Math-3

Lesson 1-1

Relations and Functions
And the Linear Function

Relation: A “mapping” or pairing of input values to output values.

Function: A relation where each input has exactly one output.

Describe how a relation is

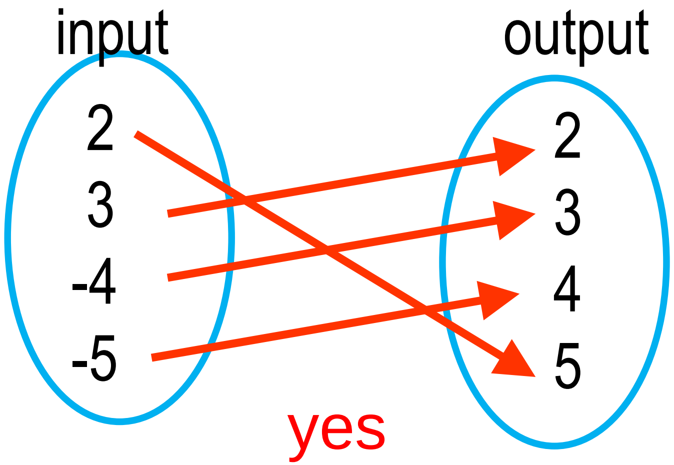
1) Similar to a function.

Both have inputs matched to outputs.

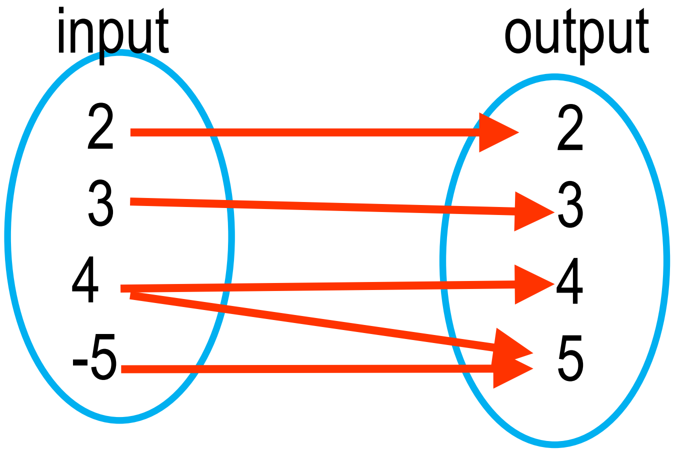
1) Different from a function?

One input to a relation can be matched with two or more outputs but one input to a function can only be matched to one output.

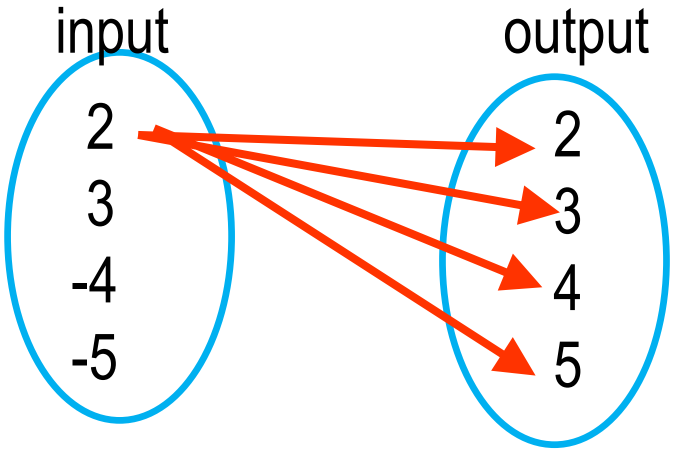
Is it a function?



yes

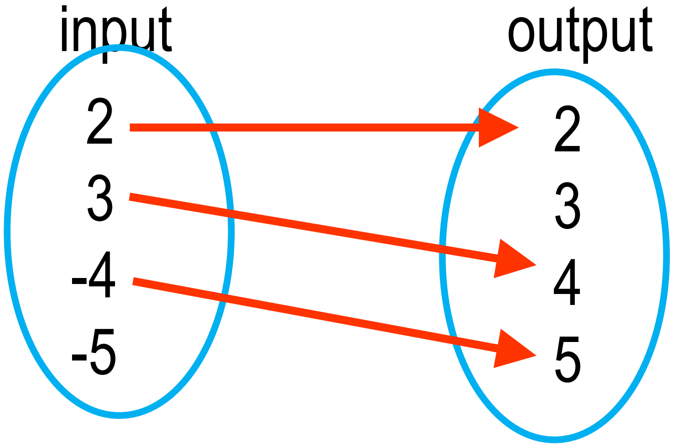


Relation but NOT a function since input value ‘4’ has 2 outputs.

