Math-3A

Lesson 8-5

Logarithmic and Exponential Modeling

Rewrite the following numbers as a power of "e"

$2 \qquad e^x = 2 \qquad x = \ln 2 \qquad e^{0.693} \approx 2$

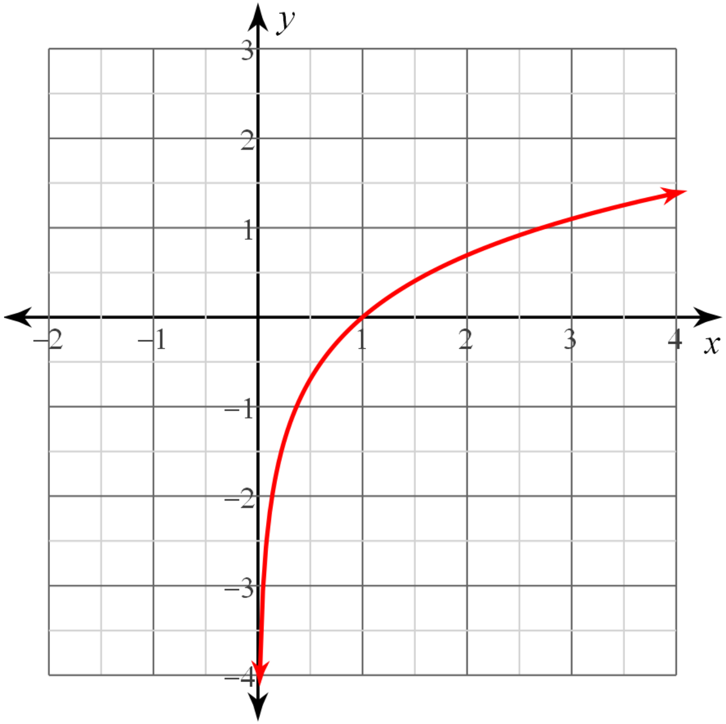
$1.05 \qquad \approx e^{0.049}$

$0.98 \qquad \approx e^{-0.020}$

$0.5 \qquad \approx e^{-0.693}$

-3 Not possible: Why?

$\ln(-3)$ doesn't exist



$y = \ln x$

We can rewrite the base of any exponential as a power of 'e'.

$$y = 2^x$$

$$y = \left(e^k\right)^x$$

$$e^k = 2$$

$$k = \ln 2$$

$$k \approx 0.693$$

$$y = e^{kx}$$

$$e^{0.693} \approx 2$$

$$y = e^{0.693x}$$

Rewrite the following as base 'e' exponential equations.

$$y = 4^x \quad = e^{1.386x}$$

$$y = 1.1^x \quad = e^{0.095x}$$

$$y = 1.01^x \quad = e^{0.010x}$$

$$y = 0.85^x \quad = e^{-0.163x}$$

$$y = 0.25^x \quad = e^{-1.386x}$$

How can you distinguish between growth and decay for…

A base "b" exponential?

$$y = b^x$$

$0 < b < 1$ decay

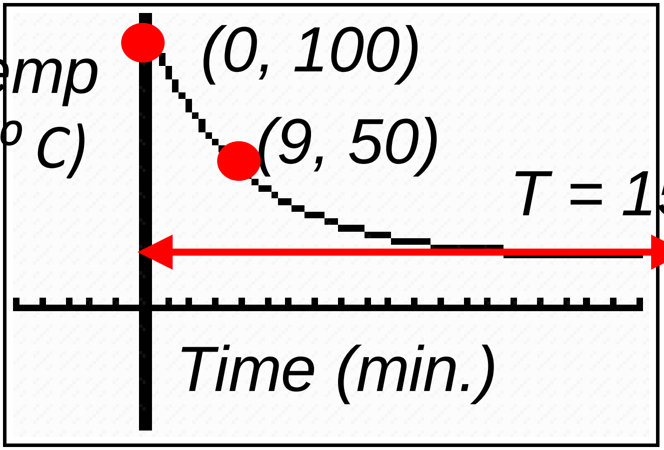
$b > 1$ growth

A base "e" exponential?

$$y = e^{kx}$$

$k < 0$ decay

$k > 0$ growth



Boiling water (100º C) is taken off the stove to cool in a room at 15º C. After 9 minutes, the water's temperature is 50 C.

