Math-3

Lesson 1-6

Modeling Cooling

with the

Exponential Function

What is the equation of the graph? Passes through (0, 5) and (-1, 6)

$$g(x) = ab^x + k$$

1) horizontal asymptote $y = 2$

$k = 2$ $\quad y = ab^x + 2$

2) y-intercept $(0, 5)$

Substitute (0, 5) into the equation.

$5 = ab^0 + 2$ $\quad a = 3$

$y = 3b^x + 2$

3) "Nice" x-y pair $(-1, 6)$

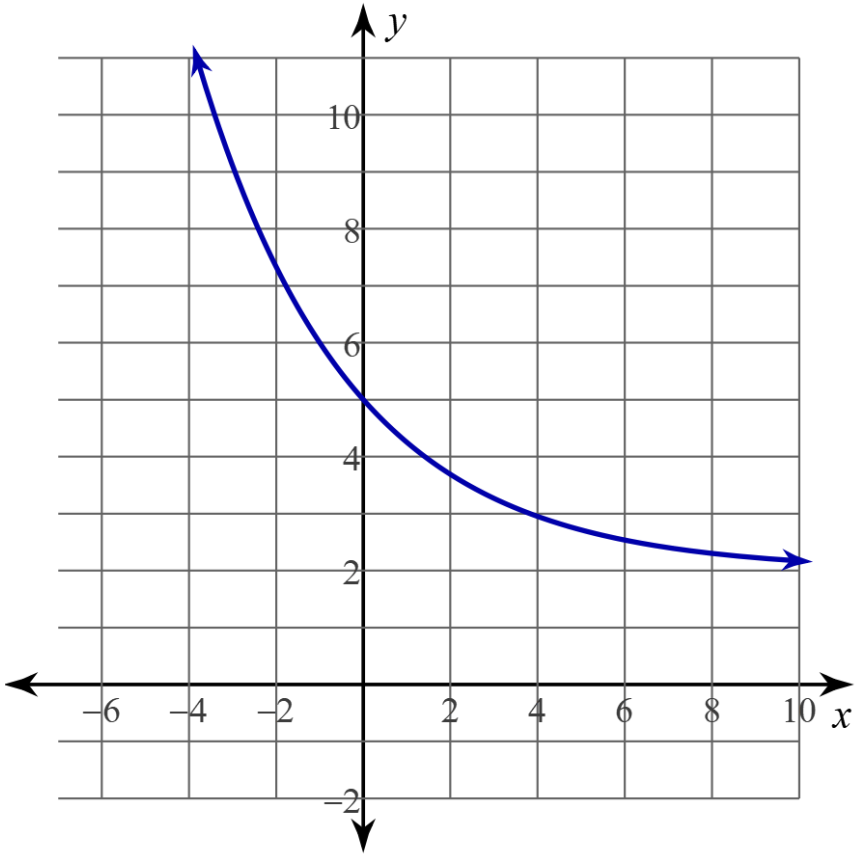
Substitute (-1, 6) into the equation.

$6 = 3b^{-1} + 2$

$4 = 3b^{-1}$

$\frac{4}{3} = b^{-1}$ $\quad b = \frac{3}{4}$

$$y = 3\left(\frac{3}{4}\right)^x + 2$$



$$g(x) = ab^x + k$$

1) Horizontal Asymptote: y = 0

$g(x) = ab^x + k$ $\quad k = 0$

Equation: $y = ab^x$

2) y-intercept: $(0,3)$

$3 = ab^0 \quad a = 3$

Equation: $y = 3b^x$

3) An x-y pair (preferably with x = 1)