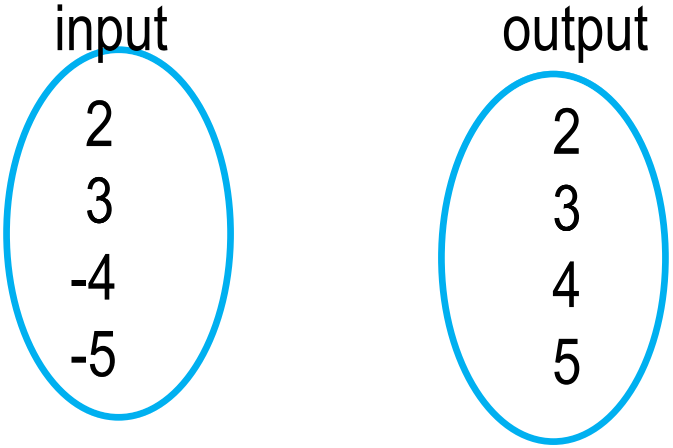


No: input value ‘2’ has more than one output

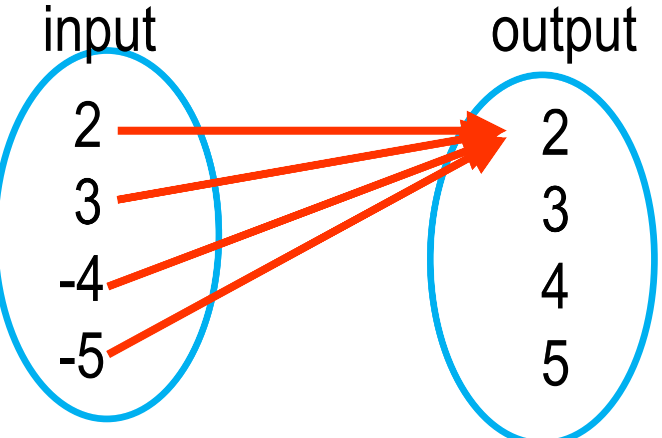
Is it a function?



yes



No (There aren’t any pairings of inputs to outputs.)



Yes Each input has exactly one output (even though it’s the same output)

There are at least 6 ways to show a relation between input and output values.

Ordered Pairs: (2, 4), (3, 2), (-4, 3)

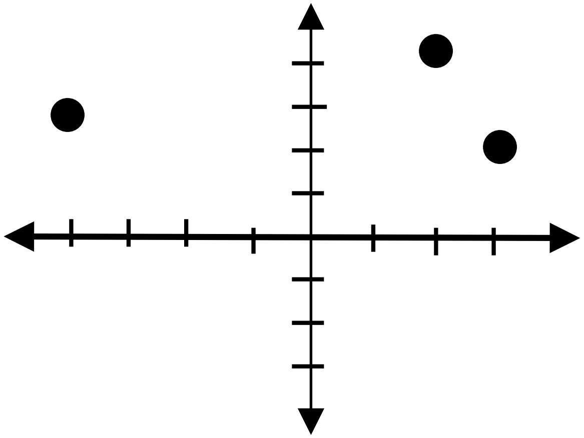
Data table:

x	2	3	-4
y	4	2	3

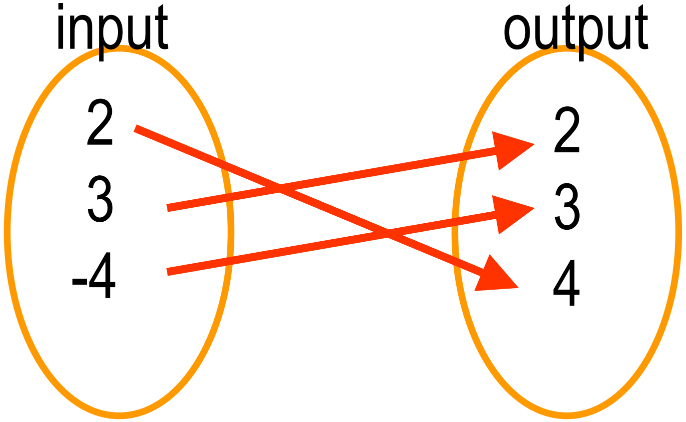
Equation: $y = 2x + 1$

Function notation: $f(2) = 4$

Graph:



Mapping



Are all of these representations the same?

