Math-3
Lesson 1-5

Exponential Function

The “Parent” Exponential Function

$$y = b^x$$

exponent ← x; base ← b

$y = 2^x$ **(base 2 exponential function)**

$y = 3^x$ **(base 3 exponential function)**

$y = \left(\frac{1}{2}\right)^{x}$ **(base 1/2 exponential function)**

The base MUST BE <u>positive</u> and CANNOT equal 1.

$$b = (0, 1) \cup (1, \infty)$$

Fill in the output values of the table and graph the points.

$$f(x) = 2^x$$

Growth Factor is the base of the exponential

x	$2^{(\ )}$	y
-2	2^{-2}	0.25
-1	2^{-1}	0.5
0	2^{0}	1
1	2^{1}	2
2	2^{2}	4

* 2

* 2

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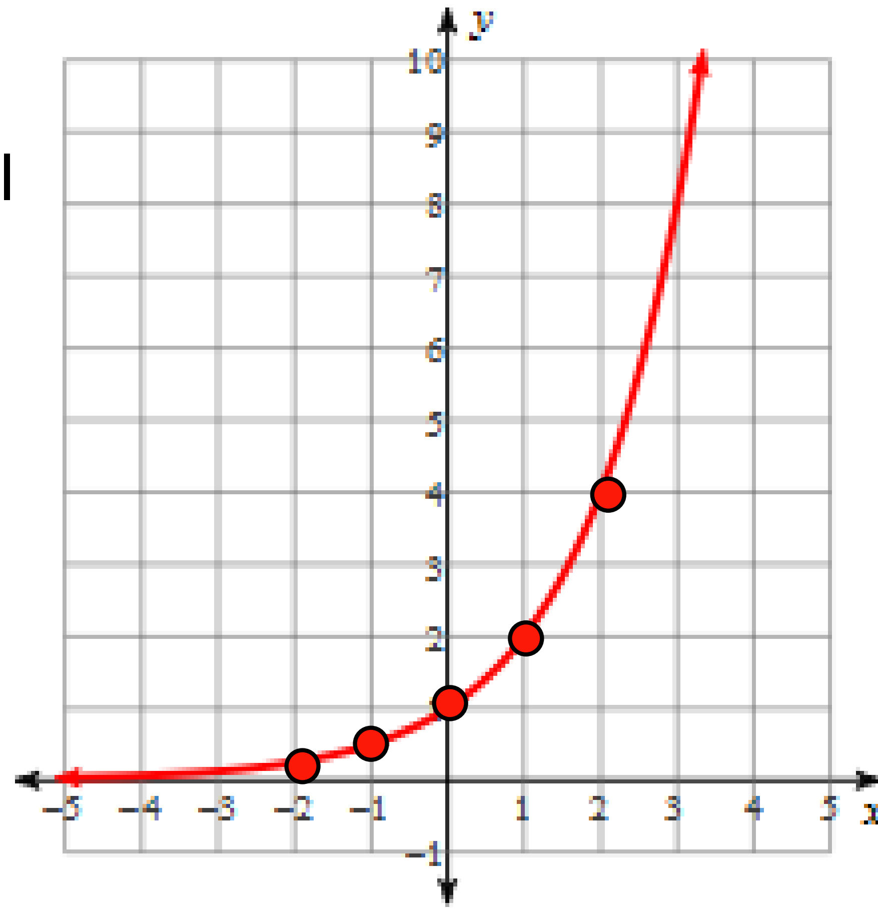


$$\left(\frac{2}{1}\right)^{-2} = \left(\frac{1}{2}\right)^{2} = \frac{1}{4} = 0.25$$

"negative exponent property"

$$2^0 = 1$$

"zero exponent property"

Exponential Function $f(x) = 2^x$

Will the 'y' value ever reach zero (on the *left end* of the graph)?

As the denominator gets bigger and bigger, the decimal version of the fraction gets smaller and smaller.

x	$2^{(\ )}$	y
-1	$2^{(-1)}$	$1/2$
-2	$2^{(-2)}$	$1/4$
-3	$2^{(-3)}$	$1/8$
-4	$2^{(-4)}$	$1/16$
-5	$2^{(-5)}$	$1/32$

$f(-1) = 1/2$

$f(-2) = 1/4$

$f(-3) = 1/8$

$f(-4) = 1/16$

$f(-5) = 1/32$

'y' gets closer and closer to zero but never reaches zero.

Horizontal Asymptote: a horizontal line the graph approaches but never reaches.