$(2, 15)$

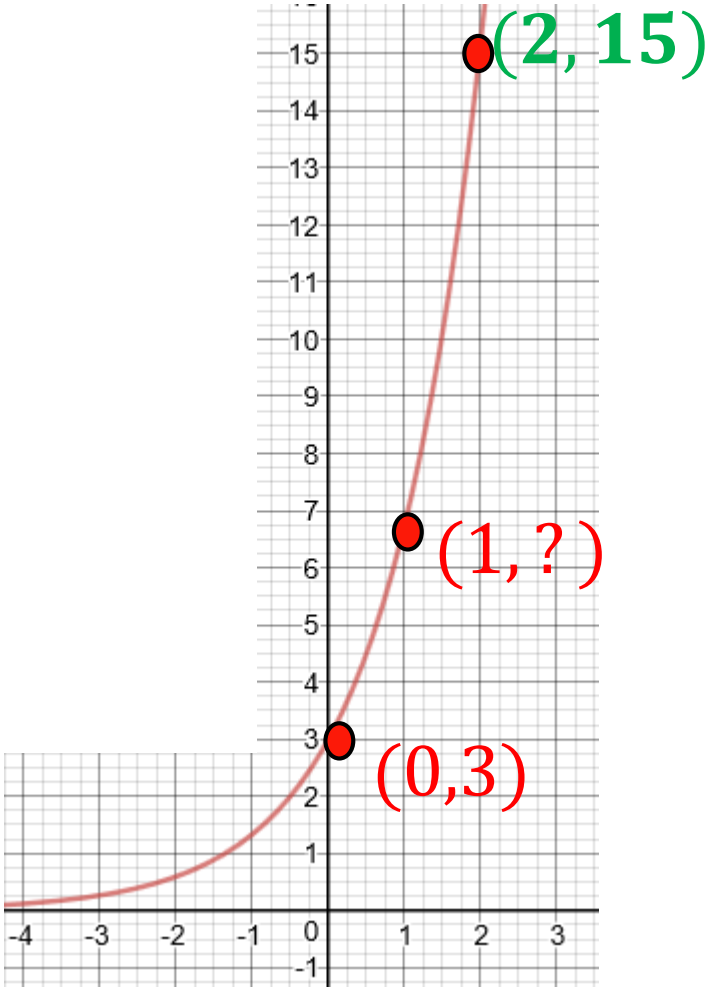
$15 = 3b^2$

$5 = b^2$

$\sqrt{b^2} = \sqrt{5}$

$b = 2.236$

$y = 3(2.236)^x$



$$g(x) = ab^x + k$$

1) Horizontal Asymptote: y = 0

$g(x) = ab^x + k$ $\quad k = 0$

Equation: $y = ab^x$

2) y-intercept: $(0,3)$

$3 = ab^0 \quad a = 3$

Equation: $y = 3b^x$

3) An x-y pair (preferably with x = 1)

