SM3-A

Lesson 9-2

Co-terminal Angles, Radian Measure,

45-45-90 Triangles, 30-60-90 Triangles

Angles are classified based upon their degree measure.

Acute angle: $0° < m\measuredangle < 90°$

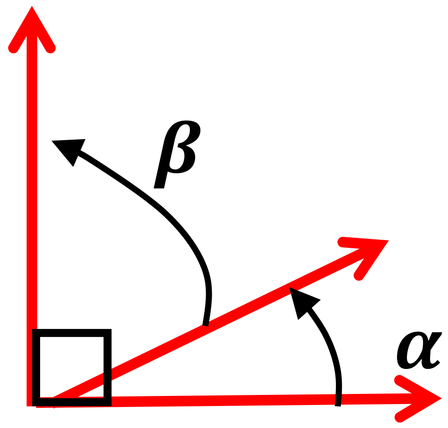
Right angle: $m\measuredangle = 90°$

Obtuse angle: $90° < m\measuredangle < 180°$

Straight angle: $m\measuredangle = 180°$

Adjacent angles share a common endpoint and one side.

Complimentary angles: two angles whose sum is 90 degrees. They need not be adjacent.

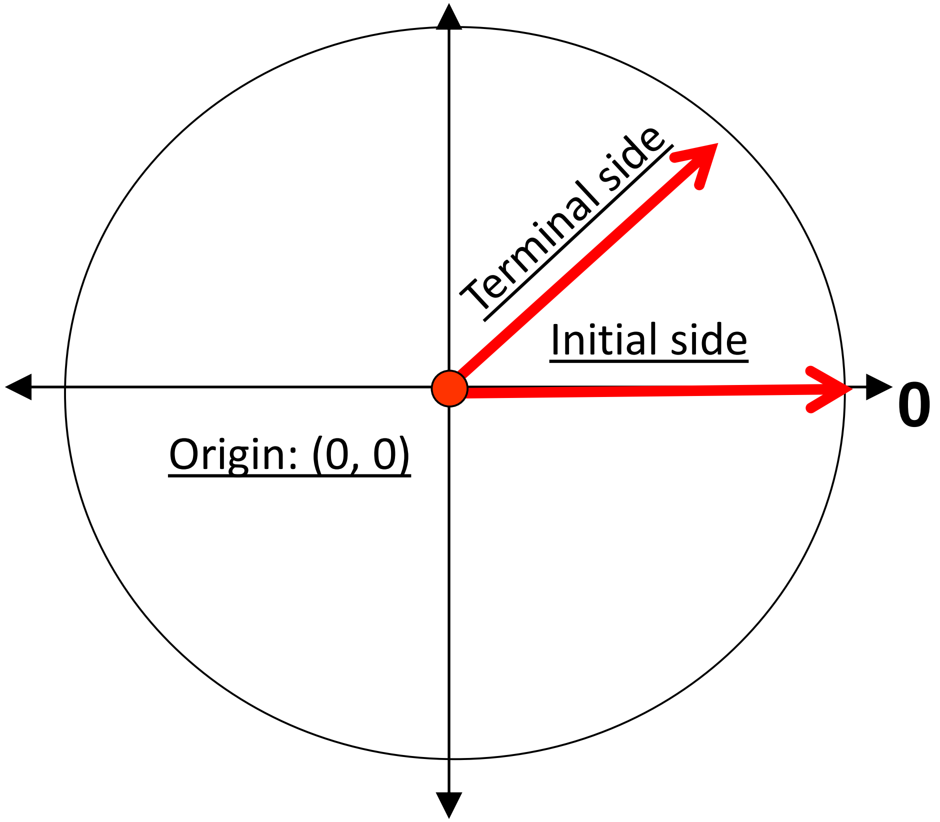


Angle: two rays with a common end point

Initial side of the angle: Angles graphed on the x-y plane, have their vertex at the origin. The initial side always points in the positive ‘x’ direction.

