Math-3A
Lesson 9-1

Trigonometric Ratios for Right Triangles

Do it right now!

1. Obtain a the following:
   a) Paper triangle from Mr. Long.
   b) Protractor
   c) Ruler
   d) Instruction sheet

Triangle Similarity: Same shape (not same size)

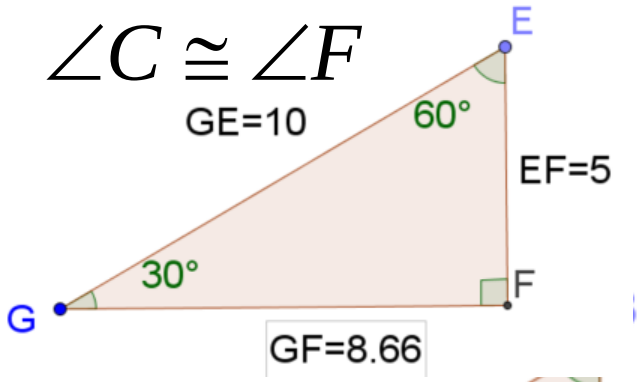
Shape results from three pairs of congruent angles.

$\angle A \cong \angle G$

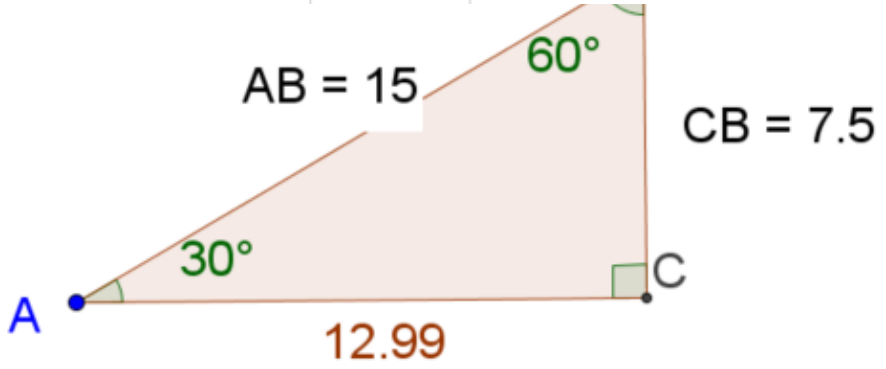
$\angle B \cong \angle E$

$\angle C \cong \angle F$

Similarity results in the ratios of corresponding sides always equaling the same number.



$$\frac{AB}{GE} = \frac{BC}{EF} = \frac{AC}{GF} = \frac{15}{10} = \frac{3}{2}$$

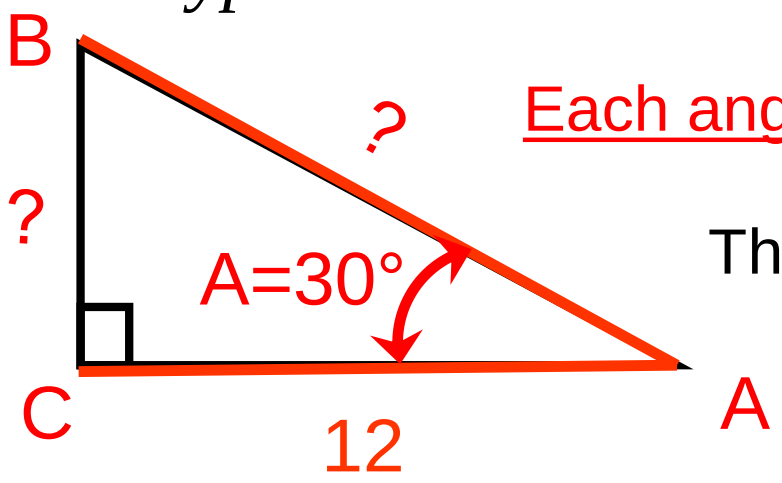


We call this number the "scale factor"

What did we learn from our activity?