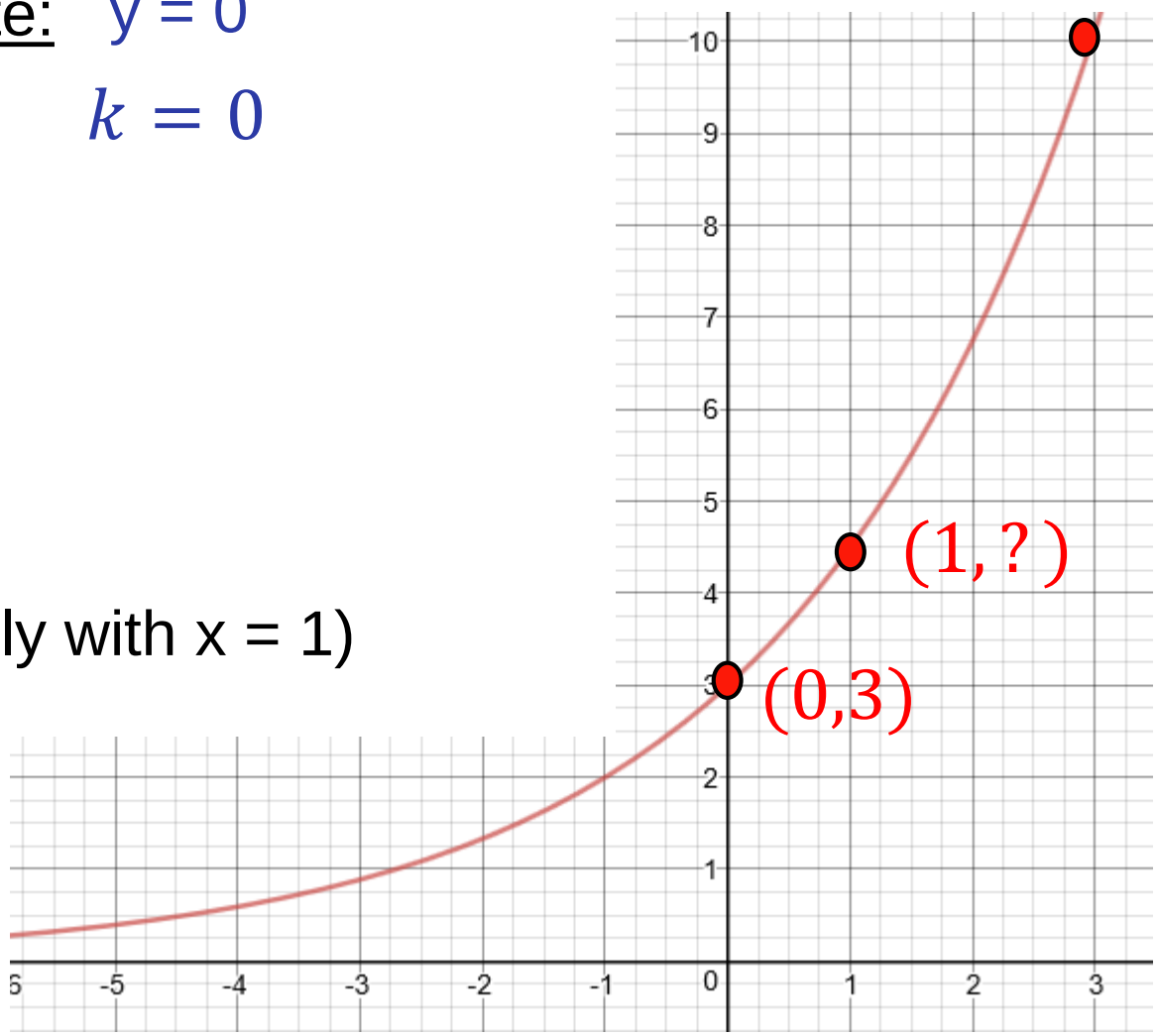
$(3, 10)$

$10 = 3b^3$

$3.333 = b^3$

$\sqrt[3]{b^3} = \sqrt[3]{3.3333}$

$b = 1.4938$



$y = 3(1.4938)^x$

Quantity: a category of measurements in the real world.

Unit of Measure: the unit that is used to measure a quantity.

Examples of quantities:	Examples of units of measure:
Height	(Height) → inches, feet, miles
Weight	(Weight) → pounds, kilograms
Temperature	(Temperature) → degrees Fahrenheit or Celsius

Suppose boiling water (212º F) is taken off the stove to cool in a room that is at 70 F.

Your turn: draw a graph of what you think the temperature will look like as time passes by (temperature as a function of time).

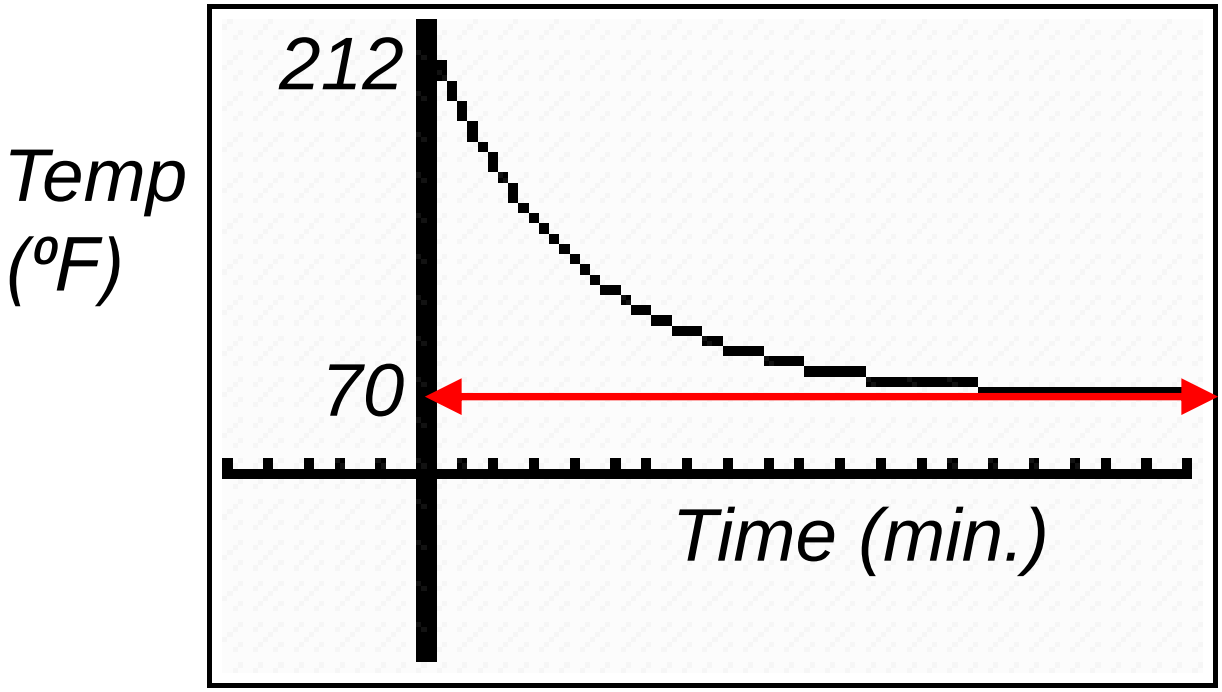
Label the x-axis and y-axis with quantity and unit of measure