$$f(x) = 2^x$$

$y = 0$

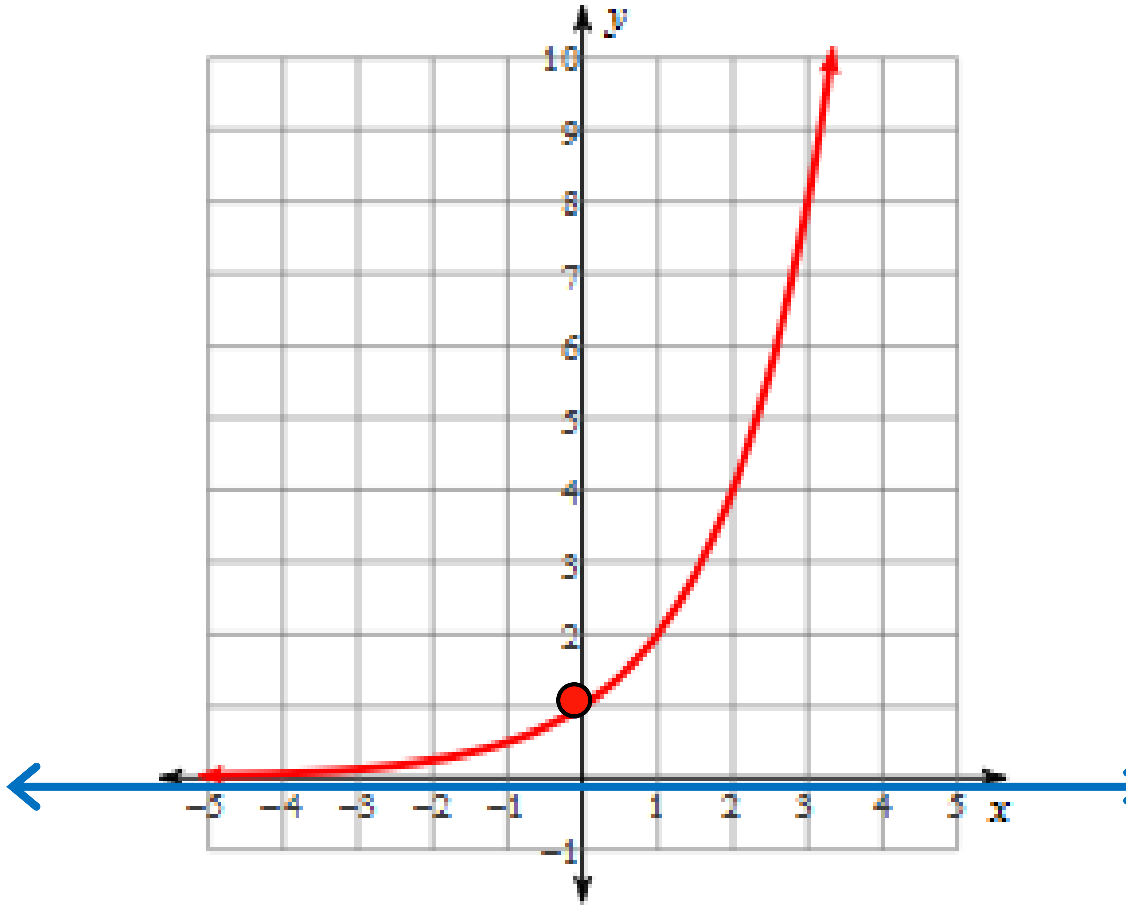
$Domain = ?$

$x = (-\infty, \infty)$

$range = ?$

$y = (0, \infty)$

Parentheses on left end of the interval → range *does include y = 0*



y-intercept = ?

$f(0) = y\ \ intercept$

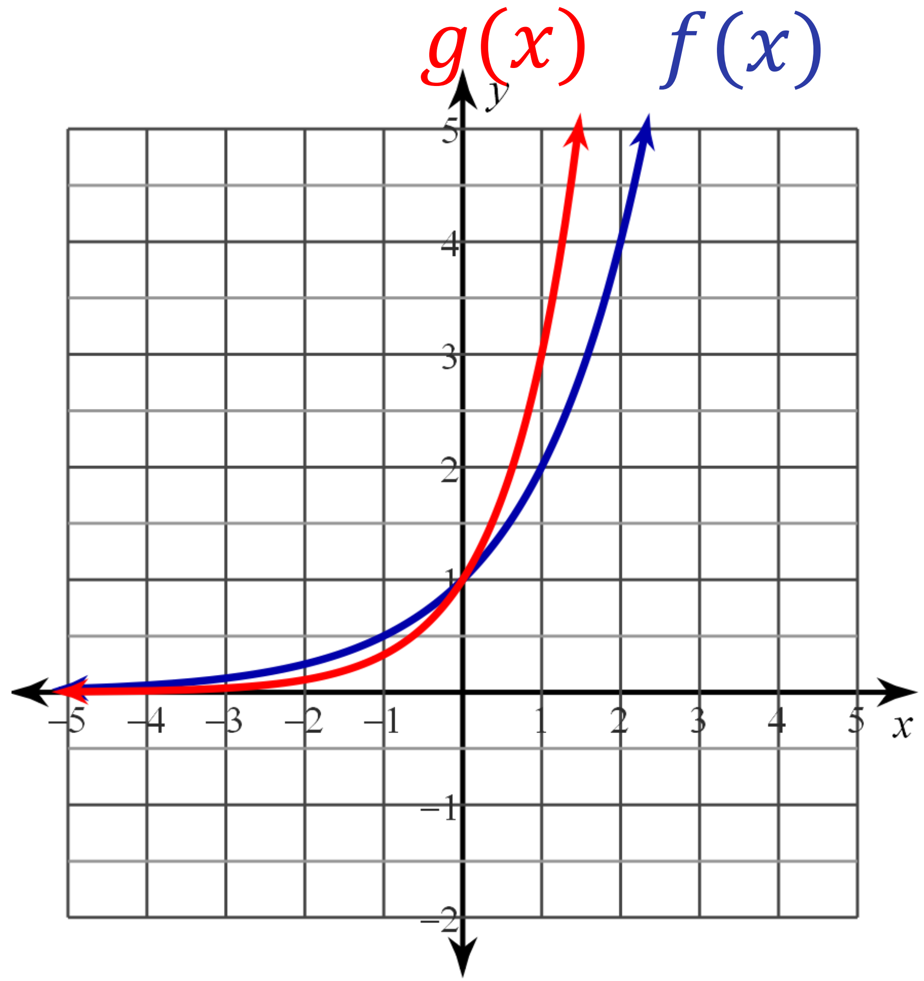
$f(0) = 2^0 = 1$

<u>Exponential Growth</u>: the graph is increasing (as you go from left to right the graph goes upward). Growth occurs when the base of the exponential is greater than 1.

$$y = b^x$$

‘b’ > 1 → growth

$$f(x) = 2^x \qquad g(x) = 3^x$$



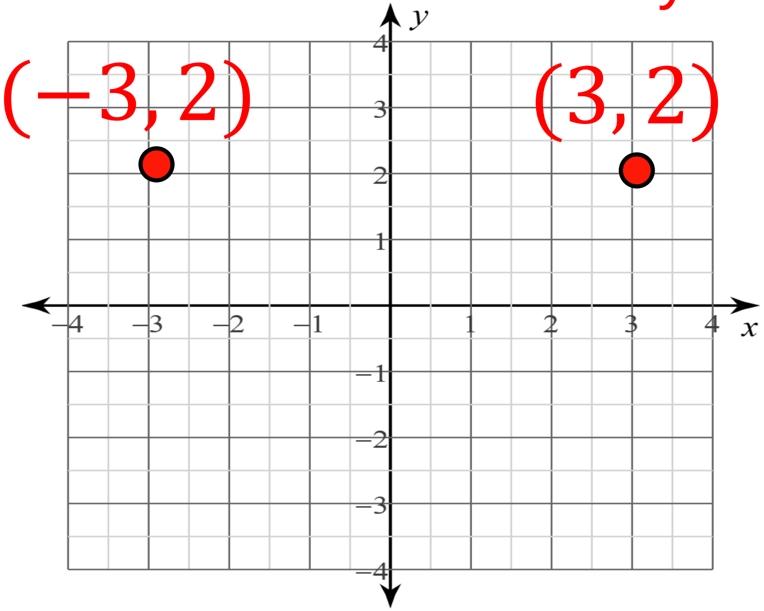
<u>Why</u> do both graphs have the same y-intercept?