$\frac{opp}{hyp} = ?$ Opposite of ***what?***

$\frac{opp \text{ of } \angle A}{hyp} = ? = \frac{1}{2} = 0.5$



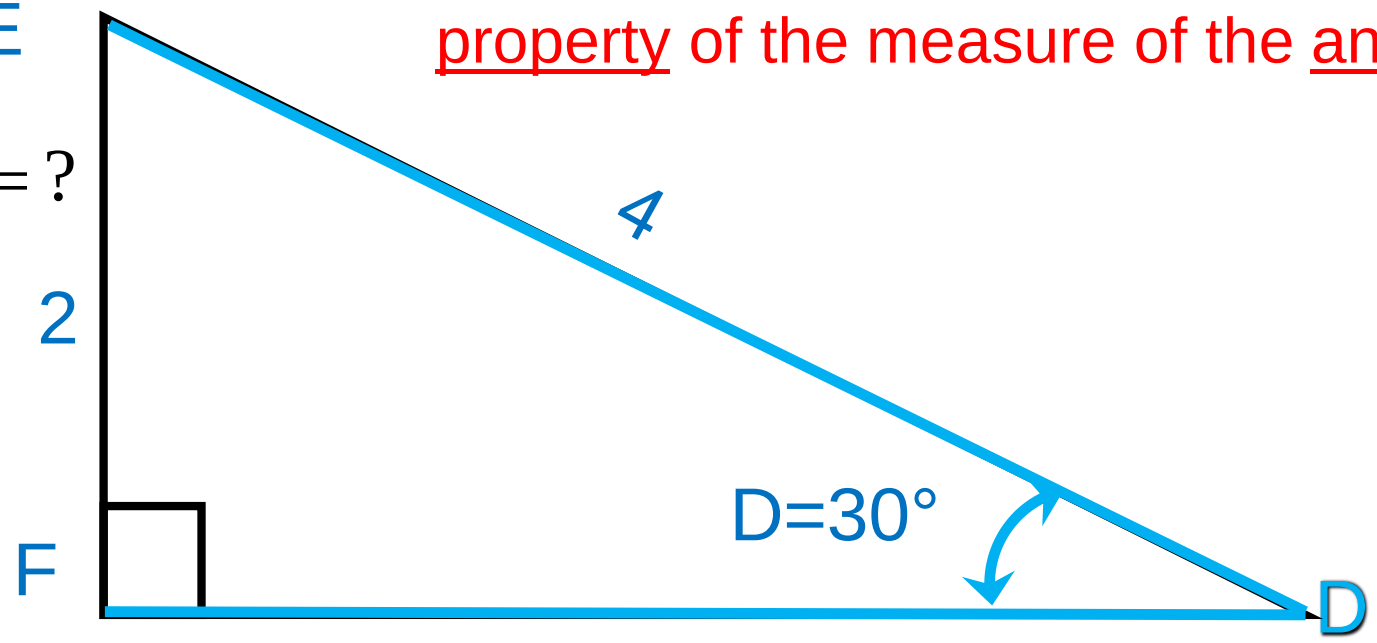
Each angle measure has its own unique ratio.

The size of the triangle does not matter.

The numerical value of this ratio is a property of the measure of the angle

$\frac{opp \text{ of } \angle D}{hyp} = ?$

$= \frac{2}{4} = 0.5$



"Ratios" are decimal form (not in fraction form).

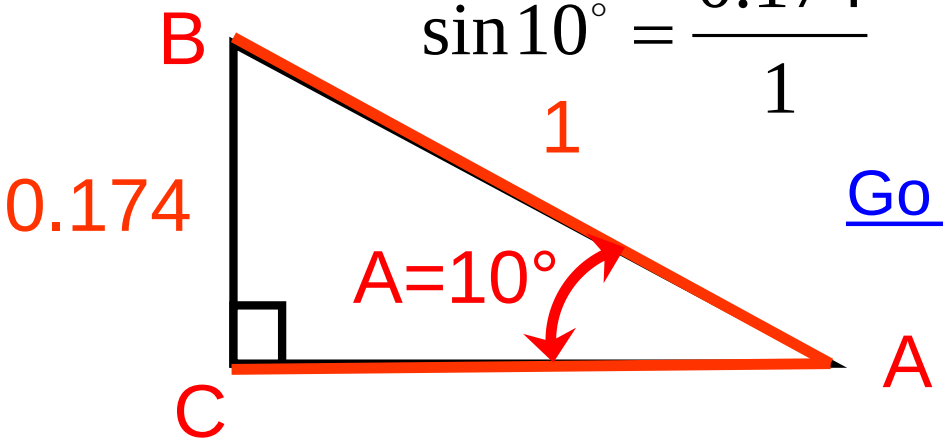
These "Ratios" are unique numbers for each angle; they are ***Properties of the angle.***