At what temperature does it start?

Does the temperature go down forever?

What temperature will the water end up at?

Will it take hours, or minutes, or seconds to cool down?



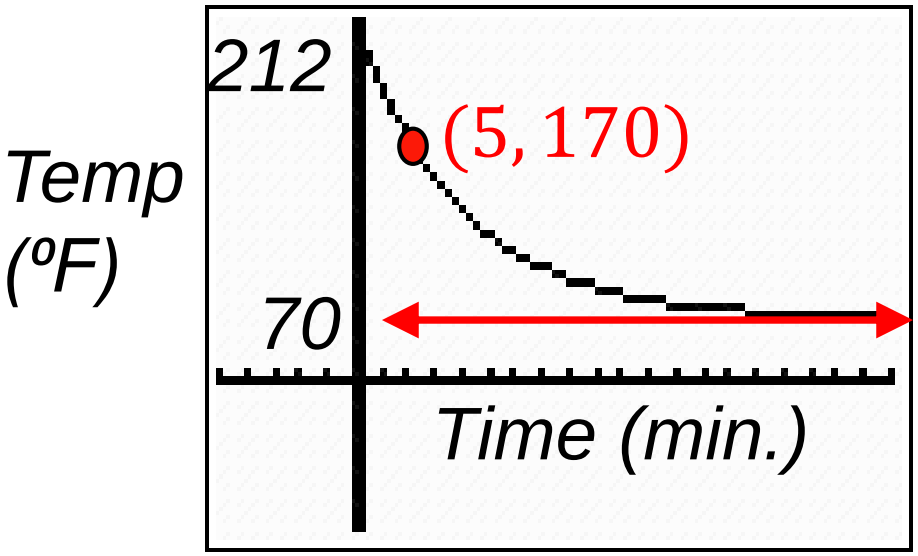
$$T(t) = ab^t + k$$

Horizontal asymptote

VSF

Growth (decay) factor

Initial Value: the y-intercept



$$T(t) = ab^t + k$$

Step 1: horizontal asymptote

$$k = 70$$

$$T(t) = ab^t + 70$$

Step 2: y-intercept → (0, 212)

$$212 = ab^0 + 70$$

$$a = 212 - 70 = 142$$

$$T(t) = 142b^t + 70$$

Step 3: need an x-y pair to plug in

After 5 minutes, the temperature is 170 F. → (5, 170)