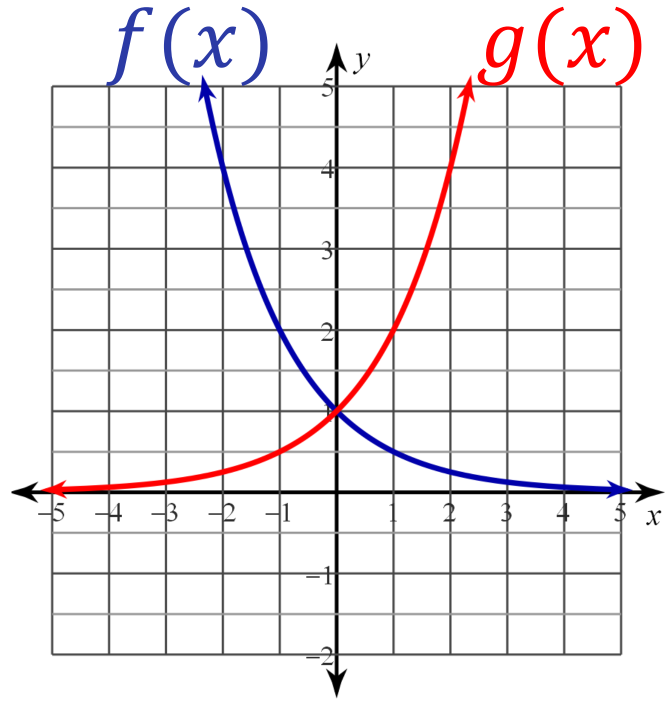
$$f(0) = 2^0 = 1$$

$$g(0) = 3^0 = 1$$

All exponential “*<u>parent functions</u>*” have (0, 1) as the y-intercept.

→ Replacing ‘x’ with ‘(-x)’ causes a reflection across the y-axis





$g(x) = 2^x$ → Reflect across the y-axis

$f(x) = 2^{-x}$

$f(x) = (2^{-1})^x$

Exponent of a Power Property of Exponents

$f(x) = \left(\frac{1}{2}\right)^x$

Negative Exponent Property of Exponents

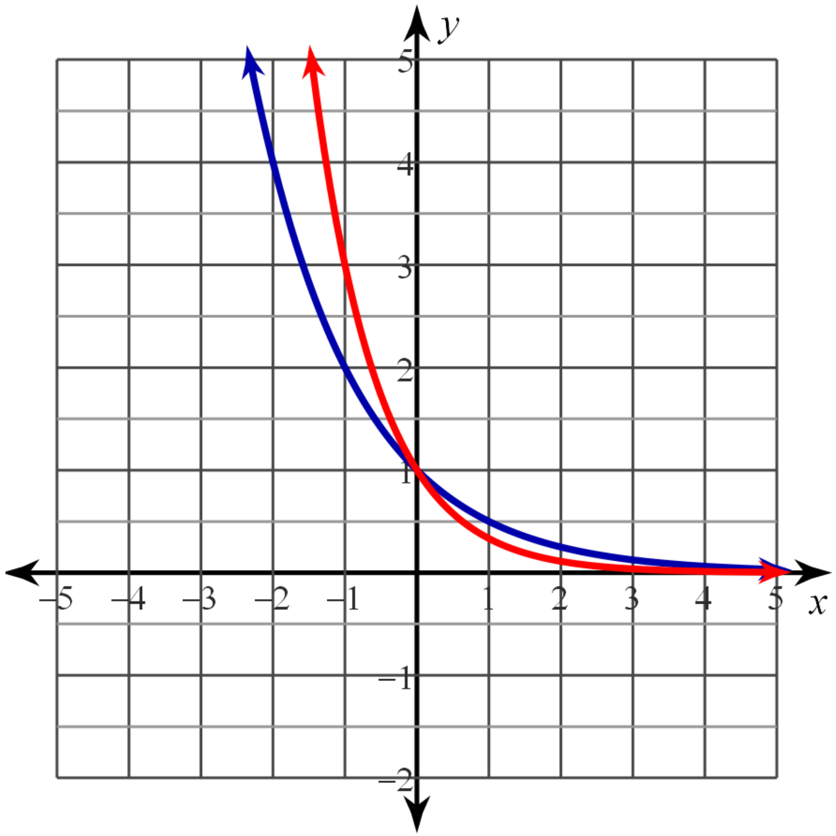
Exponential Decay: the graph is decreasing (as you go from left to right the graph goes downward). This occurs when the base of the exponential is between 0 and 1.

$$y = b^x$$

0 < 'b' < 1 → decay

$$f(x) = \left(\frac{1}{2}\right)^x$$

$$g(x) = \left(\frac{1}{3}\right)^x$$



$f(x) = b^x$ Can the base be zero? b ≠ 0

$g(x) = (0)^x$

x	y
-1	$\frac{1}{0} = ??$
0	???
1	0

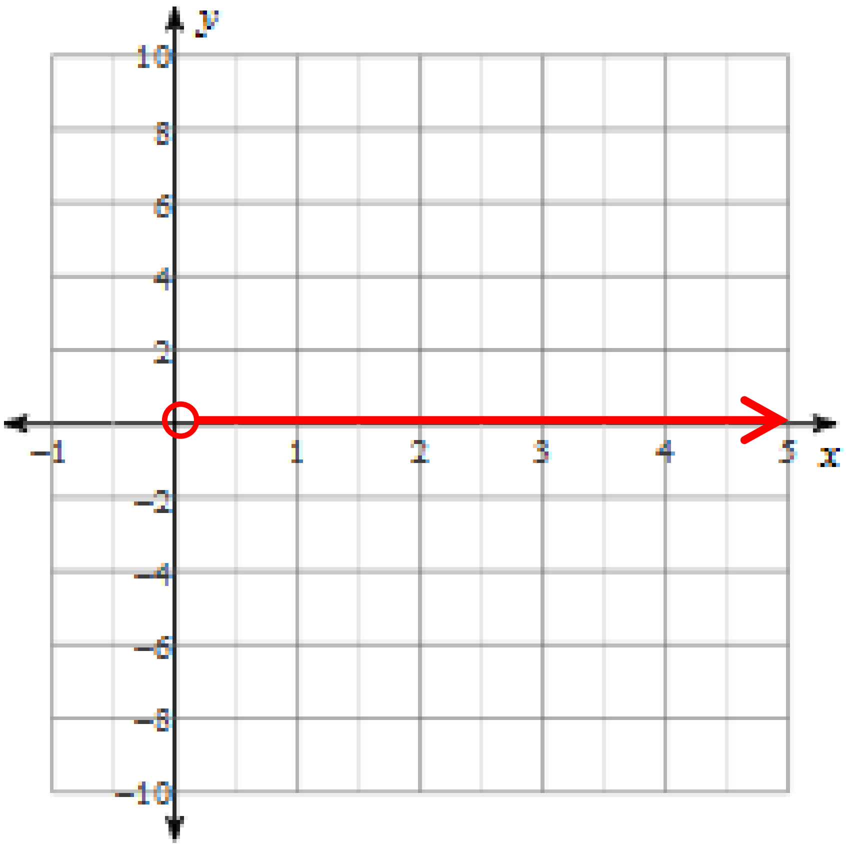
$$g(-1) = \left(\frac{0}{1}\right)^{-1}$$

$$g(-1) = \left(\frac{1}{0}\right)^{1}$$

$$g(0) = (0)^0$$

$$= 0? \quad = 1?$$

$$g(1) = (0)^1$$



→ negative number input values are not "mapped" to an output value.