Angle	$\frac{opp}{hyp}$
10°	0.1736
20°	0.3420
30°	0.5
43.9°	0.6934
60°	0.8660

Radian	Degree	Sine	Cosine	Tan
0.000	0	0.000	1.000	0.0
0.017	1	0.017	1.000	0.0
0.035	2	0.035	0.999	0.0
0.052	3	0.052	0.999	0.0
0.070	4	0.070	0.998	0.0
0.087	5	0.087	0.996	0.0
0.105	6	0.105	0.995	0.1
0.122	7	0.122	0.993	0.1
0.140	8	0.139	0.990	0.1
0.157	9	0.156	0.988	0.1
0.175	10	0.174	0.985	0.1
0.192	11	0.191	0.982	0.1
0.209	12	0.208	0.978	0.2

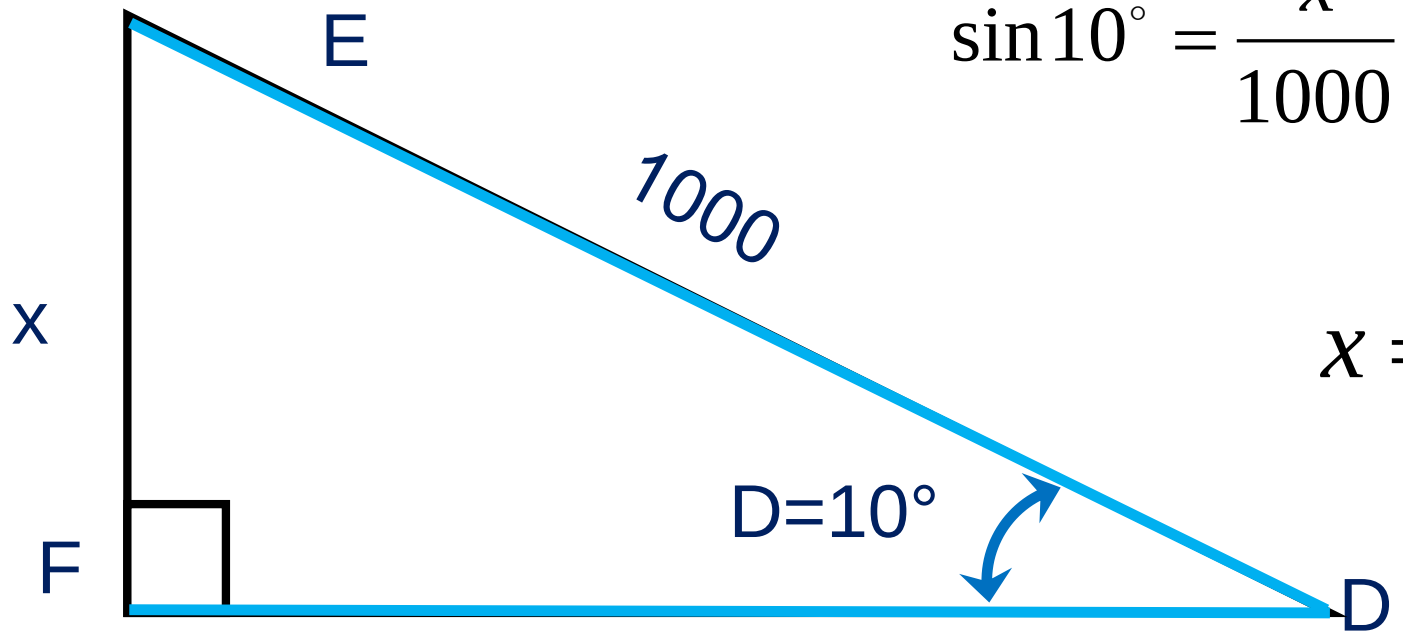
$$\text{sine A} = \frac{opp}{hyp}$$

$$\sin 10^\circ = 0.174$$



$$\sin 10^\circ = \frac{0.174}{1}$$

Go to Geogebra (trig ratios)

