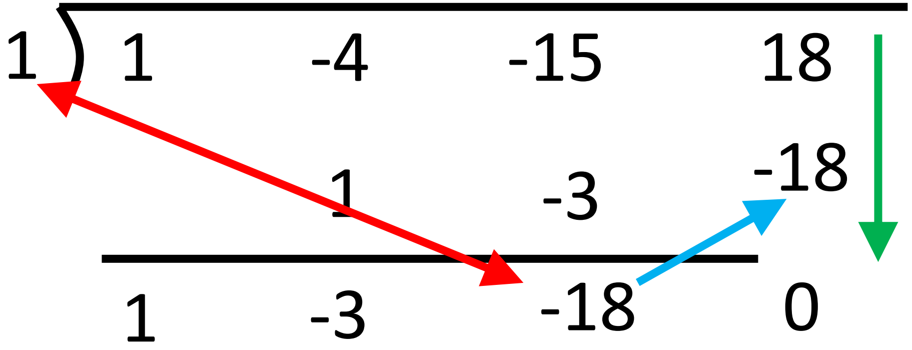
7th step: Multiply the “zero” by the second number

8th step: Write the product under the next term to the right.

9th step: add the next column downward

Is there an easier way to do this? Yes!

$$x-1\overline{)\,x^3-4x^2-15x+18}$$



$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

10th step: Multiply the “zero” by the 3rd number

11th step: Write the product under the next term to the right

12th step: add the next column downward

$$\begin{array}{r} x^2-3x-18 \\ x-1\overline{\smash{)}\,x^3-4x^2-15x+18} \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & \boxed{0} \end{array}$$

remainder

$$x^3-4x^2-15x+18 \div (x-1) \boxed{= x^2-3x-18}$$

Because the remainder = 0, then (x – 1) is a factor AND x = 1 is a zero of the original polynomial!

Is (x + 3) a factor of f(x)?

$$f(x) = 3x^3 - x^2 - 20x - 12$$

$$\begin{array}{r|rrrr} -3 & 3 & -1 & -20 & -12 \\ & & & & \\ \hline & & & & \end{array}$$

Is (x + 2) a factor of f(x)?

$$f(x) = 3x^3 - x^2 - 20x - 12$$

-2) 3 -1 -20 -12

The "gotcha" of Synthetic Division is that you must account for the missing terms of the polynomial (whose coefficients = 0).

$$x^2-4=(x+2)(x-2)$$

$$x-2\,\overline{\big)\;x^2-4\;}$$

$$\begin{array}{r|rr} 2 & 1 & -4 \\ & & 2 \\ \hline & 1 & -2 \end{array}$$

remainder not = 0

$$x-2\,\overline{\big)\;x^2+0x-4\;}$$

$$\begin{array}{r|rrr} 2 & 1 & 0 & -4 \\ & & 2 & 4 \\ \hline & 1 & 2 & 0 \end{array}$$

remainder = 0

Another "gotcha" of Synthetic Division is that you can divide by *linear factors whose lead coefficient = 1.*

$$x-2\,\overline{\smash{)}\,x^3-x^2+x-1}$$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & 1 & -1 \\ & & 2 & 2 & 6 \\ \hline & 1 & 1 & 3 & \boxed{5} \end{array}$$

The remainder of division is the output value when the input is the zero of the divisor!

remainder = 5

$$(x^3-x^2+x-1)\div(x-2) = x^2+x+3+\frac{5}{x-2}$$

Now here is something that is really cool.