→ input value "0" has an ambiguous output value

→ Any positive number input value is "mapped" to "0"

Can the ‘base’ be negative?

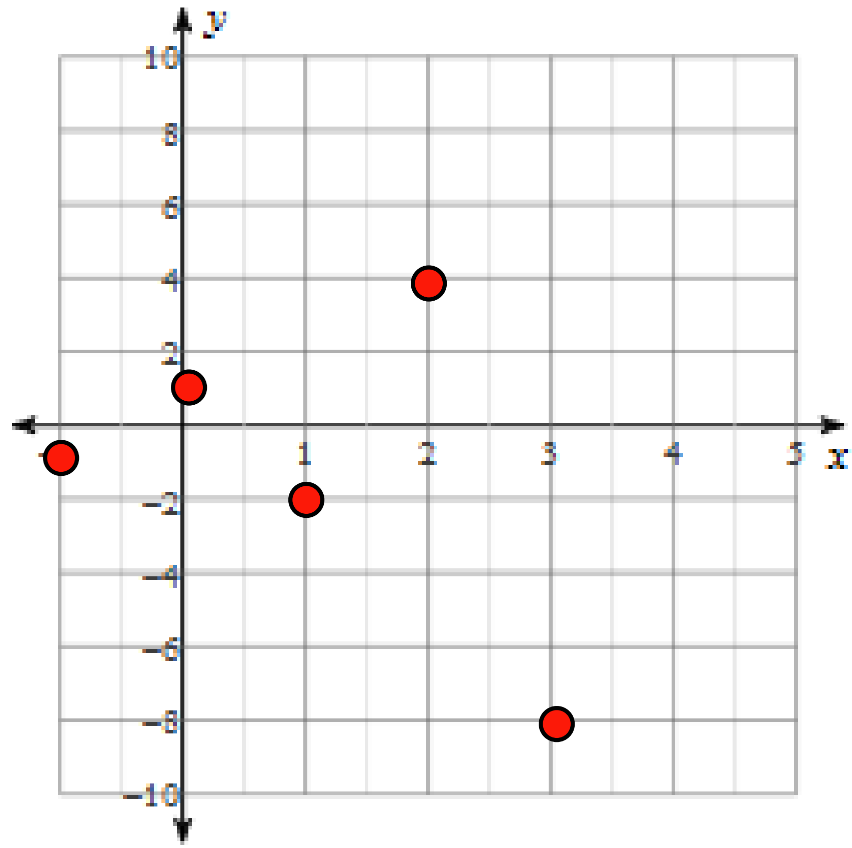
$$f(x) = ab^x$$

‘b’ > 1 → growth

0 < ‘b’ < 1 → decay

$$g(x) = (-2)^x$$

x	y	
-1	-0.5	$(-2)^{-1}$
0	1	$(-2)^{0}$
$\frac{1}{2}$	*i* = ?	$(-2)^{\frac{1}{2}} = \sqrt{-2}$
1	-2	$(-2)^{1}$
2	4	$(-2)^{2}$
3	-8	$(-2)^{3}$



b ≠ negative numbers

Can the base be 1?