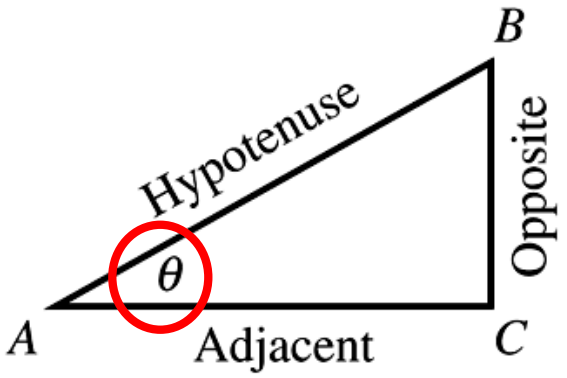


$$\sin 10^\circ = \frac{x}{1000}$$

$$x = 174$$

What are the "code words" for the ratios?

SOH-CAH-TOA



Key point: sine of an angle
(measured in degrees or radians)

$$\sin 30^\circ = \frac{\text{opposite (length)}}{\text{hypotenuse (length)}}$$

$$\sin\theta = \frac{\text{opposite (length)}}{\text{hypotenuse (length)}}$$

$$\cos\theta = \frac{\text{adjacent (length)}}{\text{hypotenuse (length)}}$$

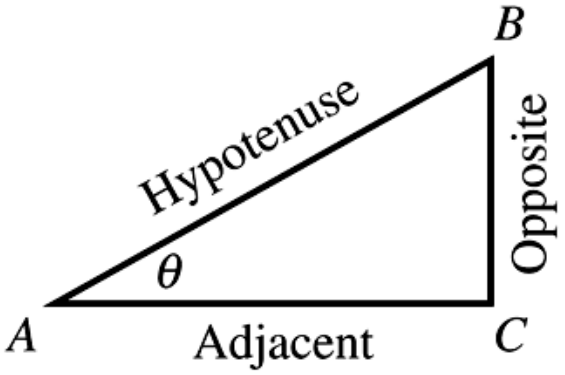
$$\tan\theta = \frac{\text{opposite (length)}}{\text{adjacent (length)}}$$

The ratio is a property of the angle. We must know the angle measure to find the correct ratio.

These only work for right triangles!!!

Trigonometric Functions

Shot your cow:
"Sha – Cho – Cao"



Notice that these ratios are *reciprocals* of the sine, cosine, and tangent ratios.

$\sin\theta \rightarrow \csc\theta$

$\cos\theta \rightarrow \sec\theta$

"S" goes with "C"

$\sec A = \frac{h}{a}$ $\quad \frac{\text{hypotenuse}}{\text{adjacent}}$

$\csc A = \frac{h}{o}$ $\quad \frac{\text{hypotenuse}}{\text{opposite}}$

$\cot A = \frac{a}{o}$ $\quad \frac{\text{adjacent}}{\text{opposite}}$

The sine ratio is the reciprocal of the cosecant ratio.