$$f(x)=x^3-x^2+x-1$$

$$f(2)=?$$

$$f(2)=(2)^3-(2)^2+(2)-1$$

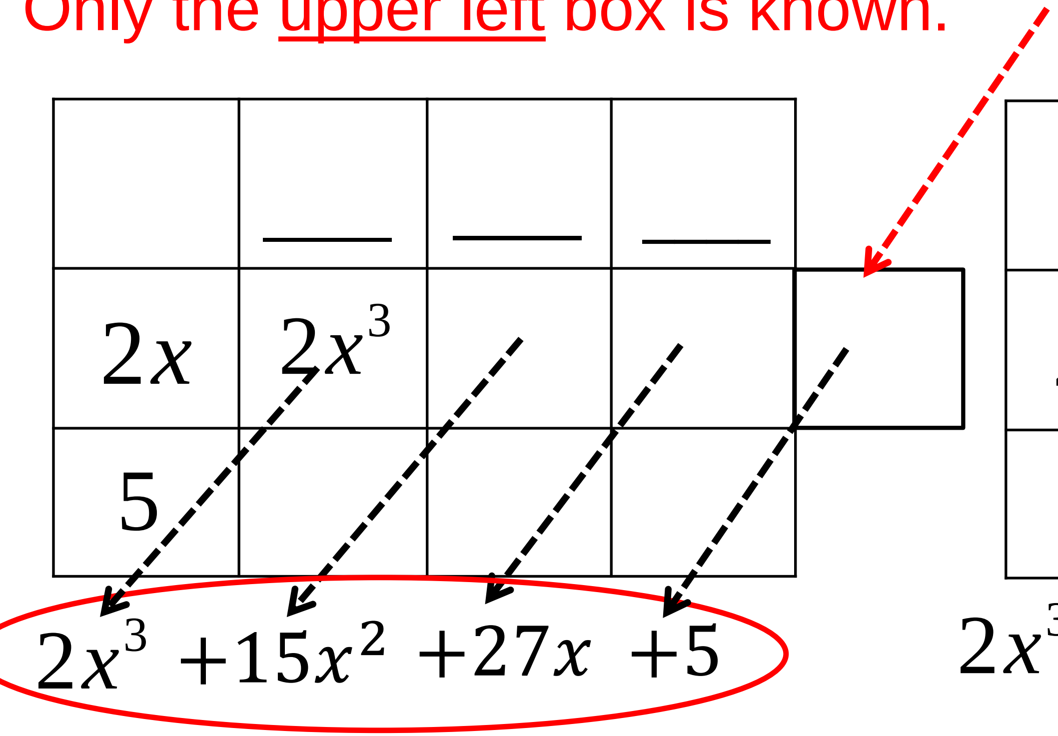
$$\boxed{f(2)=5}$$

Polynomial Division Box Method

$$(2x^3+15x^2+27x+5)\div(2x+5)$$

Only the upper left box is known.

Remainder



Diagonals have “like terms”