Vocabulary

Domain: the set made up of all of the input values that have corresponding output values.

Range: the set made up of all of the corresponding output values.

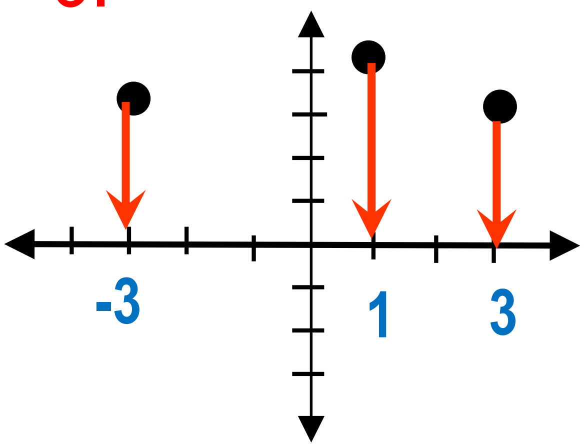
Identify the Domain

1. (2, 4), (3, 5), (-4, 2)

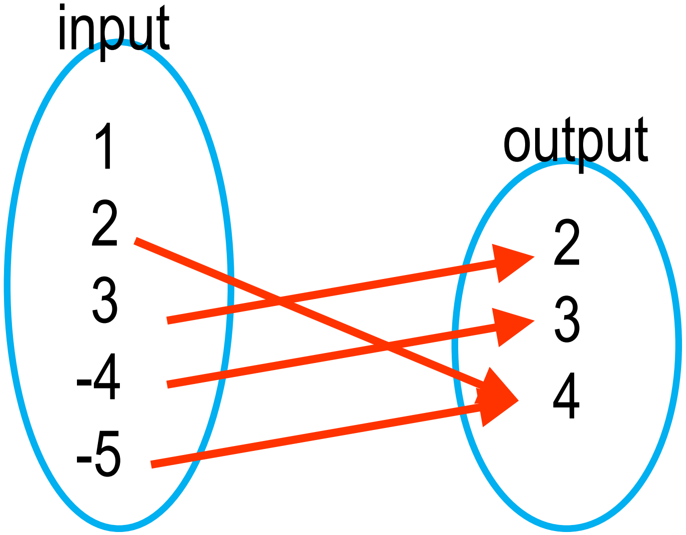
2.

x	6	9	-2
y	4	7	3

3.



4. **2, 3, -4, -5**



What are 6 ways you can show a relation between input and output ?

Ordered Pairs

Data table

Equation

Graph

Function notation: f(2) = 4

Mapping

$y = f(x)$ Function Notation

When we say “y is a function of x” we mean:

We are “doing math” (performing mathematical operations) on the input value ‘x’ to determine the corresponding output value ‘y’.

Which of the following equations is “ ’y’ a function of x”?

$$x = \frac{1}{2}y - 3$$

$$\boxed{y = 2x + 6}$$

We are performing operations on the input value ‘x’ to get the output value ‘y’.

In the equation, “x” is just place holder for the values that we “plug in” (substitute) into the equation in place of “x”.

$$y = 2x - 1$$

We replace ‘x’ (the place-holder) with a parentheses. Then we substitute into the parentheses the input value then simplify.

$$y = 2(\quad) - 1$$

x	0	1	2
y	-1	1	3

$$y = 2(0) - 1$$
$$y = -1$$

$$y = 2(1) - 1$$
$$y = 1$$

$$y = 2(2) - 1$$
$$y = 3$$

Equation → table

Using the equation form of the function, fill in the missing values in the table to convert the equation into a table of values.

$$y = 3x + 4$$

x	0	1	2
y	4	7	10

$$y = 4x - 2$$

x	0	1	2
y	-2	2	6

$$y = 5x + 3$$

x	0	1	2
y	3	8	13

What do you notice when comparing the constant term in the equation to the numbers in the table?