Write the modeling equation as a base 'b' exponential.

$$T(t) = a(b)^t + k$$

1) Horizontal Asymptote

$$T(t) = a(b)^t + 15$$

2) y-intercept

$$100 = a(b)^0 + 15$$

$$a = 85$$

3) "nice point"

$$50 = 85(b)^9 + 15$$

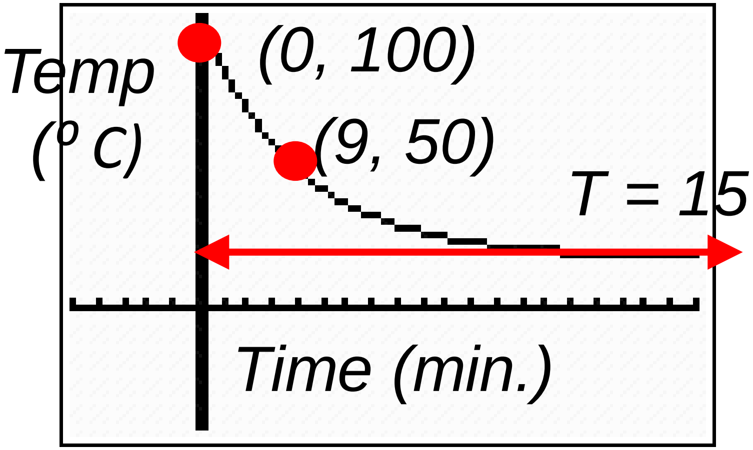
$$\left(\frac{50-15}{85}\right) = (b)^9$$

$$\left(\frac{50-15}{85}\right)^{1/9} = b$$

$$b = 0.906$$

4) Final equation

$$\boxed{T(t) = 85(0.906)^t + 15}$$



Boiling water (100º C) is taken off the stove to cool in a room at 15º C. After 9 minutes, the water's temperature is 50 C.

Write the modeling equation as a base 'e' exponential.

$$T(t) = 85(0.906)^t + 15$$

$$e^k = 0.906$$

$$\ln 0.906 = k$$

$$k = -0.0987$$

$$T(t) = 85e^{-0.0987t} + 15$$

A hard-boiled egg at temperature 212° F is placed in 60° F water to cool. 5 minutes later the temperature of the egg is 95° F. When will the egg be 75°C?

A cake taken out of the oven at temperature of 350° F. It is placed on in a room with an ambient temperature of 70°F to cool. Ten minutes later the temperature of the cake is 150°F. When will the cake be cool enough to put the frosting on (90°F) ?

Sound Intensity: the rate that energy is deposited on a surface by sound.

$$\frac{Energy/time}{area}$$

Unit of measure: $\frac{watt}{m^2}$

Lowest measureable sound intensity: 10^{-12} w/m^2

Sound intensity that causes pain: 10 w/m^2

10 is 1 trillion times larger than 10^{-12}

What is the sound intensity of ….?

Threshold of hearing	10^{-12} w/m^2	0.000000000001
breathing	$10^{-9.5}$ w/m^2	0.0000000003
Threshold of pain	10^{0} w/m^2	1.0
Firecracker by ear	10^{5} w/m^2	100,000.
Pistol by ear	10^{6} w/m^2	1,000,000.

These numbers <u>don't give us a good "feel"</u> for loudness.

So we use something more useful: "Loudness".

Have you heard of "dB" (decibels)?

Loudness

Intensity of the sound in watts/sq meter

$$L(I) = 10\log\frac{I}{10^{-12}}$$

Loudness of the sound (in decibels) as a function of the sound intensity

Intensity of sound at the threshold of hearing

The logarand is the ratio of the actual sound intensity compared to the minimum detectable sound intensity

Don't freak out! This is just a simple log equation. But you must be able to handle properties of exponents!

What is the sound intensity of ….?

Threshold of hearing	10^{-12} w/m^2	O dB
breathing	$10^{-9.5}$ w/m^2	25 dB
Threshold of pain	10^{0} w/m^2	120 dB
Firecracker by ear	10^{5} w/m^2	170 dB
Pistol by ear	10^{6} w/m^2	180 dB

Which units are easier?