$$f(x) = ab^x$$

$$g(x) = (1)^x$$

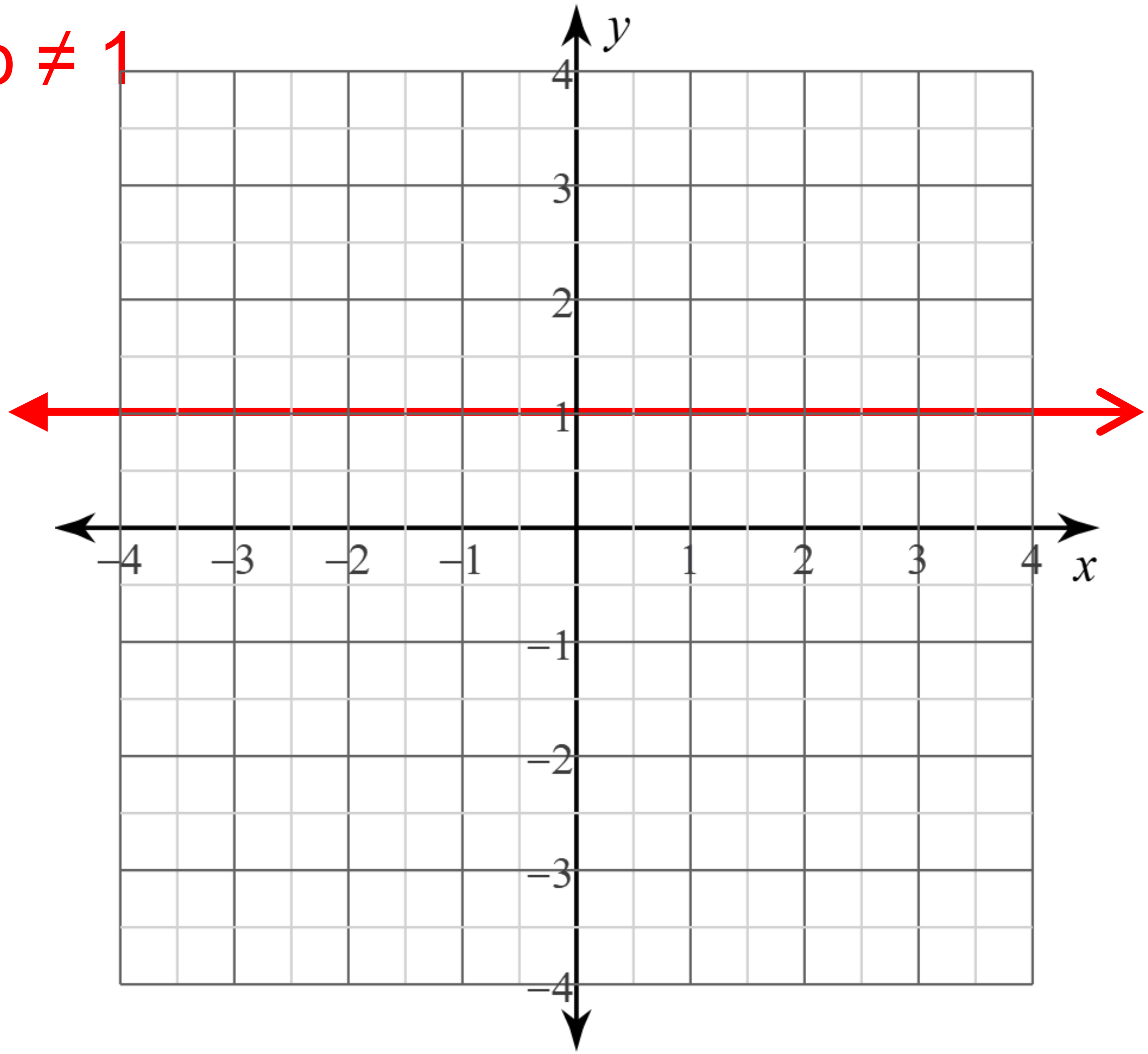
b ≠ 1

x	y
-1	1
0	1
1	1

$(1)^{-1}$

$(1)^0$

1^1



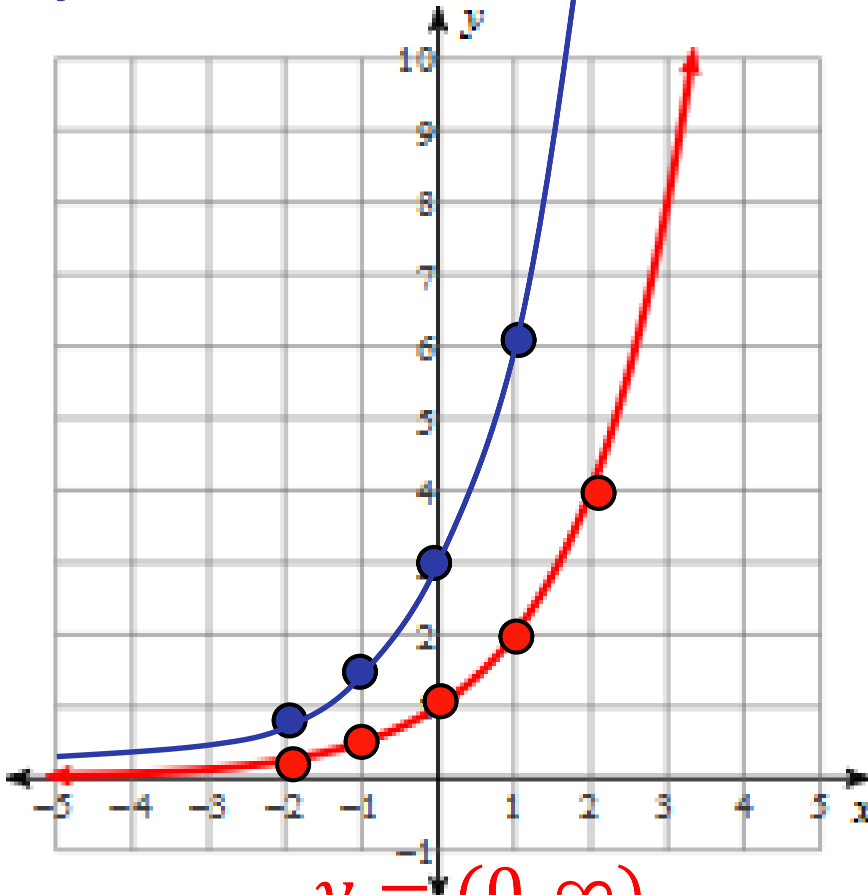
$0 < b < 1, \text{ OR } \text{b} > 1$

$b = (0,1) \cup (1, \infty)$

$f(x) = 2^x$ $g(x) = 3(2)^x$

Vertically stretched by a factor of 3

x	$2^{(\ )}$	f(x)	$g(x)$
-2	2^{-2}	0.25	0.75
-1	2^{-1}	0.5	1.5
0	2^0	1	3
1	2^1	2	6
2	2^2	4	12



Horizontal asymptote: $y = 0$ $y = 0$

$range = ?$ $y = (0, \infty)$ $y = (0, \infty)$

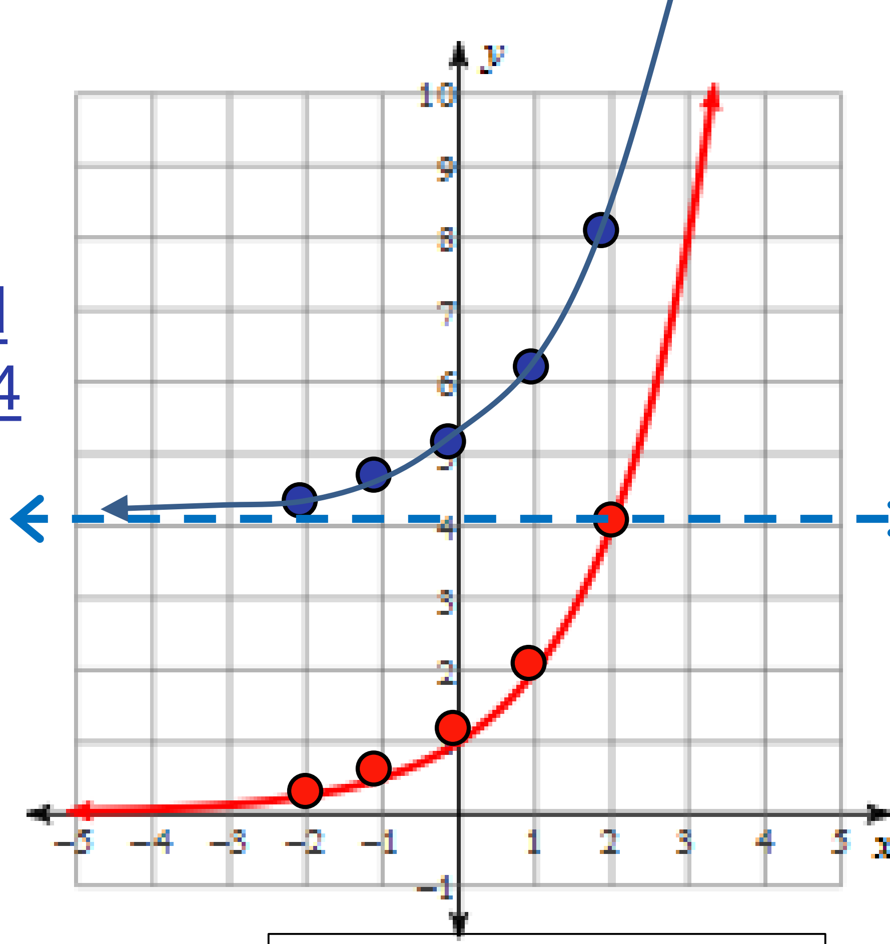
$Domain = ?$ $x = (-\infty, \infty)$ $x = (-\infty, \infty)$

y-intercept = ? $f(x): (0, 1)$ $g(x): (0, 3)$

$f(x) = 2^x$ $\quad$ $k(x) = 2^x + 4$

x	$2^{(\ )}$	f(x)	$k(x)$
-2	2^{-2}	0.25	4.25
-1	2^{-1}	0.5	4.5
0	2^{0}	1	5
1	2^{1}	2	6
2	2^{2}	4	8

