Math-3

Lesson 2-2

Factoring Quadratics with Lead Coefficient Not = 1,

Irrational and Complex Conjugates

When factoring a quadratic expression, what if there is no common factor AND the lead coefficient is NOT equal to 1?

$$ax^2 + bx + c$$

(These come from multiplying binomials that also do not have lead coefficients of 1.)

$$(2x + 1)(x + 3)$$

Use the "box method" to multiply the binomials

$$2x^2 + 7x + 3$$

	x	3
2x	$2x^2$	6x
1	x	3

Notice a nice pattern when you multiply this out ("simplify")

$$(2x+1)(x+3)$$

$$\boxed{2x^2+7x+3}$$

Left times left is left

Right times right is right

"right plus right" *does not* add up to 7, but notice something.

$$6x$$

$$(2x+1)(x+3)$$

$$x$$

$$6x+x=7x$$

$$2x^2+7x+3$$

$$2*3=6$$

Are there any other factors of 6 that add up to 7?

$$1+6=7$$

$$6=1*6$$

$2 * 15 = 30$

$$2x^2 + 13x + 15$$

$10 + 3 = 13$

$30 = 10 * 3$

Are there any *other* factors of 30 that add up to 13?

This tells us to break 13x into 10x + 3x

$$2x^2 + 13x + 15$$

$$2x^2 + 10x + 3x + 15$$

These are all of the terms in “the box”

	x	5
2x	$2x^2$	10x
3	3x	15

What is the bottom-left term in the box?

x*(3) = 3x

What is the top-right term in the box?

2x*(5) = 10x

Final check: 3*5 = 15?

Factored form:

$$2x^2 + 13x + 15$$

$$\rightarrow (2x + 3)(x + 5)$$

$$4 * 10 = 40$$

$$4x^2 + 13x + 10$$

$$8 + 5 = 13$$

$$40 = 8 * 5$$

Other factors of 40 that add up to 13?

This tells us to break 13x into 8x + 5x

$$4x^2 + 13x + 10$$

$$4x^2 + 8x + 5x + 10$$

Factored form:

$$4x^2 + 13x + 10$$
$$\rightarrow (x + 2)(4x + 5)$$

These are all of the terms in "the box"

	4x	5
x	$4x^2$	5x
2	8x	10

Since $4x^2$ can be factored 2 ways, look for the common factors of the 1st row.

'x' is the common factor of $4x^3$ and 5x

Look for the common factors of the 1st column

'4x' is the common factor of $4x^3$ and 8x

4x*(2) = 8x

x*(5) = 5x

Final check: 2*5 = 10?

$$3 * 8 = 24$$

$$3x^2 + 14x + 8$$

$$2 + 12 = 14$$

Other factors of 24 that add up to 14?

$$24 = 2 * 12$$

This tells us to break 14x into 2x + 12x

$$3x^2 + 14x + 8$$

$$3x^2 + 2x + 12x + 8$$

These are all of the terms in "the box"

	x	4
3x	$3x^2$	12x
2	2x	8

What is the bottom-left term in the box?

x*(2) = 2x

What is the top-right term in the box?

3x*(4) = 12x

Final check: 2*4 = 8?

Factored form:

$$3x^2 + 14x + 8$$

$$\rightarrow (3x + 2)(x + 4)$$

Factor

5*4= ___

$$5x^2 + 12x + 4$$

___ * ___ = ___

___ + ___ = 12

11*(-9) = ___

$$11x^2 + 2x - 9$$

___ * ___ = ___

___ + ___ = 2

Factor

9*10 = ____

$$9x^2 - 13x - 10$$

____*____= ____

____+____= -13

12*5 = ____

$$12x^2 - 16x + 5$$

____*____= ____

____+____= -16

Factor

____ * ____ = ____

$$6x^2 - 5x - 6$$

____ * ____ = ____

____ + ____ = ____

____ * ____ = ____

$$8x^2 - 2x - 3$$

____ * ____ = ____

____ + ____ = ____

Factor

_____ * _____ = _____

$$7x^2 - 12x - 4$$

_____ * _____ = _____

_____ + _____ = _______

_____ * _____ = _____

$$6x^2 - 29x + 9$$

_____ * _____ = _____

_____ + _____ = _____

$x^2 - 1$

"the difference of two squares"

$x^2 + 0x - 1$

Two numbers multiplied = (-1)
and added = 0

$(-1)(+1)$

$(x - 1)(x + 1)$

"***conjugate binomial pair***"

$x^2 - 1 \quad \rightarrow (x - 1)(x + 1)$

$x^2 - 4 \quad \rightarrow (x - 2)(x + 2)$

$x^2 - 9 \quad \rightarrow (x - 3)(x + 3)$

$x^2 - 16 \quad \rightarrow (x - 4)(x + 4)$

$$x^2 - 1 \quad \rightarrow (x-1)(x+1)$$

"***conjugate binomial pair***"

$$x^2 - 4 \quad \rightarrow (x-2)(x+2)$$

"***conjugate binomial pair***"

Follows this pattern

$$\rightarrow \left(\sqrt{x^2} - \sqrt{4}\right)\left(\sqrt{x^2} + \sqrt{4}\right)$$

What if the second term is not a "perfect square"?

$$x^2 - 2 \quad \rightarrow \left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)$$

"***irrational conjugate binomial pair***"

$$x^2 - 3 \quad \rightarrow \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)$$

"***irrational conjugate binomial pair***"

Use the "box method" to multiply the following binomial pairs

$(x-\sqrt{5})(x+\sqrt{5})$

	x	$-\sqrt{5}$
x		
$\sqrt{5}$		

$(x-i)(x+i)$

	x	-i
x		
i		

$(x-\sqrt{7})(x+\sqrt{7})$

	x	$-\sqrt{7}$
x		
$\sqrt{7}$		

$(x-2i)(x+2i)$

	x	-2i
x		
2i		

Imaginary Numbers

$$x = \sqrt{-1}$$

$$x^2 = \left(\sqrt{-1}\right)^2$$

$$x^2 = -1$$

Factor the following:

$$x^2 + 3$$

$$x^2 + 4$$

$$x^2 - 5$$

$$x^2 - 17$$