$$y = 3x + 4$$

$$y = 3(0) + 4$$

x	0	1	2
y	4	7	10

$$y = 4x - 2$$

$$y = 4(0) - 2$$

x	0	1	2
y	-2	2	6

$$y = 5x + 3$$

$$y = 5(0) + 3$$

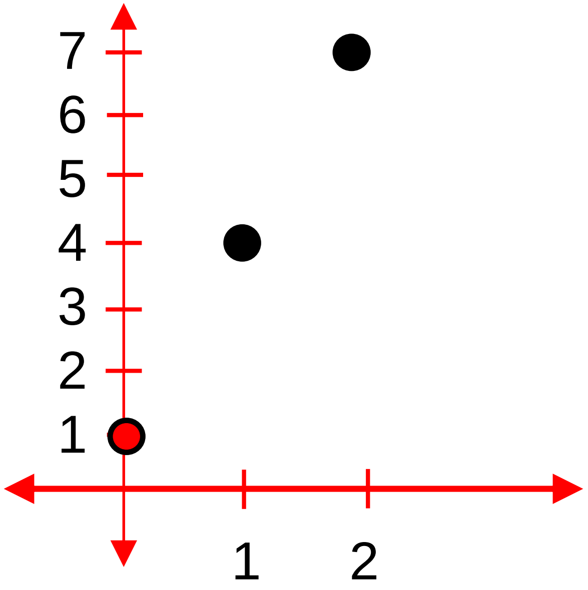
x	0	1	2
y	3	8	13

The constant term of the equation is always mapped from the input value zero.

Fill in the table then graph
x-y pairs from the table.

$y = 3x + 1$

x	0	1	2
y	1	4	7

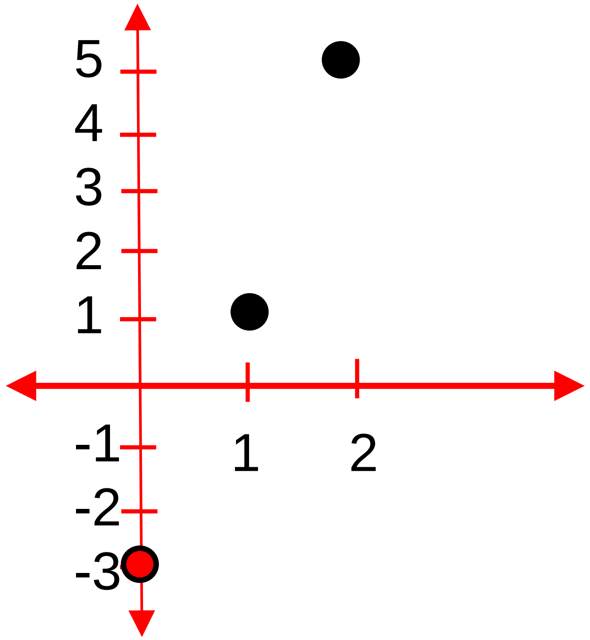


y-intercept: the x-y pair where a graph crosses the y-axis.

Solution of a two-variable equation: all x-y pairs that make the equation true.

$y = 4x - 3$

x	0	1	2
y	-3	1	5



Does the table represent the complete solution? no

Does the graph represent the complete solution? no

What do you notice when comparing the coefficient of the input variable to the numbers in the table?

$y = \boxed{3}x + 4$

x	0	1	2
y	4	7	10

+3 +3

$y = \boxed{4}x - 2$

x	0	1	2
y	-2	2	6

+4 +4

$y = \boxed{5}x + 3$

x	0	1	2
y	3	8	13

+5 +5

If the input value changes by one, the coefficient of the input variable is the change in ‘y’ between adjacent terms in the table.

Why isn't the change in 'y' between adjacent terms equal to the coefficient of 'x'?

Fill in the tables.

We changed the input value to 'x' by '2' for each adjacent value in the table instead of '1'.

$y = 2x + 1$

+2 +2

x	0	2	4
y	1	5	9

+4 +4

$y = 3x - 5$

+2 +2

x	0	2	4
y	-5	1	7

+6 +6

$y = 4x + 2$

+2 +2

x	0	2	4
y	2	10	18

+8 +8

How can you use the change in 'x' and the change in 'y' in the tables to calculate the coefficient of 'x'?

<u>Delta</u> a Greek letter (that looks like a triangle) used in engineering and math to denote “change.”