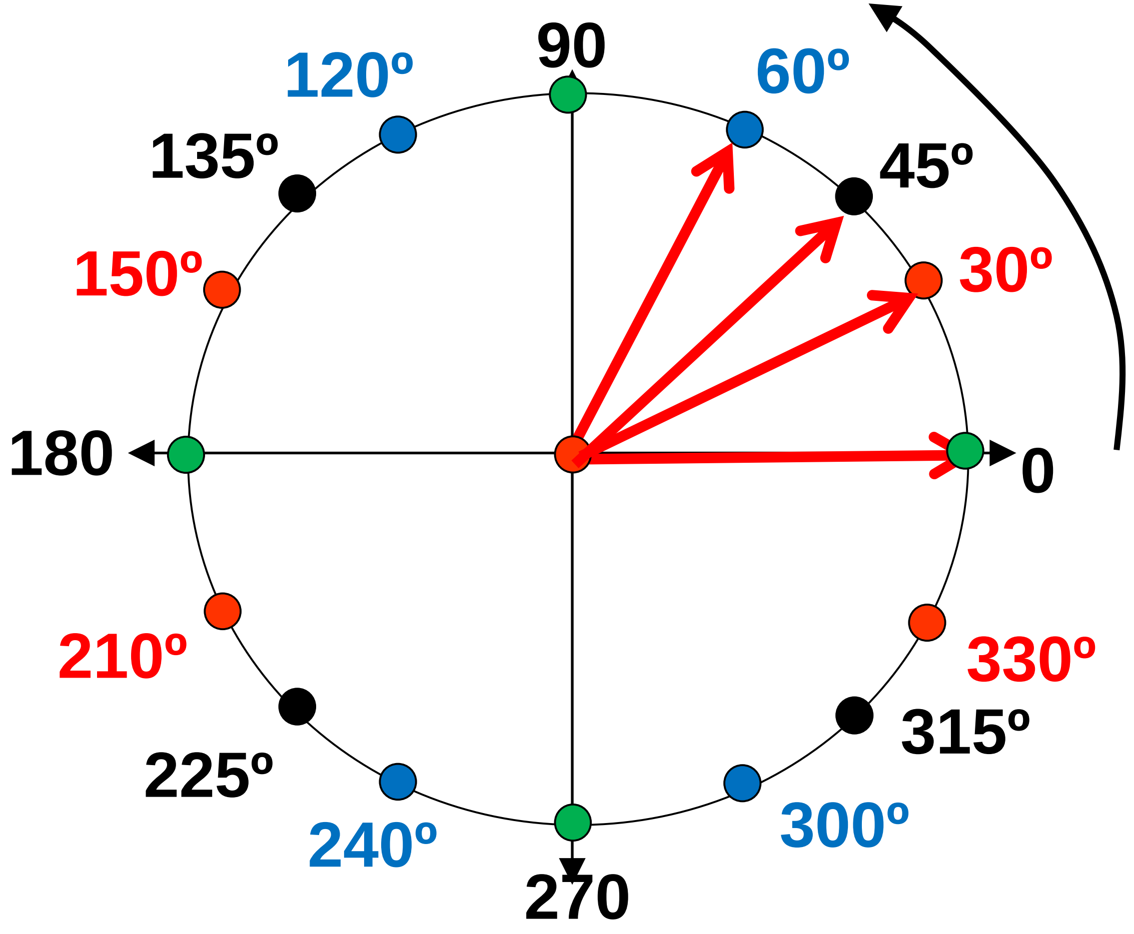
Terminal side of the angle: points in any direction from the origin.

We think of these angles as “opening” as the terminal side moves around the circle.



