Math-3

Lesson 2-7

Difference of Cubes

Sum of Cubes

Quadratic Form

Convert to standard form:

$$y = (x - 3)(x^2 + 3x + 9)$$

$$y = x^3 - 27$$

	x	-3
x^2	x^3	$-3x^2$
$3x$	$3x^2$	$-9x$
9	$9x$	-27

$$y = x^3 + 0x^2 + 0x - 27$$

There are NO x^2 terms
and NO ‘x’ terms

$$y = x^3 - 1$$

<u>Factoring the Difference of cubes</u>:

The upper-left term in the box is x^3

	x	-1
ax^2	x^3	
bx		
c		

$$x^3 + 0x^2 + 0x - 1$$

Find the 1st zero.

$$0 = x^3 - 1$$

$$1 = x^3$$

$$1 = x$$

→ Comes from the factor (x – 1)

	x	-1
x^2	x^3	$-1x^2$
$1x$	$1x^2$	$-1x$
1	$1x$	-1

There are *NO* x^2 terms and *NO* 'x' terms

$$y = (x - 1)(x^2 + 1x + 1)$$

$$y = x^3 + 8$$

Factoring the Sum of cubes:

The upper-left term in the box is x^3

	x	2
ax^2	x^3	
bx		
c		

$$x^3 + 0x^2 + 0x + 8$$

Find the 1st zero.

$$0 = x^3 + 8$$

$$-8 = x^3$$

$$-2 = x$$

→ Comes from the factor (x + 2)

There are *NO* x^2 terms and *NO* 'x' terms

	x	2
x^2	x^3	$2x^2$
$-2x$	$-2x^2$	$-4x$
4	$4x$	8

$$y = (x + 2)(x^2 - 2x + 4)$$

Factor the Sum of cubes:

$$y = x^3 + 125$$

	x	5
x^2	x^3	$5x^2$
$-5x$	$-5x^2$	$-25x$
25	$25x$	125

$$y = (x+5)(ax^2 + bx + c)$$

$0x^2$ $0x$

$$y = (x+5)(x^2 - 5x + 25)$$

Factor the Difference of cubes:

$$y = x^3 - 64$$

	x	-4
x^2	x^3	$-4x^2$
$4x$	$4x^2$	$-16x$
16	$16x$	-64

$$y = (x - 4)(ax^2 + bx + c)$$

$0x^2$ $0x$

$$y = (x - 4)(x^2 + 4x + 16)$$

“Nice” Difference of Squares (of higher degree):

$$y = x^4 - 81$$

Use “m” substitution Let $m^2 = x^4$

Then $m = x^2$

$$y = m^2 - 81$$

$$y = (m + 9)(m - 9)$$ Use “m” substitution

$$y = (x^2 + 9)(x^2 - 9)$$

$$y = (x + 3)(x - 3)(x + 3i)(x - 3i)$$

Find the *zeroes*. $x =$ -3, 3, -3i, 3i

$$y = x^4 - 64$$

Find the zeroes. $y = (x-7)(x^2 - 6x + 2)$

$x = 7$, ?, ?

How do you find the zeroes of an "unfactorable" quadratic factor? Convert to vertex form then take square roots.

x =

$$y = (x-1)(x^2 + 1x + 1)$$

$x = 1$, ?, ?

$$x^2 + 1x + 1 = 0 \qquad x_{vertex} = \frac{-b}{2a} = \frac{-(1)}{2(1)} = \frac{-1}{2}$$

$$y_{vertex} = \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right) + 1 = \frac{3}{4}$$

$$0 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} \qquad -\frac{3}{4} = \left(x + \frac{1}{2}\right)^2$$

$$\pm\frac{i\sqrt{3}}{2} = x + \frac{1}{2} \qquad x = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$$

"Quadratic Form": a trinomial that looks like a quadratic but has a larger degree.

$y = x^4 + 3x^2 + 2$ Looks like → $y = m^2 + 3m^1 + 2$

Factors similarly

$y = (x^2 + 2)(x^2 + 1)$ $y = (m + 2)(m + 1)$

BUT, according to the Linear Factorization Theorem, it factors into 4 linear factors.

$$y = (x + i\sqrt{2})(x - i\sqrt{2})(x + i)(x - i)$$

Zeroes: $x = i\sqrt{2}, -i\sqrt{2}, i, -i$

Find the zeroes

$$y = x^4 - 16x^2 + 28$$

$$y = (x^2 - 14)(x^2 - 2)$$

$$y = (x + \sqrt{14})(x - \sqrt{14})(x + \sqrt{2})(x - \sqrt{2})$$

Zeroes: $x = \sqrt{14}, -\sqrt{14}, \sqrt{2}, -\sqrt{2}$

$$y = 16x^4 - 81$$

$$y = (4x^2 - 9)(4x^2 + 9)$$

$$y = (2x - 3)(2x + 3)(2x - 3i)(2x + 3i)$$

$$x = \frac{3}{2}, -\frac{3}{2}, \frac{3i}{2}, -\frac{3i}{2}$$