Δx Means the change in ‘x’

Δy Means the change in ‘y’

$\Delta x = 2$ $\Delta x = 2$

$y = \textcircled{3}x - 5$

x	0	2	4
y	-5	1	7

$\Delta y = 6$ $\Delta y = 6$

The coefficient of ‘x’ in the equation equals the change in ‘y’ of the table values divided by the change in ‘x’ of the table values.

$$3 = \frac{6}{2} = \frac{\Delta y}{\Delta x}$$

Fill in the table then graph the ordered pairs

$$y = 2x - 2$$

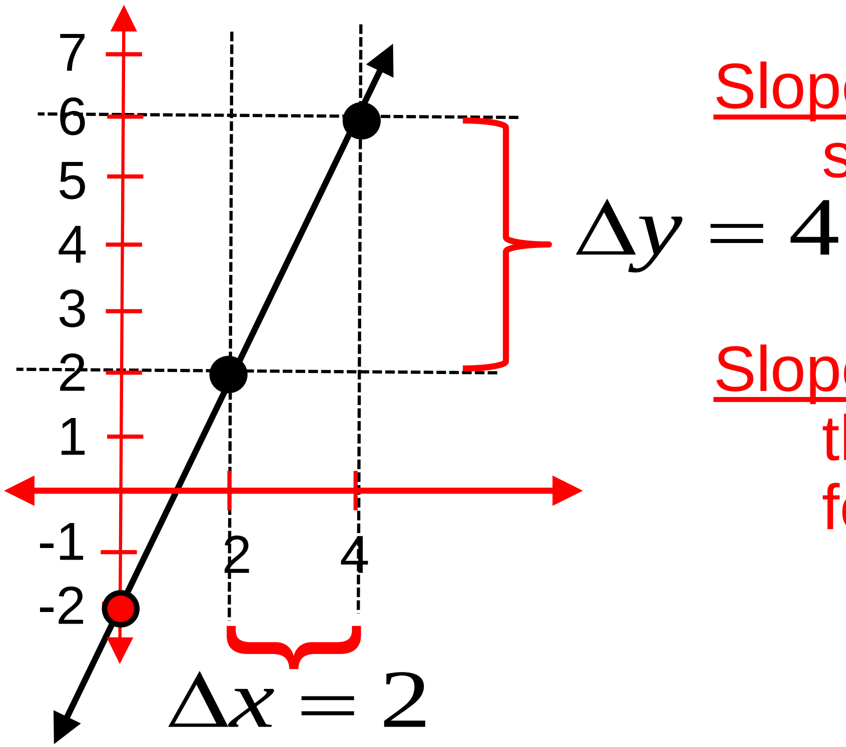
$\Delta x = 2$ $\Delta x = 2$

x	0	2	4
y	-2	2	6

$\Delta y = 4$ $\Delta y = 4$

Graphing the solution to the equation will result in infinitely points

→ they all form a line.



$$\Delta y = 4$$

$$\Delta x = 2$$

Slope (of a line) is its steepness given by $m = \dfrac{\Delta y}{\Delta x}$

Slope is the coefficient of ‘x’ when the equation is written in the form: y = mx + b

$$m = \frac{4}{2} = 2$$

Slope-intercept form of a linear equation:

the equation of a line written in the form:

$y = f(x)$

that gives the

slope of the line

and

the y-value where the graph crosses the y-axis.

$$y = mx + b$$

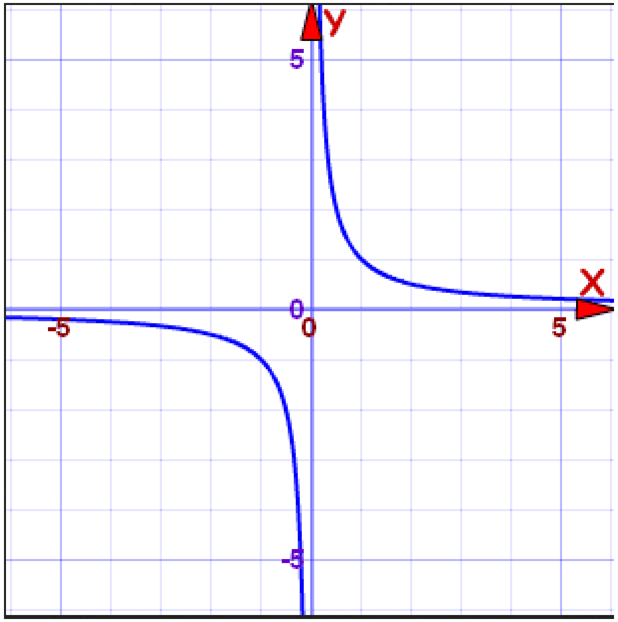
$$y = 3x + 2$$

Slope = 3

y-intercept: (0, 2)

Determining if the relation is linear.