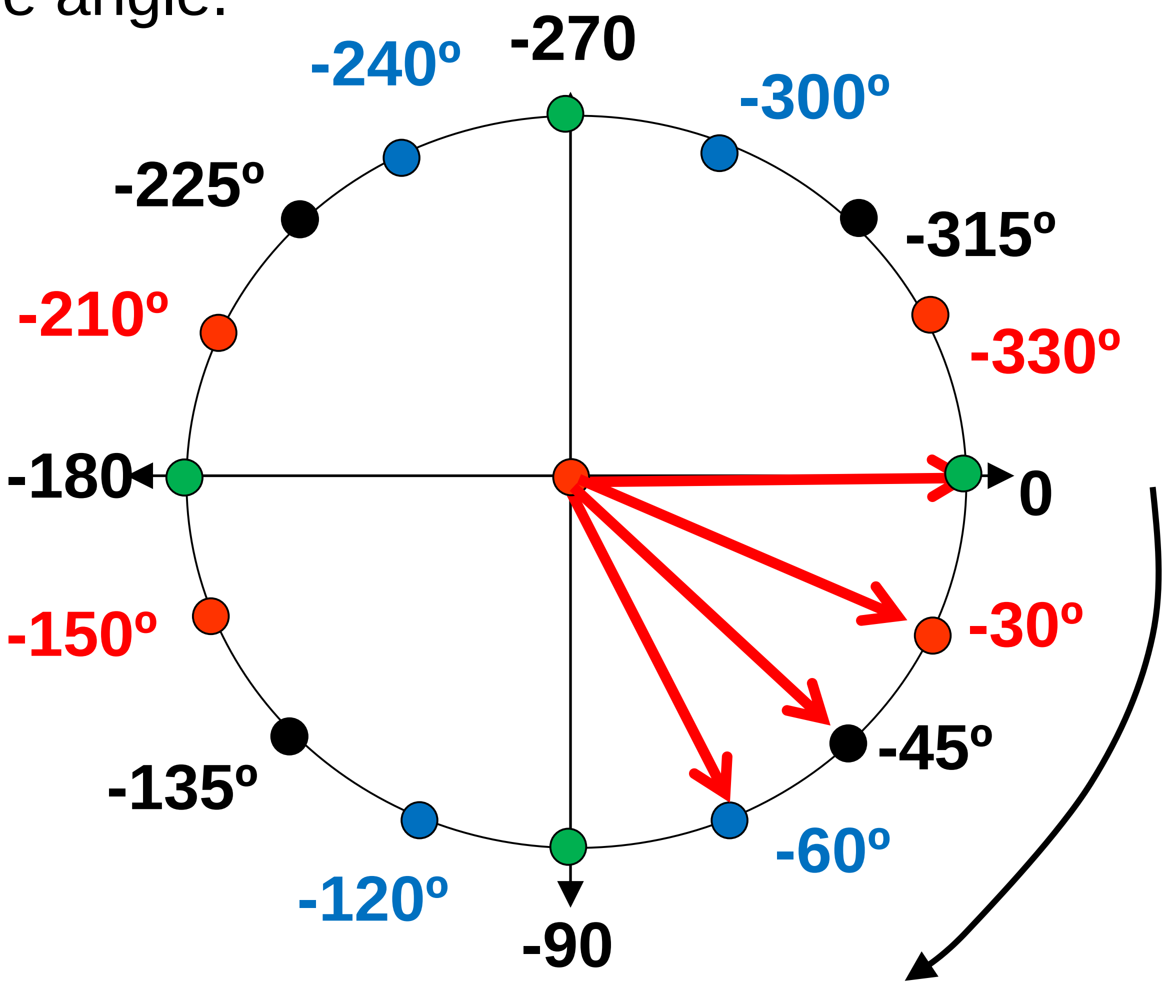
Positive Angles: The terminal side opened in the counter-clockwise direction from the initial side of the angle.

Angle measure: is put at the end of the terminal side rather than inside the angle.