Shifted UP by 4



Horizontal asymptote: $f(x)\!: y = 0$ $\quad$ $g(x)\!: y = 4$

$Domain = ?$ $\quad$ $x = (-\infty, \infty)$ $\quad$ $x = (-\infty, \infty)$

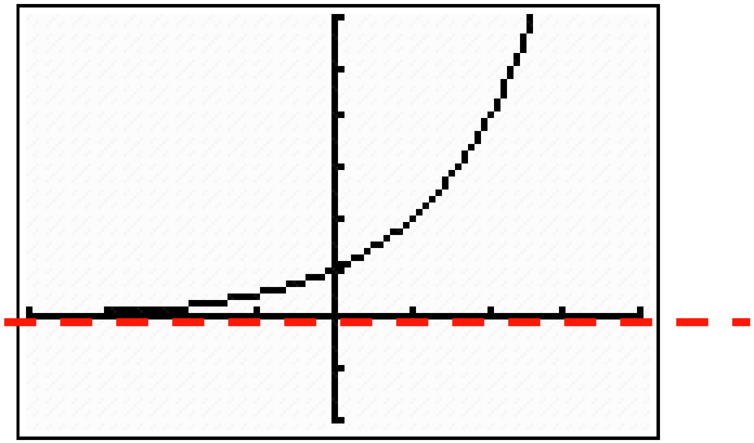
$range = ?$ $\quad$ $f(x)\!: y = (0, \infty)$ $\quad$ $g(x)\!: y = (4, \infty)$

y-intercept = ? $\quad$ $f(x)\!: (0, 1)$ $\quad$ $g(x)\!: (0, 5)$

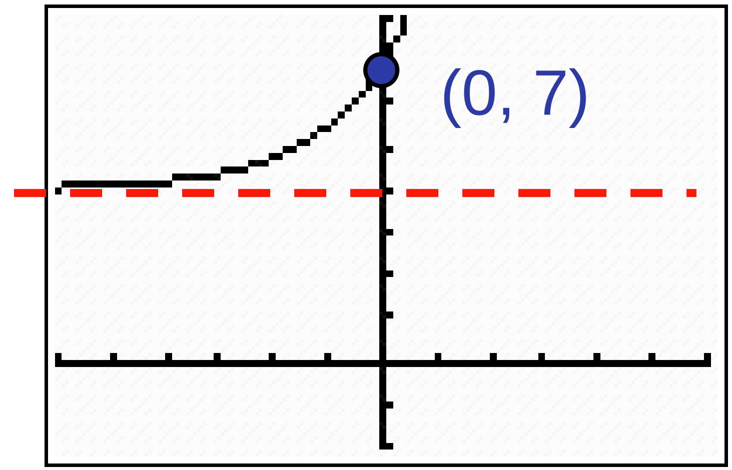
Transformations of the Exponential Function

VSF= 3　　　Up 4 shift

$$h(x) = 3(2)^x + 4$$



$f(x) = 2^x$　Base-2 Exponential Parent Function



y = 4

Transformation Form of the Exponential Function

$$y = ab^x + k$$

VSF

Growth Factor (the base of the exponential)

vertical shift and horizontal Asymptote

y-intercept: (0, a + k)

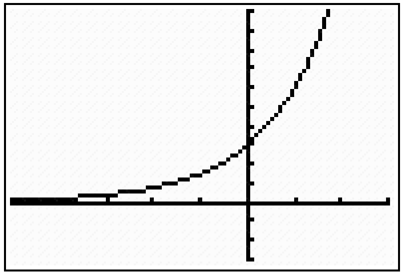
$$h(0) = 3(2)^0 + 4$$

$$h(0) = 7$$

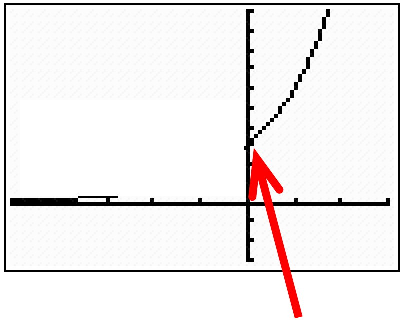
Horiz. asympt: y = k)

y = 4

<u>Initial Value</u>: (of the exponential) is the <u>vertical stretch factor</u> (*for problems with no up/down shifts)*



If in input is time ("stopwatch time") the <u>initial value</u> occurs when t = 0.



$$f(t) = 3(2)^t$$

Domain: x = [0, ∞)

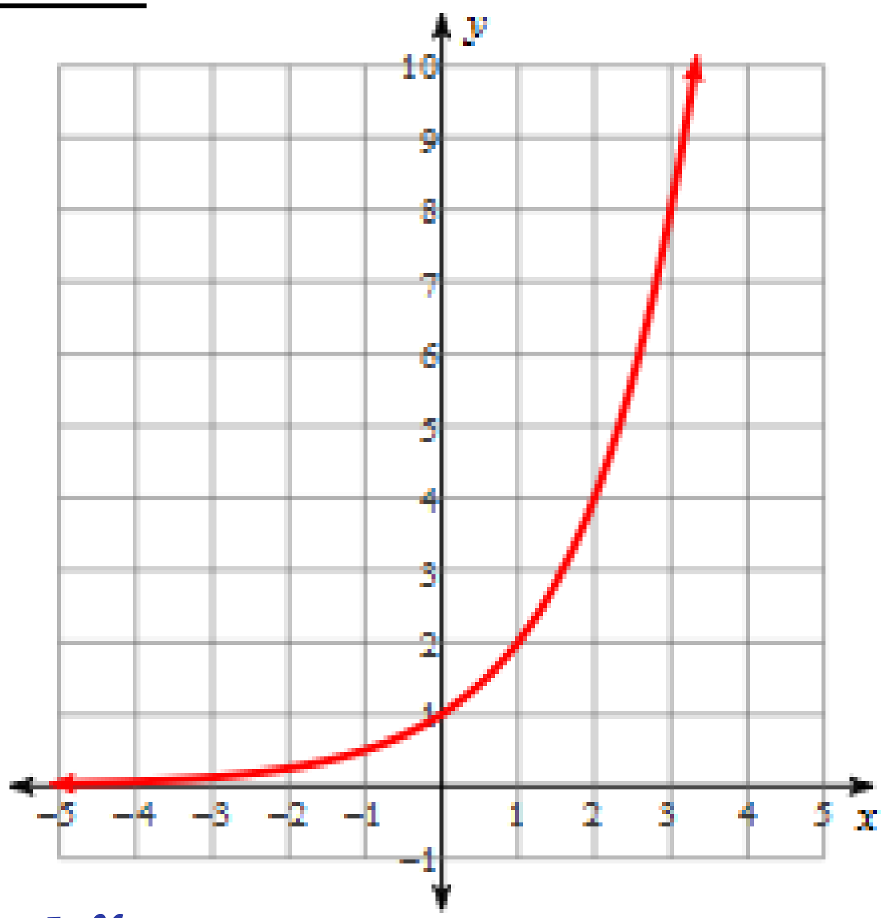
$$f(0) = 3(2)^0 = ?$$

$$f(0) = 3$$

What is the initial value of: $f(t) = 0.5(3)^t$

What is the equation of the graph?