Graph: is it linear?



The slope (steepness) needs to be constant.

Data table:

Is the data linear? The slope is constant → the graph of the points will be linear.

change in

x = 2

$\Delta x = 2$

$\Delta x = 2$

$\Delta x = 2$

$\Delta x = 2$

$\Delta x = 2$

$\Delta x = 2$

$\Delta x = 2$

x	f(x)
-2	-7
0	-5
2	-3
4	-1
6	1
8	3
10	5
12	7
14	9

$\Delta y = 2$

$\Delta y = 2$

$\Delta y = 2$

$\Delta y = 2$

$\Delta y = 2$

$\Delta y = 2$

$\Delta y = 2$

$\Delta y = 2$

Your turn: Which data set is linear?

A

x	f(x)
0	0
1	1
2	1.4
3	1.7
4	2.0
5	2.2
6	2.4
7	2.6
8	2.8
9	3

B

x	g(x)
-4	32
-3	18
-2	8
-1	2
0	0
1	2
2	8
3	18
4	32

[✓ C]

x	f(x)
-4	-7
-3	-5
-2	-3
-1	-1
0	1
1	3
2	5
3	7
4	9

Your turn: Is the data linear? If so, what is the equation that “fits” the data? $y = mx + b$

What does this number represent on the graph?

The output value ‘y’ when input value x = 0.

x	f(x)
-4	-7
-3	-5
-2	-3
-1	-1
0	1
1	3
2	5
3	7
4	9

$\Delta x = 1$ $\Delta y = 2$

$$y = m(0) + b \qquad y = b$$

The y-intercept always has ax x-value of zero.

$$(0, b) \qquad b = 1$$

Substitute b = 1 into the general equation.

$$y = mx + 1$$

What is the slope?

$$m = \frac{\Delta y}{\Delta x} \qquad m = \frac{2}{1}$$

Substitute m = 2 into the general equation.

$$y = 2x + 1$$

Another way to do it:

$$y = mx + b \qquad b = 1 \qquad y = mx + 1$$

x	f(x)
-4	-7
-3	-5
-2	-3
-1	-1
0	1
1	3
2	5
3	7
4	9

Every x-y pair is a solution of the equation → makes the equation true.

Substitute any x-y pair in for ‘x’ and ‘y’ in the equation.

$$3 = m(1) + 1$$

Solve for ‘m’.

$$m = 2$$

We know ‘m’ and ‘b’ → we know the equation that corresponds to the table.

$$y = 2x + 1$$

What is the equation of the line?