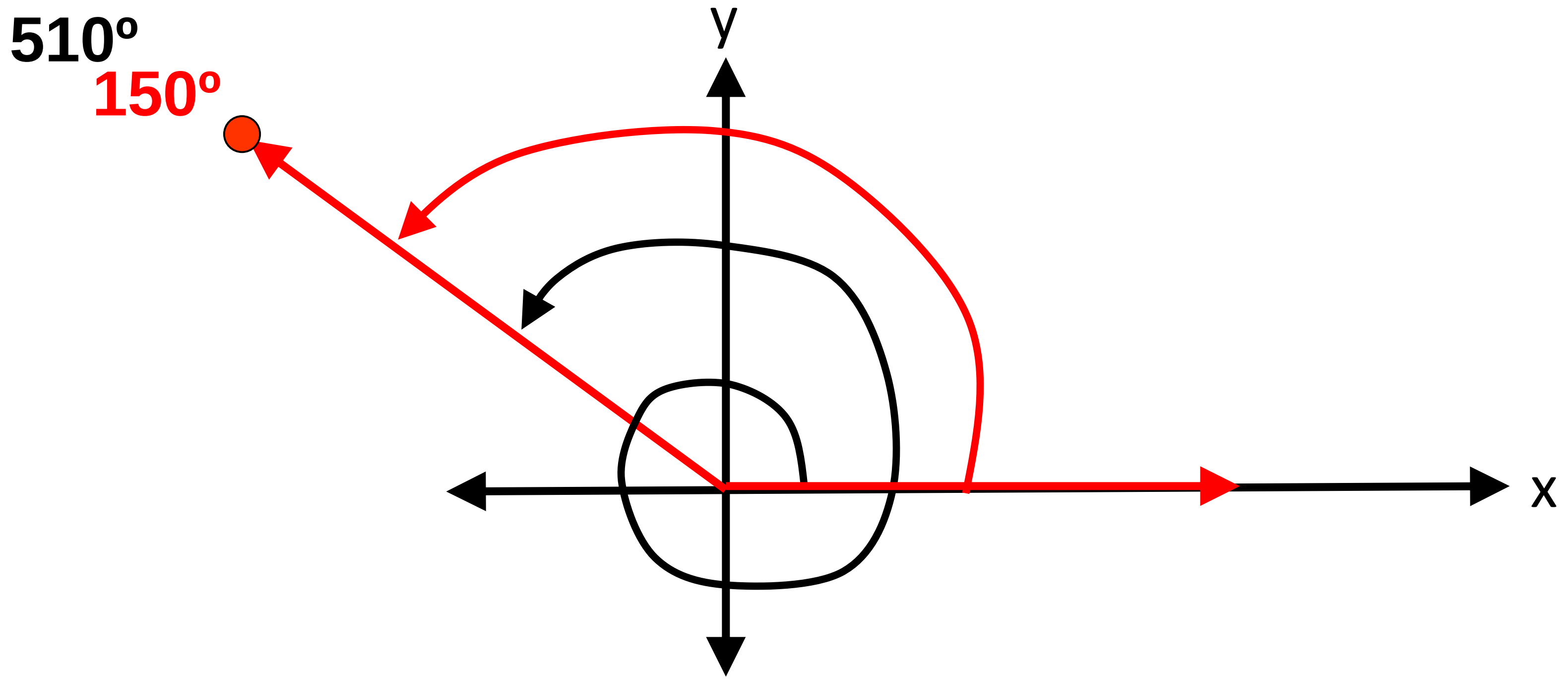


<u>Negative Angles</u>: The terminal side opened in the clockwise direction from the initial side of the angle.

<u>Large Angles</u>: The terminal side may make more than one revolution. An angle whose terminal side makes one complete revolution *<u>and then</u>* an additional 30 degree would have a measure of 390 degrees.



Co-terminal Angles: may have different measures but they share both the initial and terminal sides.



In general: co-terminal angles have measures defined by:

$$m\angle\theta = m\angle\beta \pm 360n$$

where ‘n’ is the number of revolutions beyond the 1st one.

Determining If angles are co-terminal:. $m\angle\theta = m\angle\beta \pm 360n$

$$m\angle\theta = 1550$$

$$m\angle\beta = 1550 - 360 = 1190$$

$$m\angle\beta = 1550 - 2(360) = 830$$

$$m\angle\beta = 1550 - 3(360) = 470$$

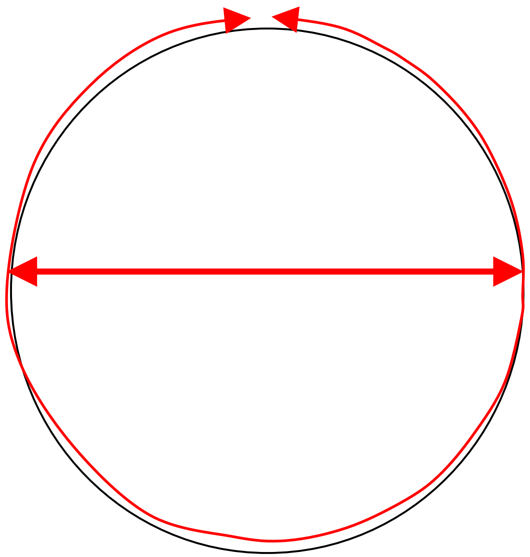
$$m\angle\beta = 1550 - 4(360) = 100$$

$$m\angle\beta = 1550 - 5(360) = -250$$

A typical co-terminal angle problem: will give the measure of an angle then ask for a different positive and negative co-terminal angle.

