SM3-A THEOREMS 3-1 (Analyze Polynomials)

Fundamental Theorem of Algebra: If a polynomial has a degree of "n", then the polynomial has "n" zeroes (provided that repeat zeroes, called "multiplicities" are counted separately).

$$y = 6x^{4} + 42x^{3} + 96x^{2} + 28x + 48 \qquad y = 6(x+4)(x+3)(x-2i)(x+2i)$$

x = -4, -3, 2i, -2i

"4th Degree" → 4 zeroes (either real and/or imaginary)

How can you tell if there are zeroes that are multiplicities?

$$y = 3(x-2)^3(x+4)^2(x-\sqrt{5})(x+\sqrt{5})(x-3i)(x+3i)$$

$(x-2)^3 \rightarrow (x-2)(x-2)(x-2)$ x = 2 (multiplicity 3)

$(x+4)^2 \rightarrow (x+4)(x+4)$ x = 4 (multiplicity 2)

$(x-\sqrt{5})(x+\sqrt{5})(x-3i)(x+3i)$ 4 single multiplicity zeroes

Linear Factorization Theorem: If a polynomial has a degree of "n", then the polynomial can be factored into "n" linear factors.

$$y = 6x^{4} + 42x^{3} + 96x^{2} + 28x + 48 \rightarrow y = 6(x+4)(x+3)(x-2i)(x+2i)$$

**Since each linear factor has one zero, these two theorems are almost saying the same thing.

Complex Conjugates Theorem If f(x) is a polynomial and if (x + bi) is a factor (–bi is a zero) then its complex conjugate, (x - bi) is also a factor (and +bi is a zero) of f(x).

Example: $0 = x^2 + 4 \quad \rightarrow 0 = (x - 2i)(x + 2i)$

x = 2i, x = 2i

Example: $0 = x^4 + 5x^3 + 13x^2 + 45x + 36$

$0 = (x + 4)(x + 1)(x - 3i)(x + 3i)$

x = -4, -1, *3i,* *-3i*

Irrational Roots Theorem If f(x) is a polynomial and if $(x - \sqrt{b})$ is a factor of the polynomial ($\rightarrow \sqrt{b}$ is a zero) then its irrational conjugate $(x + \sqrt{b})$ is also a factor of the polynomial ($\rightarrow \sqrt{b}$ is also a zero).

Example: $0 = x^2 - 3 \quad \rightarrow 0 = (x - \sqrt{3})(x + \sqrt{3})$

$x = \sqrt{3}, \quad -\sqrt{3}$

Example: $0 = x^4 - x^2 - 20 \quad \rightarrow 0 = (x + 2i)(x - 2i)(x - \sqrt{5})(x + \sqrt{5})$

$x = -2i, \quad 2i, \quad \sqrt{5}, \quad -\sqrt{5}$