$$y = mx + b$$

$$b = 2$$

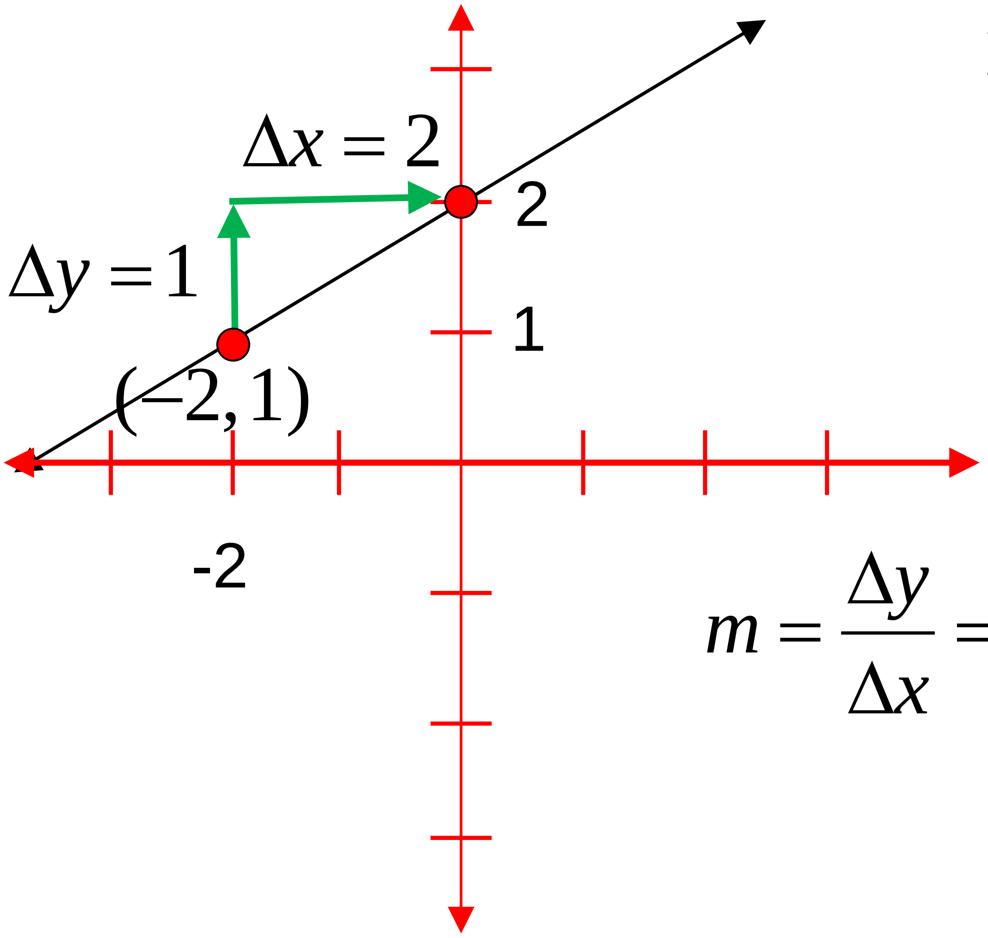
$$y = mx + 2$$

$$(x, y) = (-2, 1)$$

$$1 = m(-2) + 2$$

$$-1 = m(-2)$$

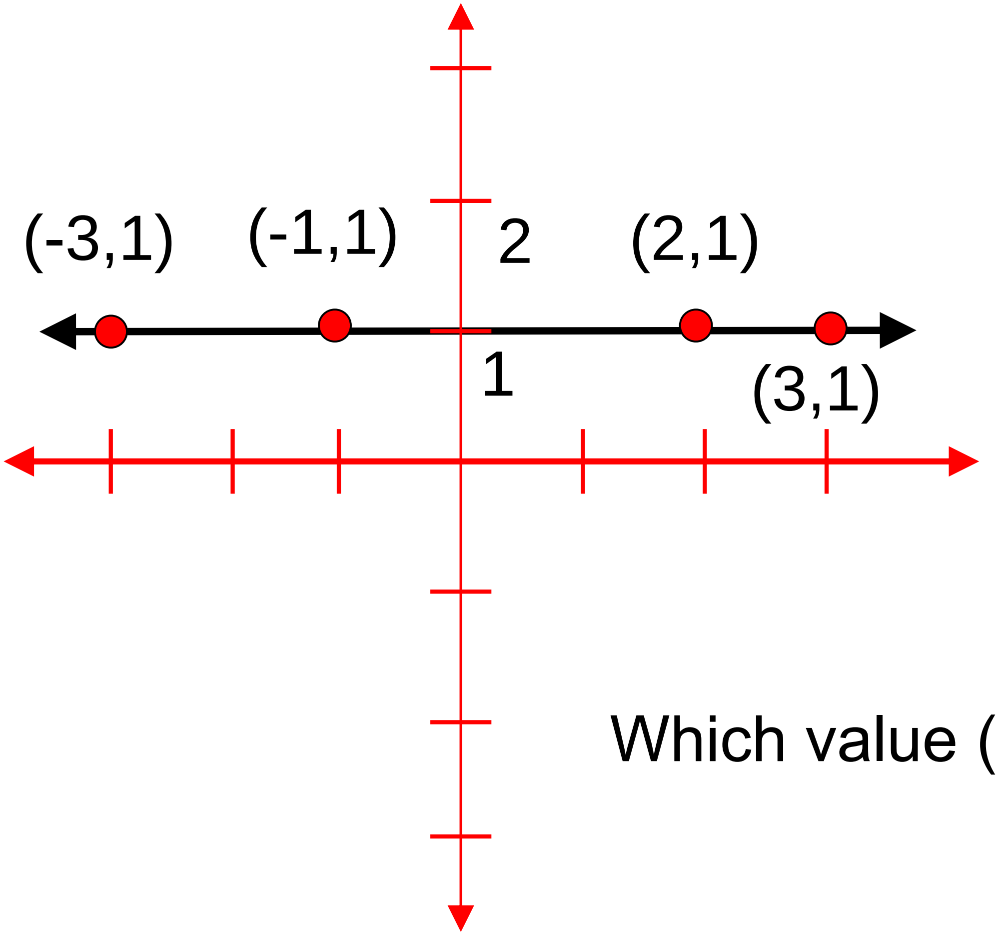
$$m = \frac{-1}{-2} = \frac{1}{2}$$



$$m = \frac{\Delta y}{\Delta x} = \frac{1}{2}$$

$$y = \frac{1}{2}x + 2$$

What is the equation of the line?



$$\boxed{y = 1}$$

or

$$x = 1$$

?