<u>Pi</u>: an <u>irrational number</u> that is the ratio of the distance <u>around</u> the circle to the distance <u>across</u> the circle.

$$\pi = \frac{C}{D}$$



$$\pi = \frac{C}{2r} \qquad C = 2\pi r$$

$$\boxed{\text{radian measure} = \frac{arc\ \text{length}}{\text{radius}}}$$

$$\text{radian measure (of a circle)} = \frac{circumference}{\text{r}}$$

$$\text{radian measure (of a circle)} = \frac{2\pi\cancel{r}}{\cancel{\text{r}}} = 2\pi$$

Units of radians = inches/inches

Radian measure has <u>no units</u>! (nice)

What is the radian measure of an angle that is ½ of the circle?

$$whole\ \text{circle} = 2\pi \text{ radians}$$

Radians vs. Degrees

$$360^{\circ} = 2\pi \text{ (radians)}$$

$$half\ \text{circle} = \pi \text{ radians}$$

Radians vs. Degrees

$$180^{\circ} = \pi \text{ (radians)}$$

Convert between radians and degrees using a "proportion".

$$\boxed{\frac{angle_{degrees}}{360} = \frac{angle_{radians}}{2\pi}}$$

$\frac{7}{8}\pi$

$$\frac{angle_{degrees}}{360} = \frac{{}^{7}/_{8}\cancel{\pi}}{2\cancel{\pi}}$$

$$360 * \frac{angle_{degrees}}{360} = 0.4375 \quad * 360$$

$$angle_{degrees} = 157.5°$$

$\boxed{180° = \pi \text{ radians}}$

Divide both sides by 180

$$\frac{180^\circ}{180^\circ} = \frac{\pi}{180^\circ}$$

$$1 = \frac{\pi}{180^\circ}$$

$$\left(\frac{\pi}{180^o}\right)$$

Divide both sides by π

$$\frac{180^\circ}{\pi} = \frac{\pi}{\pi}$$

$$1 = \frac{180^\circ}{\pi}$$

$$\left(\frac{180^o}{\pi}\right)$$

These are "unit conversion factors"

Unit Conversion factor: a ratio of equal measurements in different units that allow conversion of a one type of unit to another (feet → inches, degrees → radians, radians → degrees etc.)

When you multiply a number by one of these factors, (you are multiplying by "1") but the units are converted.

Converting from Degrees to Radian Measure

$$\cancel{140^{o}}\left(\frac{\pi}{\cancel{180^{o}}}\right) = \frac{140}{180}\pi = \frac{14}{18}\pi = \frac{7}{9}\pi$$

Converting from Radian Measure to Degrees

$$\frac{\cancel{\pi}}{2}\left(\frac{180^{o}}{\cancel{\pi}}\right) = 90^{\circ}$$

$$\frac{11}{3}\pi = ?$$

$$270^{o} = ?$$

Determining If angles are co-terminal:. $m\angle\theta = m\angle\beta \pm 2\pi * n$

$$m\angle\theta = \frac{\pi}{4}$$

$$m\angle\beta = \frac{\pi}{4} - 2\pi = \frac{\pi}{4} - \frac{8\pi}{4} = \frac{-7\pi}{4}$$

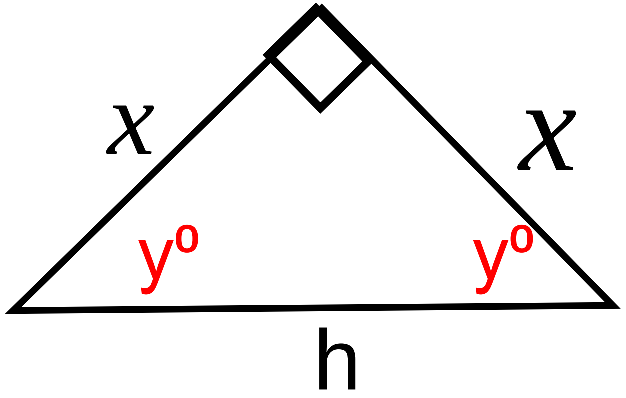
$$m\angle\beta = \frac{\pi}{4} - 2(2\pi) = \frac{\pi}{4} - \frac{16\pi}{4} = \frac{-15\pi}{4}$$

$$m\angle\beta = \frac{\pi}{4} + 2\pi = \frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4}$$

$$m\angle\beta = \frac{\pi}{4} + 2(2\pi) = \frac{\pi}{4} + \frac{16\pi}{4} = \frac{17\pi}{4}$$

A typical co-terminal angle problem: will give the measure of an angle then ask for a different positive and negative co-terminal angle.