$$170 = 142b^5 + 70$$

$$100 = 142b^5$$

$$\frac{100}{142} = b^5$$

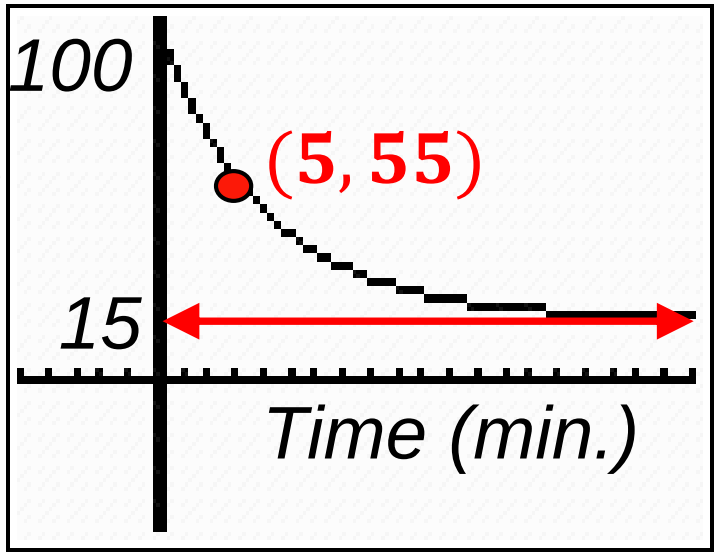
$$b^5 = 0.704$$

$$\sqrt[5]{b^5} = \sqrt[5]{0.704}$$

$$b = 0.932$$

$$T(t) = 142(0.932)^t + 70$$

A hard-boiled egg at temperature 100°C is placed in 15°C water to cool. Five minutes later the temperature of the egg is 55°C. What will be the temperature after 10 minutes?



$$T(t) = ab^t + k$$

Step 1: horizontal asymptote

$$k = 15$$

$$T(t) = ab^t + 15$$

Step 2: y-intercept → (0, 100)

$$100 = ab^0 + 15$$

$$a = 100 - 15 = 85$$

$$T(t) = 85b^t + 15$$

Step 3: an x-y pair to plug in → (5, 55)

$$55 = 85b^5 + 15$$

$$40 = 85b^5$$

$$\frac{40}{85} = b^5$$

$$b^5 = 0.471$$

$$\sqrt[5]{b^5} = \sqrt[5]{0.471}$$

$$b = 0.86$$

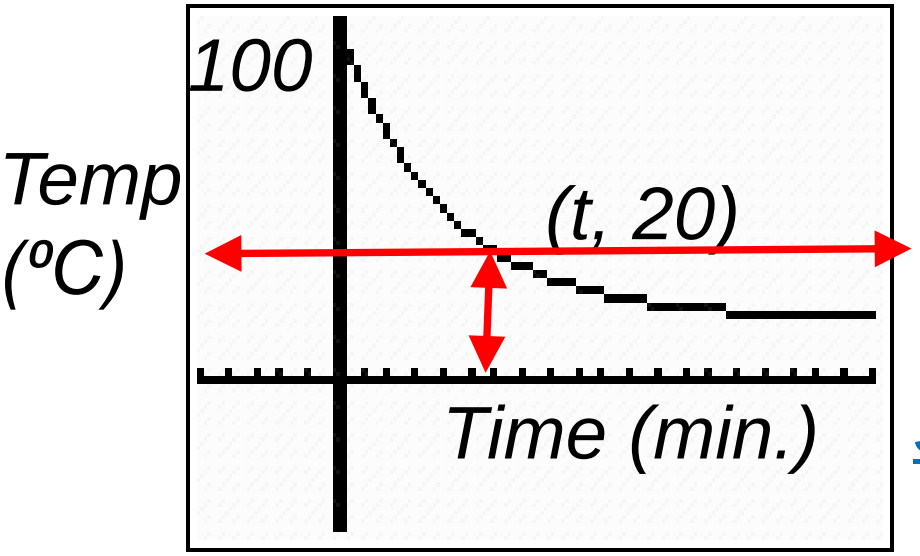
$$T(t) = 85(0.86)^t + 15$$

$$T(10) = 85(0.86)^{10} + 15$$

$$T(10) = 33.8\ C^0$$

When will the temperature reach 20 C?

$$T(t) = 85(0.86)^t + 15$$



Solve by graphing

Step 1: graph your equation

$$y = 85(0.86)^x + 15$$

Step 2: graph the equation

$$y = 20$$

Step 3: find the point of intersection

$$\text{Time} = 18.8\ minuts$$