1) Start with the general transformation equation. $g(x) = ab^x + k$



2) Find the value of 'k' (this is the horizontal asymptote).

$$g(x) = ab^x + k$$

Horizontal asymptote: y = 0

$$k = 0$$

Rewrite the equation.

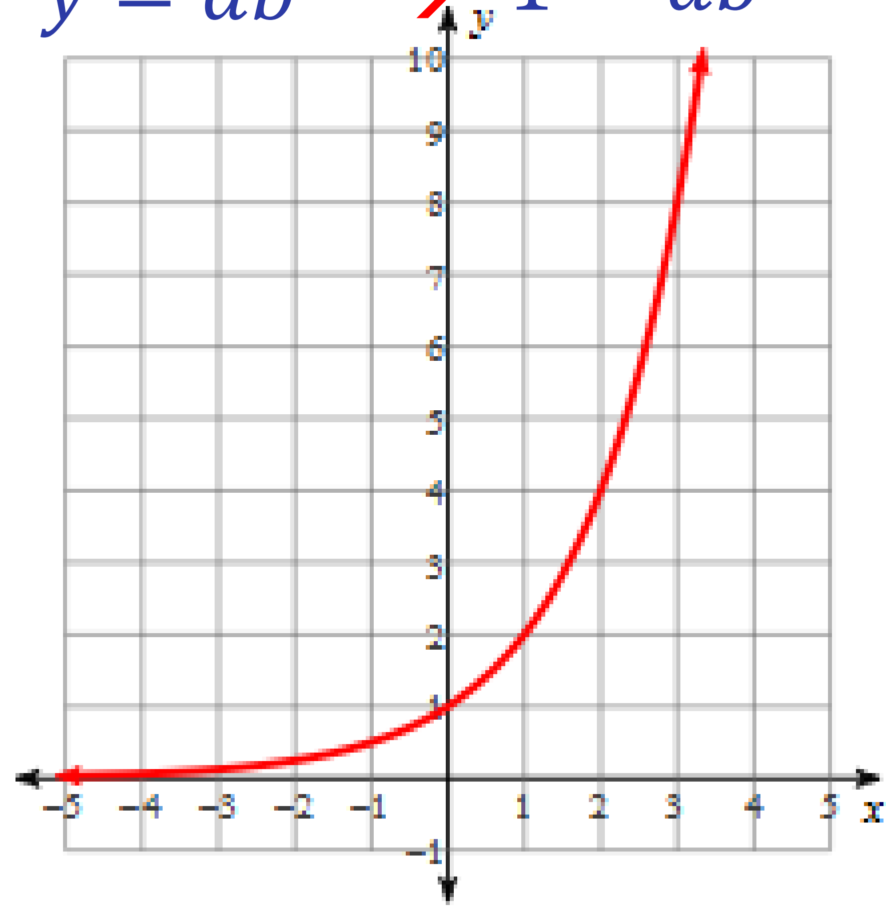
$$g(x) = ab^x + k \rightarrow y = ab^x$$

$$g(x) = ab^x + k \rightarrow y = ab^x$$

3) <u>y-intercept</u>

Substitute (0, 1) into the equation. $y = ab^x \rightarrow 1 = ab^0$

Solve for ‘a’ $a = 1$



<u>Rewrite the equation</u>

$$y = ab^x \rightarrow \boxed{y = b^x}$$

4) <u>Substitute a “nice” x-y pair</u> from the graph into the equation.

Substitute (1, 2) into the equation.

$$y = b^x \rightarrow 2 = b^1$$

Solve for ‘b’ $b = 2$

<u>Rewrite the equation</u>

$$\boxed{y = 2^x}$$

Converting the graph into an equation

1) Start with $g(x) = ab^x + k$

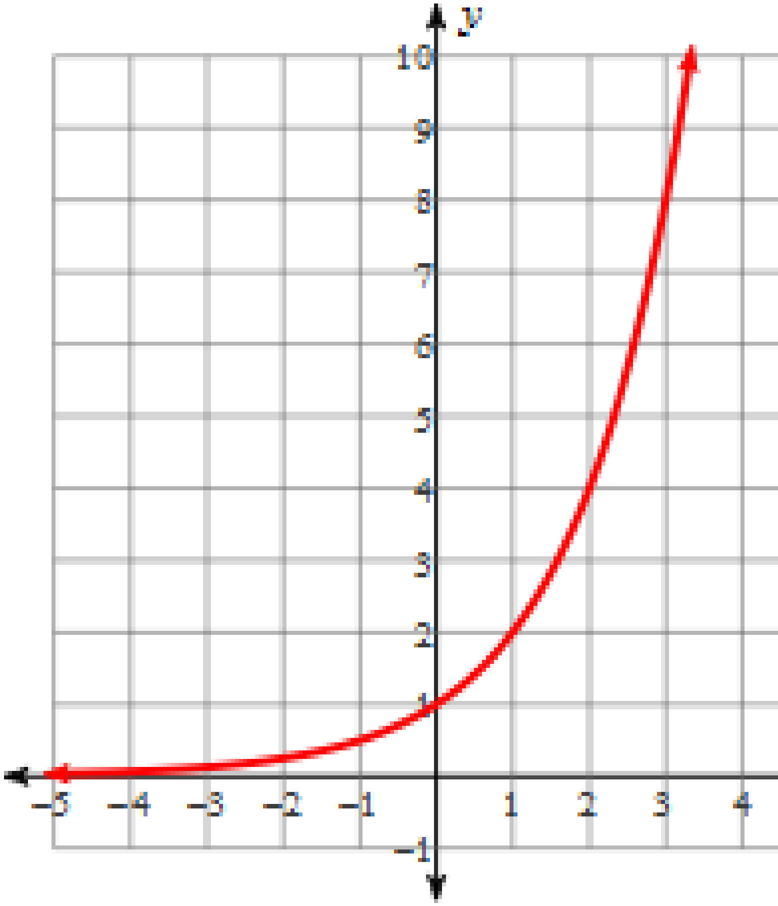
2) Find the value of 'k' (horizontal asymptote). $k = 0$