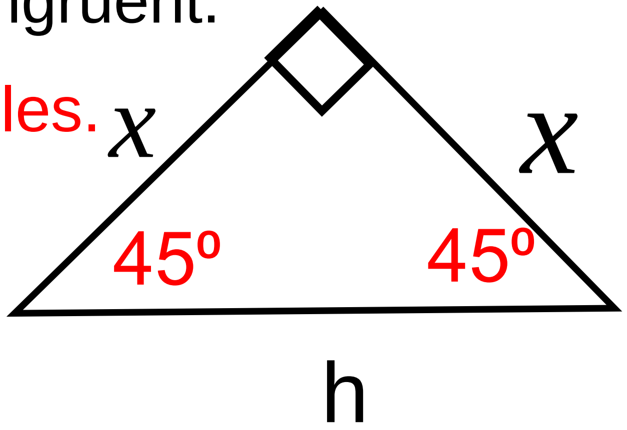
<u>Isosceles Right Triangle</u>: a right triangle with two sides that are congruent.



1) Find the measures of the base angles.

$$y^\circ + y^\circ + 90 = 180$$

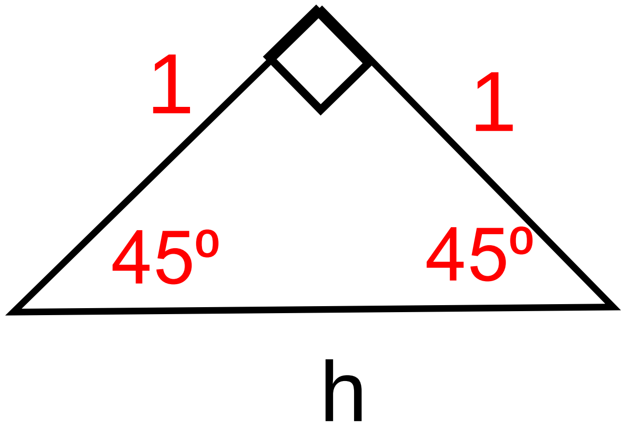
$$2y^\circ = 90 \quad y = 45^\circ$$



2) "X" can be any number. To make it ***really*** easy, lets just make x = 1.

3) *<u>Solve for 'h'.</u>*

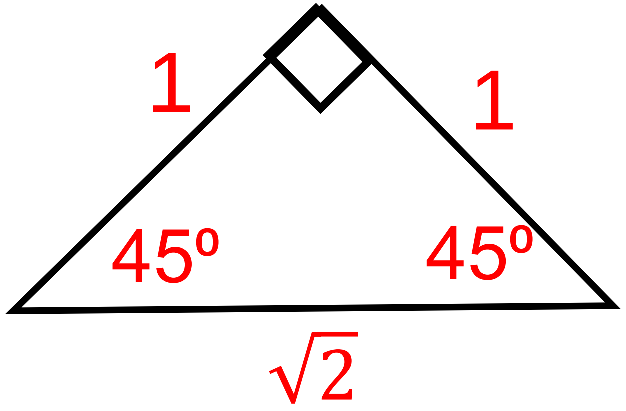
$$a^2 + b^2 = c^2$$



$$1^2 + 1^2 = c^2$$

$$2 = c^2$$

$$c = \sqrt{2}$$



We start with an *__Equilateral Triangle__* (all 3 sides and all 3 angles are congruent)

We construct an angle bisector.