	x^2	5x	1	
2x	$2x^3$	10x²	2x	0
5	5x²	25x	5	

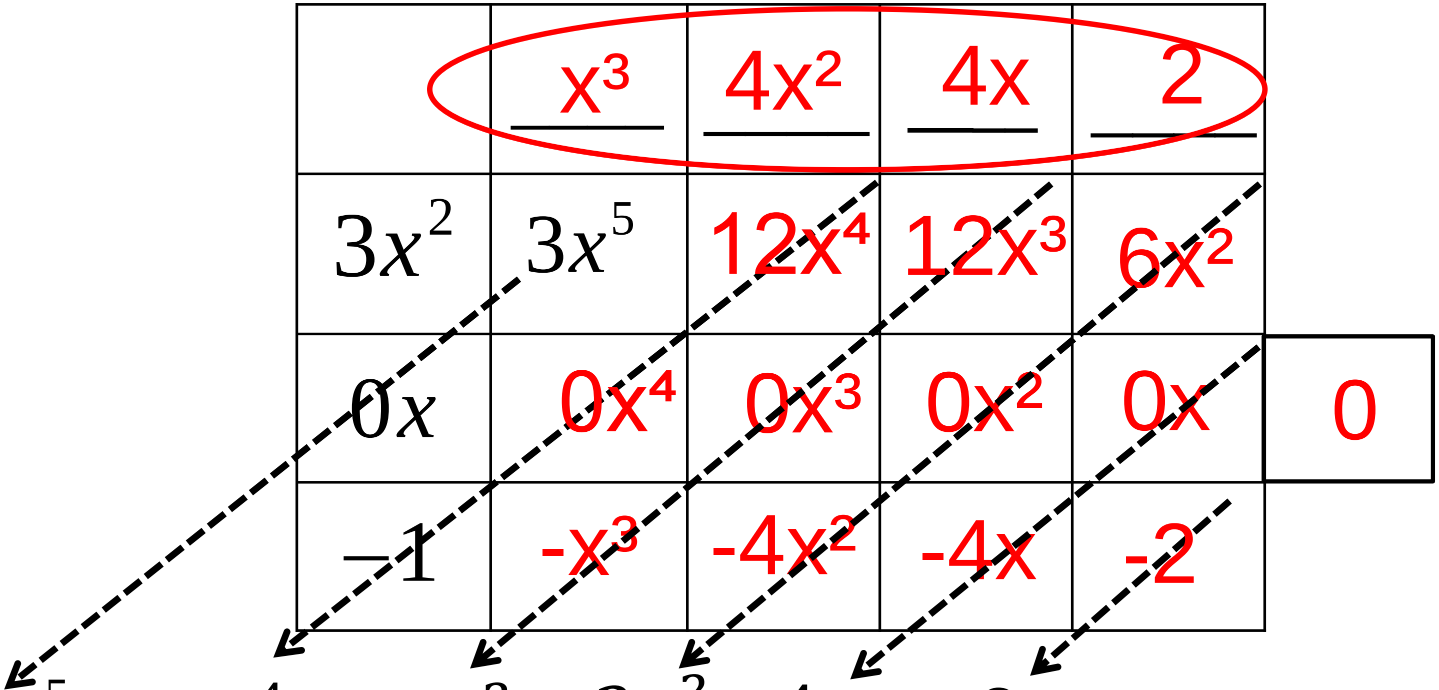
$$2x^3+15x^2+27x+5$$

$$(2x^3+15x^2+27x+5)\div(2x+5)=x^2+5x+1$$

Division of Polynomials Box Method

$$(3x^5 + 12x^4 + 11x^3 + 2x^2 - 4x - 2) \div (3x^2 - 1)$$

Only the upper left box is known. Diagonals have “like terms”



	$\underline{x^3}$	$\underline{4x^2}$	$\underline{4x}$	$\underline{2}$	
$3x^2$	$3x^5$	$12x^4$	$12x^3$	$6x^2$	
$0x$	$0x^4$	$0x^3$	$0x^2$	$0x$	0
-1	$-x^3$	$-4x^2$	$-4x$	-2	

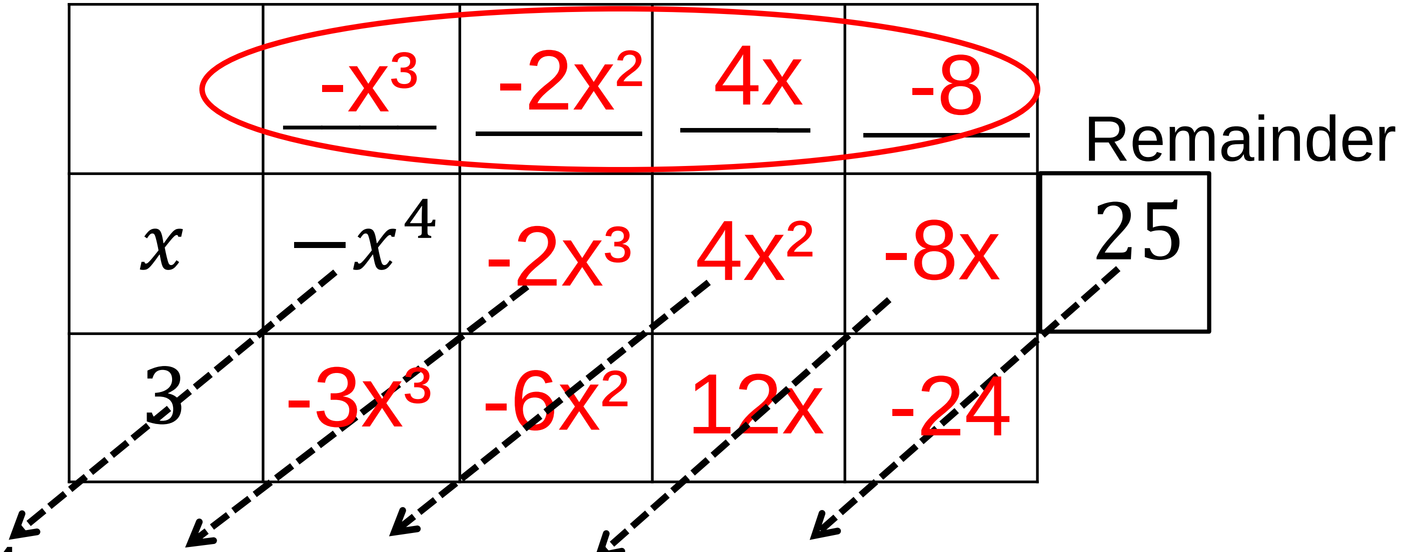
$$3x^5 + 12x^4 + 11x^3 + 2x^2 - 4x \quad -2$$