Which value (x or y) is always ‘1’?

Your turn: What is the equation that fits the data?

$$y = mx + b \qquad b = -3$$

x	f(x)
-4	-9
-2	-6
0	-3
2	0
4	3
6	6
8	9
10	12
12	15

$\Delta x = 2$

$\Delta y = 3$

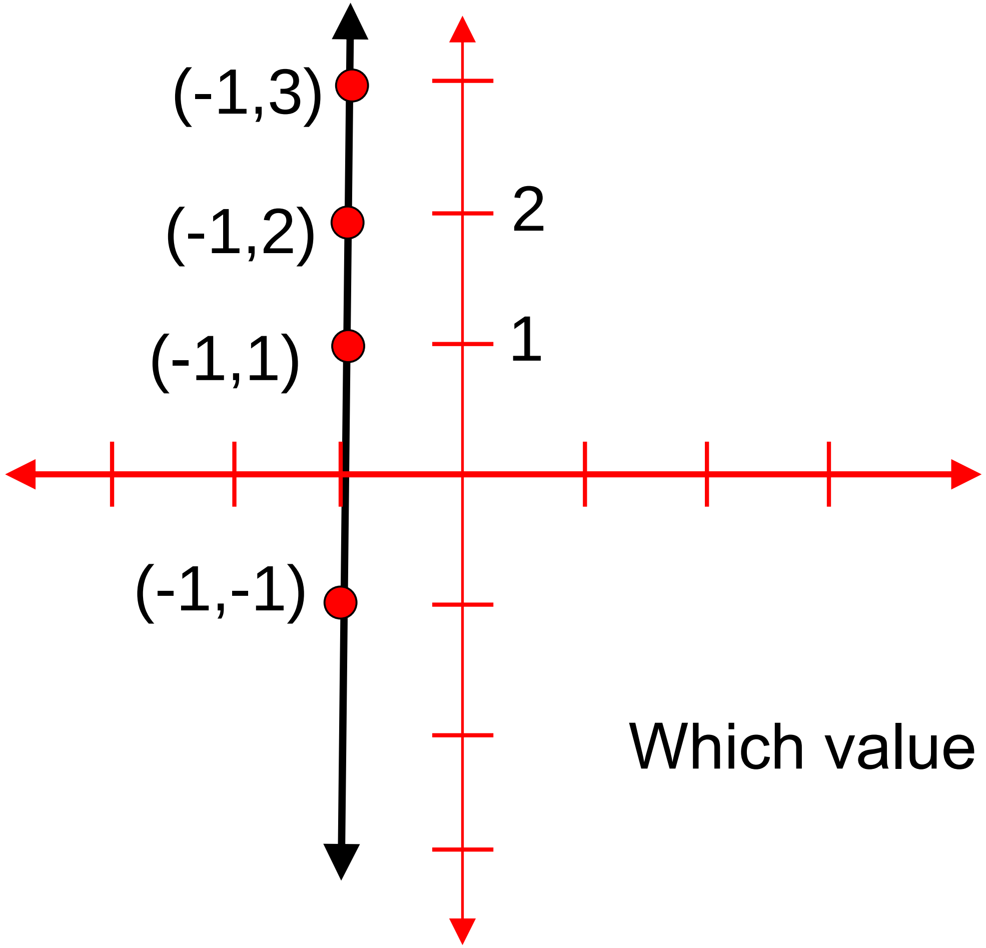
$$y = mx - 3$$

$$0 = m(2) - 3 \qquad m = \frac{3}{2}$$

$$y = \frac{3}{2}x - 3$$

$$m = \frac{\Delta y}{\Delta x} = \frac{3}{2}$$

What is the equation of the line?



$$y = -1$$

or

$$\boxed{x = -1}$$

?

Which value (x or y) is always ‘-1’?

What is the equation of the line?

$$y = -2x + 1$$