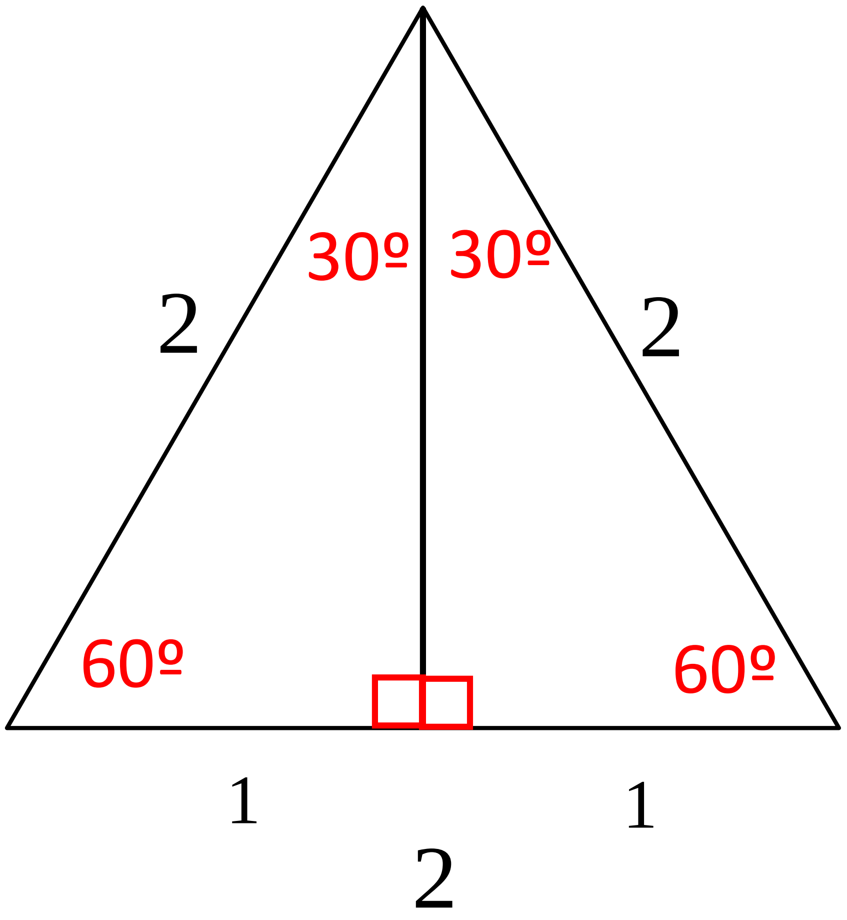
Are the two triangles congruent?

Yes: by ASA

__CPCTC__ (all remaining corresponding pairs of angles and sides are congruent).

Length = 1 and length = 1

Bottom legs (of the right triangles) are congruent so each is ½ the total of the original triangle's bottom length).



We now have a 30-60-90 triangle.

Solve for ‘x’.

