Math-3
Lesson 3-8
Solve Rational Equations

Solution to an equation: the value of the variables or unknown value that makes the equation “true”.

Equivalent equation: has the same solution as the original equation:

$$4x+2=10 \qquad 4x=8$$

The solution to both equations is x = 2.

They are equivalent equations.

Solving Rational Equations

Method #1: Obtain common denominators for each term

Method #2: Determine what the common denominator would be then multiply by that value.

What does solve a single variable equation mean?

$$3x + 2 = 11$$

Find the value of the variable that makes the equation "true."

What is a factor?

A number that is being multiplied by another number.

Method 1: Obtain a common denominator

$$(4x + 5) * \frac{3}{x+1} = \frac{9}{4x+5} * (x + 1)$$

$$(4x + 5) * \frac{3}{x+1} = \frac{9}{4x+5} * (x + 1)$$

Multiply both sides by the common denominator

$$\frac{(4x+5)(x+1)\ 3(4x+5)}{(4x+5)(x+1)} = \frac{9(x+1)(4x+5)(x+1)}{(4x+5)(x+1)}$$

Solve: $3(4x+5) = 9(x+1)$

$$\boxed{x = -2}$$

Identify the excluded value then solve.

$x \neq -2$

$$\frac{(x+2)}{(x+2)} * \frac{9}{5} = \frac{4}{x+2} * \frac{5}{5}$$

$$\frac{5(x+2)}{1} * \frac{9(x+2)}{5(x+2)} = \frac{20}{5(x+2)} * \frac{5(x+2)}{1}$$

$$9(x+2) = 20$$

$$9x + 18 = 20$$

$$-18 \quad -18$$

$$9x = 2$$

$$\div 9 \quad \div 9$$

$$x = \frac{2}{9}$$

Identify the excluded value then solve.

$x \neq 1$

$$\frac{x}{3} + 1 = \frac{2}{x+1}$$

$$\frac{(x+1)}{(x+1)} * \frac{x}{3} + 1 * \frac{3(x+1)}{3(x+1)} = \frac{5}{x+1} * \frac{3}{3}$$

$$\frac{x(x+1)}{3(x+1)} + \frac{3(x+1)}{3(x+1)} = \frac{15}{3(x+1)}$$

$$\frac{\cancel{3(x+1)}}{1} * \frac{x(x+1)+3(x+1)}{\cancel{3(x+1)}} = \frac{15}{\cancel{3(x+1)}} * \frac{\cancel{3(x+1)}}{1}$$

$$x(x+1) + 3(x+1) = 15$$

$$x^2 + x + 3x + 3 = 15$$

$$x^2 + 4x - 12 = 0$$

$$(x+6)(x-2) = 0$$

$x = -6, 2$

Sometimes it's easier to just "undo" division by 'x'.

$$\frac{4}{x}+x=5$$

$\boxed{x \neq 1}$

$$x * \left(\frac{4}{x}+x\right)=5 * x$$

$$4+x^2=5x$$

Non-standard quadratic equation.

Put into standard form !!!

$$x^2-5x+4=0$$

$$(x-4)(x-1)=0$$

$\boxed{x=4}$ $\boxed{x=1}$

$$1+\frac{8}{x-5}=-\frac{9}{x}$$

$x \neq 0, 5$

Obtain a common denominator.

$$\frac{x(x-5)}{x(x-5)} * 1+\frac{8}{x-5} * \frac{x}{x} = -\frac{9}{x} * \frac{(x-5)}{(x-5)}$$

$$\frac{\cancel{x(x-5)}}{1} * \quad \frac{x(x-5)+8x}{\cancel{x(x-5)}} = \frac{-9(x-5)}{\cancel{x(x-5)}} \quad * \frac{\cancel{x(x-5)}}{1}$$

$$x(x-5)+8x=-9(x-5)$$

$$x^2-5x+8x=-9x+45$$

$$x^2+12x-45=0$$

$$(x+6)(x-2)=0$$

$$(x+15)(x-3)=0$$

$x = -15, 3$

Neither solution is an excluded value!

Extraneous Solution: a solution obtained algebraically that is not in the domain of the original equation.

$$\frac{2x}{x-3}+1=\frac{x^2-x}{x-3}$$

What are the excluded values?

$x \neq 3$

$$\frac{2x}{x-3}+\frac{1}{1} * \frac{(x-3)}{(x-3)} = \frac{x^2-x}{x-3}$$

$$\frac{(x-3)}{1} * \quad \frac{2x+(x-3)}{(x-3)} = \frac{x^2-x}{(x-3)} * \frac{x(x-3)}{1}$$

$$2x+(x-3)=x^2-x$$

$$0=x^2-4x+3$$

$$0=(x-3)(x-1)$$

$x = 3, 1$

x = 3 is an excluded value!

$$\frac{1}{2x} = \frac{1}{6} + \frac{x^2 - x - 12}{6x^2}$$

$x = 1 \pm \sqrt{7}$

neither are excluded values

$$\frac{1}{x^2} = \frac{1}{4x^2} - \frac{x+3}{4x^2}$$

$\boxed{x = -6}$

Solution is not an excluded value

Method #2: Multiply by the "brute force" common denominator

$$\frac{1}{x} = \frac{1}{5} + \frac{1}{4}$$

$$\frac{5 * 4 * x}{1} * \frac{1}{x} = \frac{5 * 4 * x}{1} * \frac{1}{5} + \frac{1}{4} * \frac{5 * 4 * x}{1}$$

$$20 = 4x + 5x$$

$$20 = 9x$$

$$\boxed{x = \frac{20}{9}}$$