$$(3x^5 + 12x^4 + 11x^3 + 2x^2 - 4x - 2) \div (3x^2 - 1)$$
$$= x^3 + 4x^2 + 4x + 2$$

Division with remainders

$$(-x^4 - 5x^3 - 2x^2 + 4x + 1) \div (x + 3)$$

Only the upper left box is known. Diagonals have “like terms”



	$-x^3$	$-2x^2$	$4x$	-8	Remainder
x	$-x^4$	$-2x^3$	$4x^2$	$-8x$	25
3	$-3x^3$	$-6x^2$	$12x$	-24	

$$-x^4 -5x^3 -2x^2 +4x \quad +1$$

$$(-x^4 - 5x^3 - 2x^2 + 4x + 1) \div (x + 3)$$

$$\boxed{= -x^3 - 2x^2 + 4x - 8 + \frac{25}{x + 3}}$$

Divide. $x^4 + x^3 - 4x^2 + 2x - 12 \quad \div x^2 - 2$

Divide. $2x^4 + 9x^3 - 8x^2 - 15x \ \div 2x - 3$

$$y = x^5 - 5x^4 + 12x^3 - 24x^2 + 32x - 16$$

1. Test to see if ‘1’ is a zero

$$\begin{array}{r|rrrrrr} 1 & 1 & -5 & 12 & -24 & 32 & -16 \\ & & 1 & -4 & 8 & -16 & 16 \\ \hline & 1 & -4 & 8 & -16 & 16 & \boxed{0} \end{array}$$

2. Test to see if ‘1’ is a zero (with multiplicity two)

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 8 & -16 & 16 \\ & & 1 & -3 & 5 & -11 \\ \hline & 1 & -3 & 5 & -11 & \boxed{5} \end{array}$$

‘1’ is not a repeat zero → x = 1 (multiplicity 1)

$$y = (x - 1)(x^4 - 4x^3 + 8x^2 - 16x + 16)$$

‘1’ is not a repeat zero → x = 1 (multiplicity 1)

$$\begin{array}{r|rrrrrr} 1 & 1 & -5 & 12 & -24 & 32 & -16 \\ & & 1 & -4 & 8 & -16 & 16 \\ \hline & 1 & -4 & 8 & -16 & 16 & \boxed{0} \end{array}$$

3. Test to see if ‘2’ is a zero.

$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 8 & -16 & 16 \\ & & 2 & -4 & 8 & -16 \\ \hline & 1 & -2 & 4 & -8 & \boxed{0} \end{array}$$

$$y = (x - 1)(x - 2)(x^3 - 2x^2 + 4x - 8)$$

$$\boxed{y = (x-1)(x-2)(x^3 - 2x^2 + 4x - 8)}$$

4. Try "box factoring" of the 3rd – degree polynomial factor

or continue trying synthetic division.

	x	-2
x^2	x^3	$-2x^2$
4	$4x$	-8

$$\boxed{y = (x-1)(x-2)(x^2+4)(x-2)}$$

5. List the zeroes.

x = 1, *2i, -2i, 2 (multiplicity 2)*