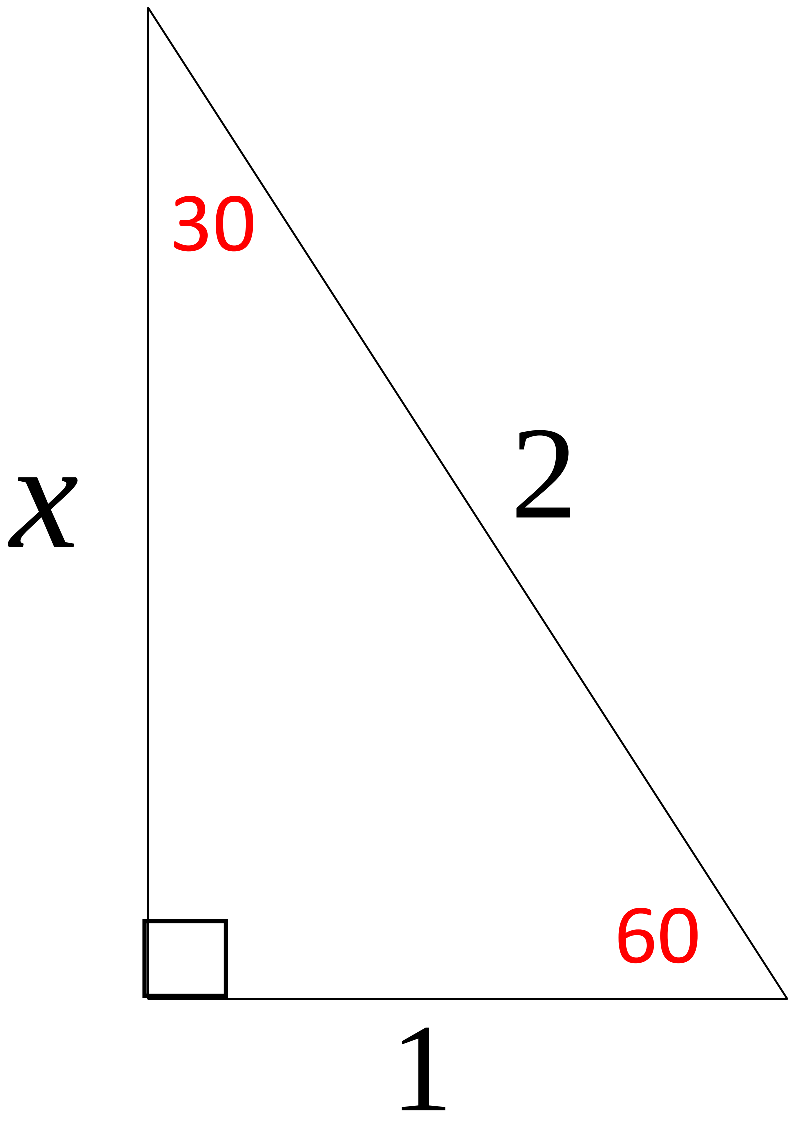
$$a^2 + b^2 = c^2$$

$$x^2 + 1^2 = 2^2$$

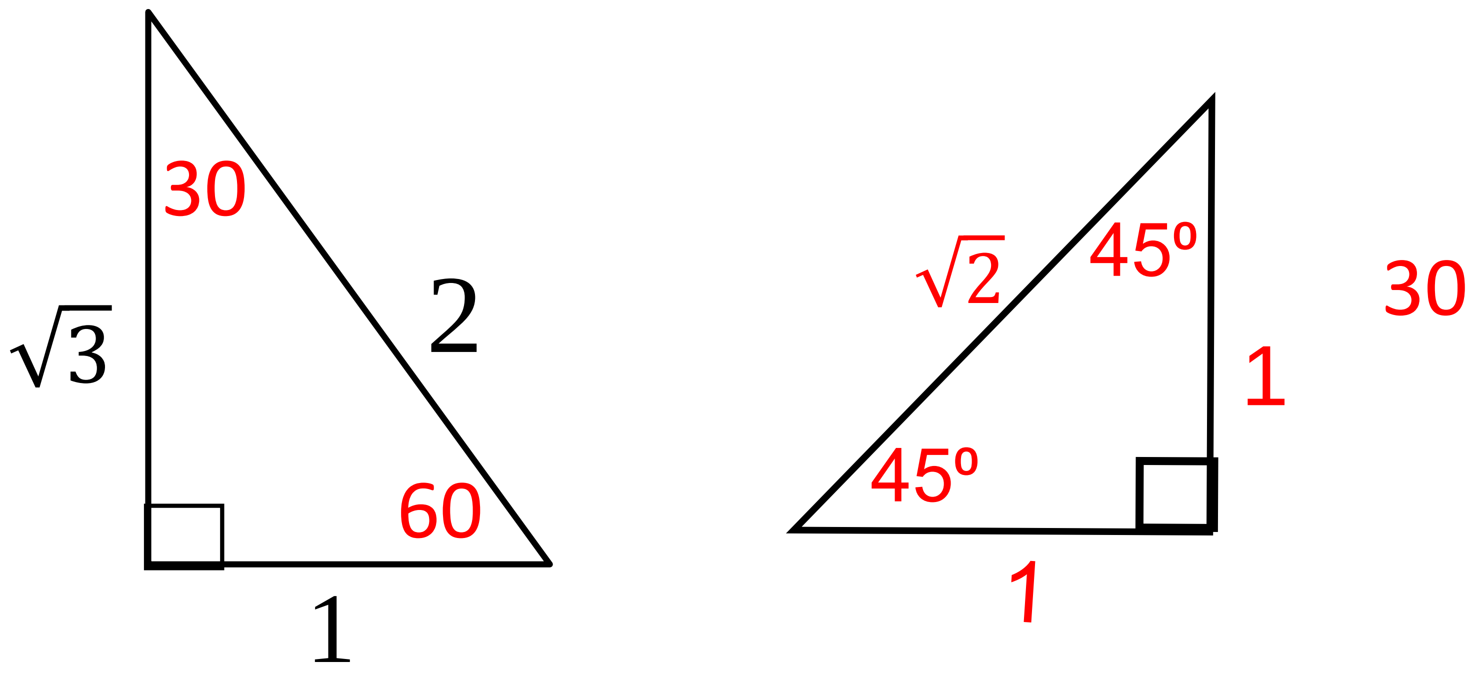
$$x^2 = 4 - 1$$

$$x^2 = 3$$

$$x = \sqrt{3}$$



“one-two-three-root”



Remember that having right triangles with a hypotenuse =1 is *"nice".*

Find the lengths of a *similar triangle* (for each) that has a hypotenuse = 1.