Math-3

Lesson 2-6

Zeroes of “NICE” 3rd Degree Polynomials

Find the zeroes of the following 3rd degree Polynomial

$y = x^3 + 5x^2 + 4x$ Set y = 0

$0 = x^3 + 5x^2 + 4x$ Factor out the common factor.

$0 = x(x^2 + 5x + 4)$ Factor the quadratic

$0 = x(x+1)(x+4)$ Identify the zeroes

0, -1, -4

“Nice” 3rd Degree Polynomial (with no constant term)

$$y = 3x^3 + 12x^2 - 36x$$

It has no constant term so it can easily be factored into ‘x’ times a quadratic factor.

$$y = 3x(x^2 + 4x - 12)$$

If the quadratic factor is “nice” we can factor that into 2 binomials.

$$y = 3x(x+6)(x-2)$$

x = 0, -6, 2

This is now “intercept form” so we can “read off” the x-intercepts. What are they?

Find the Zeroes

$$y = x^3 - 6x^2 - 16x$$

$$y = x^3 + 4x^2 - 12x$$

$$y = 2x^3 - 6x^2 - 30x$$

Find the Zeroes

$$y = x^3 + 6x^2 - 4x$$

$$y = x^3 + 4x^2 - 11x$$

$$y = 2x^3 - 16x^2 + 14x$$

“Nice” 3rd Degree Polynomial (with no constant term)

$$y = x^3 + 6x^2 + 4x + 0$$
$$0 = x^3 + 6x^2 + 4x$$

It has no constant term so it can easily be factored into ‘x’ times a quadratic factor. $0 = x(x^2 + 6x + 4)$

x = 0

What if the quadratic factor is not factorable?

$$x = \frac{-b}{2a} = \frac{-(6)}{2(1)}$$

$x = -3$

$$y = f(-3) = (-3)^2 + 6(-3) + 4$$

$y = -5$

Convert the quadratic factor into vertex form and solve.

$$0 = (x+3)^2 - 5$$
$$x = -3 \pm \sqrt{5}$$

Zeroes:
$x = 0, -3 \pm \sqrt{5}$

Factor the following “nice” 3rd degree polynomials then find the “zeroes” of the polynomial.

$$y = x^3 + 5x^2 - 14x$$

$$0 = x^3 + 5x^2 - 14x$$

$$0 = x(x^2 + 5x - 14)$$

$$0 = x(x + 7)(x - 2)$$

0, -7, 2

$$y = 3x^3 - 24x^2 + 6x$$

$$0 = 3x(x^2 - 8x + 2)$$

x = 0

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(1)}$$

$x = 4$

$$y = f(4) = (4)^2 - 8(4) + 2$$

$y = -14$ $\quad 0 = (x - 4)^2 - 14$

$$x = 4 \pm \sqrt{14}$$

Find the zeroes using "box factoring"

$$y = 4x^3 - 5x^2 + 12x - 15$$

	$4x$	-5
x^2	$4x^3$	$-5x^2$
3	$12x$	-15

$$0 = (x^2 + 3)(4x - 5)$$

$$0 = x^2 + 3 \qquad 4x - 5 = 0$$

$$-3 = x^2 \qquad 4x = 5$$

$$x = \pm i\sqrt{3} \qquad x = {}^5/_4$$

$$x = i\sqrt{3},\ -i\sqrt{3},\ {}^5/_4$$

Box Factoring works for “Nice” 3rd Degree Polynomial

$$y = ax^3 + bx^2 + cx + d$$

This has the constant term, but it has a very useful feature:

$$y = 1x^3 + 2x^2 + 2x + 4$$

What pattern do you see?

$$3rd/1st = 2/1 \qquad 4th/2nd = 4/2 = 2/1$$

If we divide the coefficients of the 1st and 3rd terms, and the coefficient of the 2nd term and the constant, we get the same number.

"factor by grouping" if it has this nice pattern.

$$y = 1x^3 + 2x^2 + 2x + 4$$

Group the 1st and last pair of terms with parentheses.

$$y = (x^3 + 2x^2) + (2x + 4)$$

Factor out the common term from the first group.

$$y = x^2(x + 2) + (2x + 4)$$

Factor out the common term from the last group.

$$y = x^2(x + 2) + 2(x + 2)$$

Common factor of (x + 2)

$$y = x^2(x+2) + 2(x+2)$$

These two binomials are now common factors of

$$x^2 \text{ and } 2$$

Factor out the common binomial term.

$$y = (x+2)(x^2+2)$$

$$x = -2,\ i\sqrt{2},\ -i\sqrt{2}$$

Apply the Complex Conjugates Theorem

An easier method is "box factoring" (if it has this nice pattern).

$$y = 1x^3 + 2x^2 + 2x + 4$$

These 4 terms are the *numbers in the box*.

Find the *common factor* of the 1st row.

Fill in the rest of the box.

	x	2
x^2	x^3	$2x^2$
2	$2x$	4

Rewrite in intercept form.

$$y = 1x^3 + 2x^2 + 2x + 4$$

$$y = (x^2 + 2)(x + 2)$$

Find the "zeroes."

$$0 = (x^2 + 2)(x + 2)$$

$$0 = x^2 + 2 \qquad 0 = x + 2$$

$$-2 = x^2 \qquad x = -2$$

$$x = \pm i\sqrt{2}$$

Find the zeroes using "box factoring"

$$y = 2x^3 + 3x^2 + 4x + 6$$

x =

$$y = 4x^3 + 8x^2 + 5x + 10$$

x =

$$y = -6x^3 + 7x^2 - 18x + 21$$

x =

What have we learned so far?

"Nice" Common Factor 3rd degree polynomial:

$$y = x^3 + 3x^2 + 2x \qquad = x(x^2 + 3x + 2)$$

$$= x(x+1)(x+2)$$

"Nice" Factor by box 3rd degree polynomial:

$$y = 2x^3 + 3x^2 + 4x + 6$$