The sound intensity values aren't nearly as useful as the decibel values.

$$L(I) = 10\log\frac{I}{10^{-12}}$$

An ambulance has a sound intensity of 10^{0} watts/sq meter

How Loud is the ambulance? (in decibels)

$$L(I) = 10\log\frac{10^{0}}{10^{-12}}$$

Properties of exponents!!!

$$= 10\log 10^{0-(-12)}$$

$$= 10\log 10^{12} \quad = 10 * 12 = 120db$$

$$L(I) = 10\log\frac{I}{10^{-12}}$$

The front row of a rock concert has a sound intensity of

$$I = 10^{-1} \text{ watts/meter}^2$$

What is the sound level in decibels on the front row of the rock concert?

$$L = 10\log\frac{10^{-1}}{10^{-12}} = 10\log 10^{11}$$

$$= 110\log 10$$

$$= 110 \text{ dB}$$

Rate: ratio of quantities

concentration: amount of a specific material compared to the total volume.

$$\frac{amount}{volume}$$

Unit of measure:

$$\frac{moles}{liter}$$

Lowest measurable concentration of hydrogen ion: $[\mathrm{H}^{+}]$

$$[H^{+}] = 10^{-14} \text{ moles/li}$$

Maximum concentration: 1 mole/li

$$1 \text{ is } 100 \text{ trillion times as large as } 10^{-14}$$

In chemistry, the acidity of a water-based solution is measured by the concentration of hydrogen ions in the solution (in moles per liter). The hydrogen-ion concentration is written $[H^+]$.

Upset stomach acid	$1\ \text{mole/li}$	1.0
Normal stomach acid	$10^{-2}\ \text{mole/li}$	0.01
rain	$10^{-5}\ \text{mole/li}$	0.00001
Sea water	$10^{-8}\ \text{mole/li}$	0.00000001
bleach	$10^{-12}\ \text{mole/li}$	0.000000000001
Sodium Hydroxide	$10^{-14}\ \text{mole/li}$	0.00000000000001

These numbers don't give us a good "feel" for acidity.

So we use something more useful: "pH".

Acidity $pH = -\log [H^+]$

Don't freak out! This is just a simple log equation.

Upset stomach acid	1 mole/li	1
Normal stomach acid	10^{-2} mole/li	2
rain	10^{-5} mole/li	5
Sea water	10^{-8} mole/li	8
bleach	10^{-12} mole/li	12
Sodium Hydroxide	10^{-14} mole/li	14

pH is a much more useful way of measuring acidity that the concentration of the hydronium ion.

Acidity

pH = - log [H^+]

The “hydronium ion concentration of a solution is

$$[H^+] = 5.7 \times 10^{-11} \text{ mole/li}$$

What is the pH of the solution? pH = - log [5.7×10^{-11}]

pH = 10.3

The pH of baking soda is 8.6.

What is the hydrogen ion concentration?

8.6 = - log [x] $x = 10^{-8.6}$ mole/li

$$x = 2.5 \times 10^{-9} \text{ mole/li}$$

Acidity

$pH = -\log [H^+]$

The pH of baking soda is 8.6.

What is the hydrogen ion concentration?

$8.6 = -\log [x]$

$$x = 10^{-8.6} \text{ mole/li}$$

$$x = 2.5 \times 10^{-9} \text{ mole/li}$$

You deposit $100 money into an account that pays 3.5% interest per year. The interest is “compounded” monthly. How much money will be in the account at the end of the 5th year?

$$A(t) = A_0(1 + r/k)^{kt} \qquad A(5) = 100(1 + 0.035/12)^{12(5)}$$

$$A(5) = \$119.09$$

What is the doubling time for this account?

$$200 = 100(1 + 0.035/12)^{12t}$$

$$2 = (1.0029)^{12t}$$

$$\log_{1.0029}(2) = 12t \qquad 239.4 = 12t \qquad t = 19.9 \text{ yrs}$$

A bank compounds interest continuously. The annual interest rate is 5.5%. How long would it take for the money in the account to triple?

$$A(t) = A_0 e^{rt}$$

$$3A_0 = A_0 e^{0.055t}$$

$$3 = e^{0.055t}$$

$$\ln 3 = 0.055t$$

$$t = 19.97 \text{ yrs}$$

The "half life" of Carbon-14 (a radioactive isotope of carbon), is 5730 years. Calculate the decay rate for carbon-14. The decay rate is the "k" of the exponent of 'e'.

$$A(t) = A_0 e^{kt}$$

$$0.5A_0 = A_0 e^{k(5730)}$$

$$0.5 = e^{5730(k)}$$

$$\ln 0.5 = 5730k$$

$$k = {}^{-0.00012}/_{yr}$$