$g(x) = ab^x + k$ → $y = ab^x$

3) Substitute the y-intercept

(0, 1) → $y = ab^x$ → $1 = ab^0$

→ a = 1 → $y = b^x$



4) Substitute a "nice" x-y pair from the graph into the equation (then simplify)

(1, 2) → $y = b^x$ → $2 = b^1$ → b = 2 → $y = 2^x$

What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

2) Find 'k'

Horizontal asymptote: y = 3

$k = 3$ $y = ab^x + 3$

3) Substitute the y-intercept

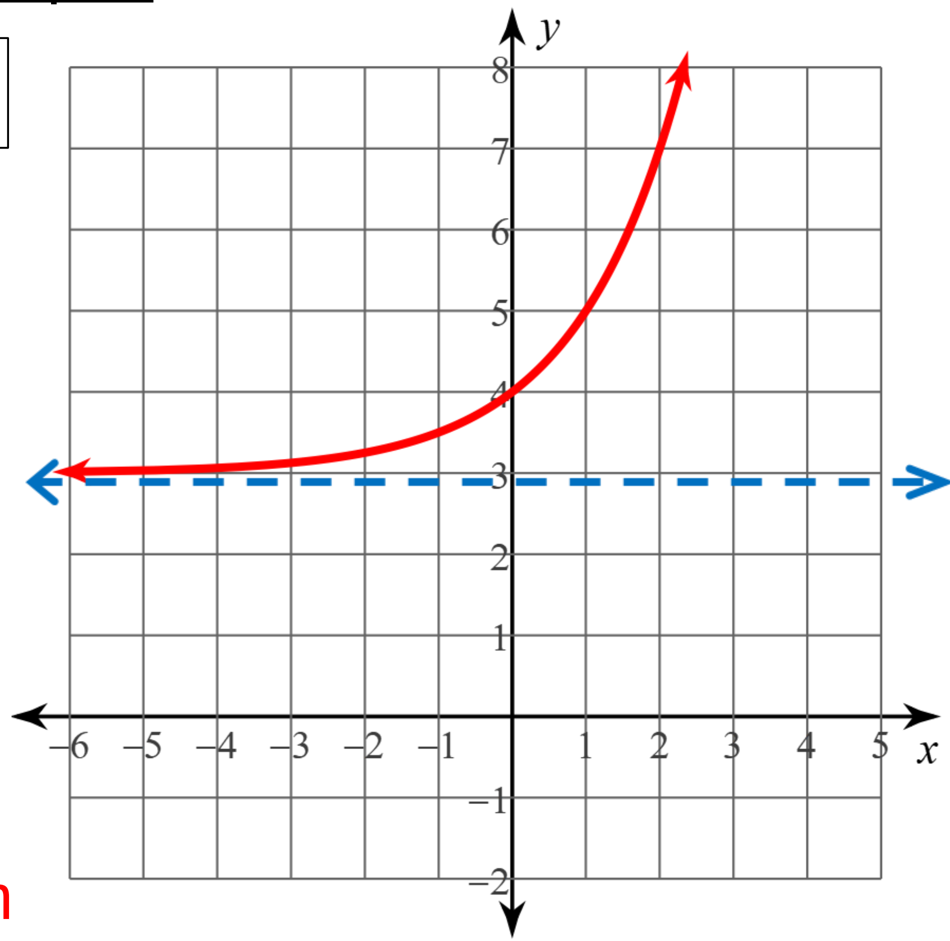
(0, 4) → $4 = ab^0 + 3$

a = 1 → $y = b^x + 3$

4) Substitute a "nice" x-y pair from the graph into the equation.

(1, 5) → $5 = b^1 + 3$ → b = 2

$y = 2^x + 3$



What is the equation of the graph?

1) Start with $g(x) = ab^x + k$

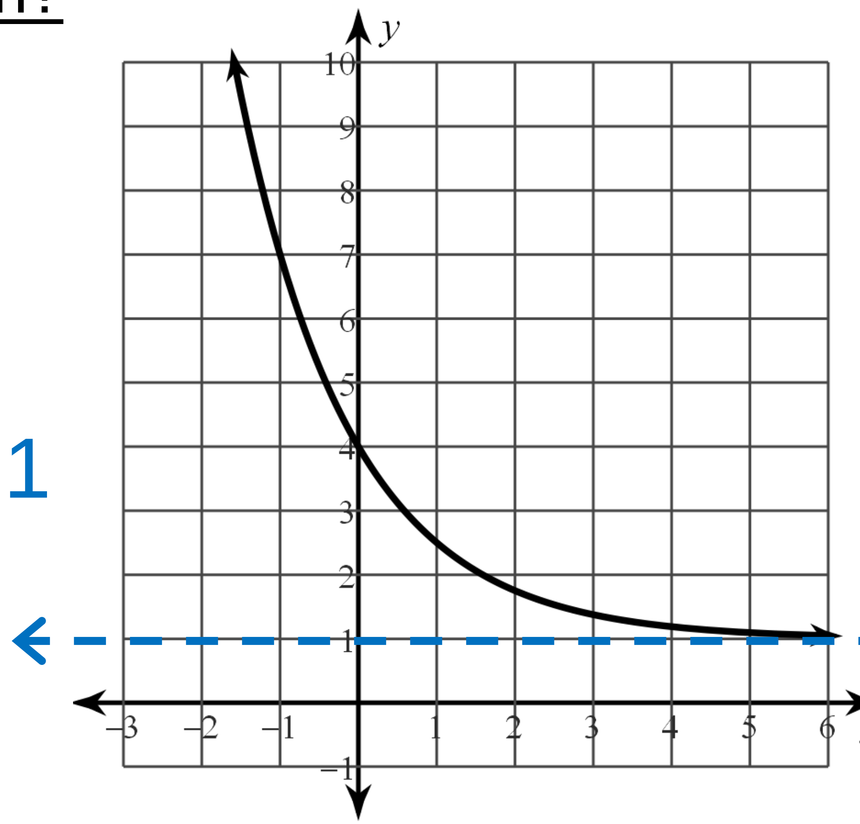
2) horizontal asymptote y = 1

$k = 1$ $y = ab^x + 1$

3) y-intercept $(0, 4)$ $4 = ab^0 + 1$

$a = 3$ $y = 3b^x + 1$

4) "Nice" x-y pair $(-1, 7)$



$7 = 3b^{-1} + 1$

$6 = 3b^{-1}$

$2 = b^{-1}$ $2 = \frac{1}{b}$ $b = \frac{1}{2}$ $y = 3\left(\frac{1}{2}\right)^x + 1$