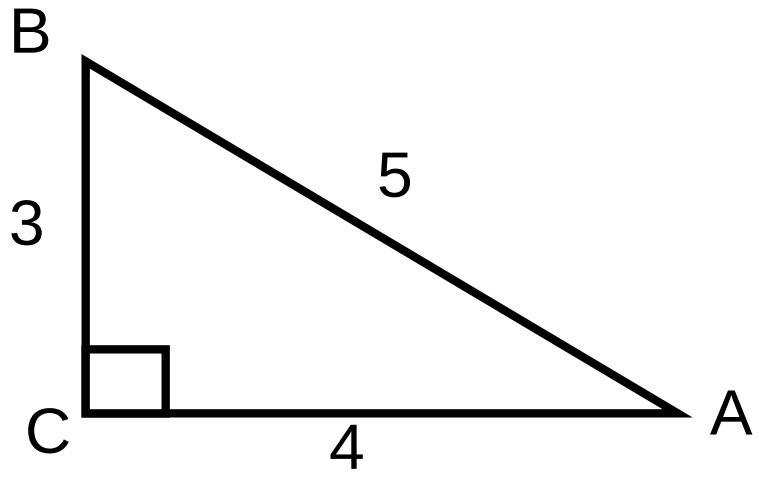
Trig Ratios

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec A = \frac{5}{4}$



$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

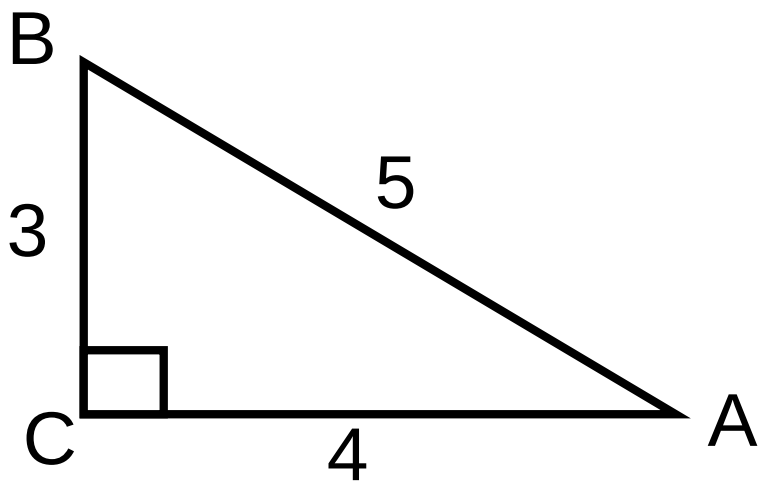
What patterns can you see?

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec A = \frac{5}{4}$



Cosine A and Secant A are reciprocals.

$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

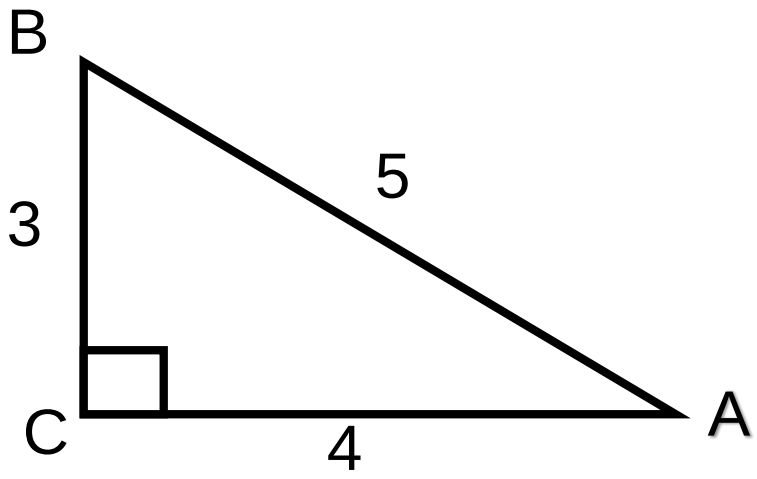
What patterns can you see?

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec A = \frac{5}{4}$



$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

Sine B and Cosecant B are reciprocals.

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

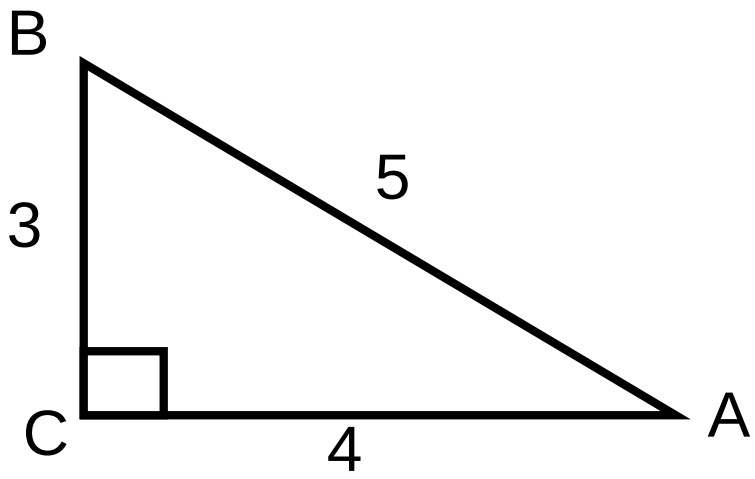
What patterns can you see?

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec B = \frac{5}{4}$



$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

Cos A = Sin B

Cos 20 = Sin 70

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

$\tan A = ?$

$\tan A = \frac{3}{4}$

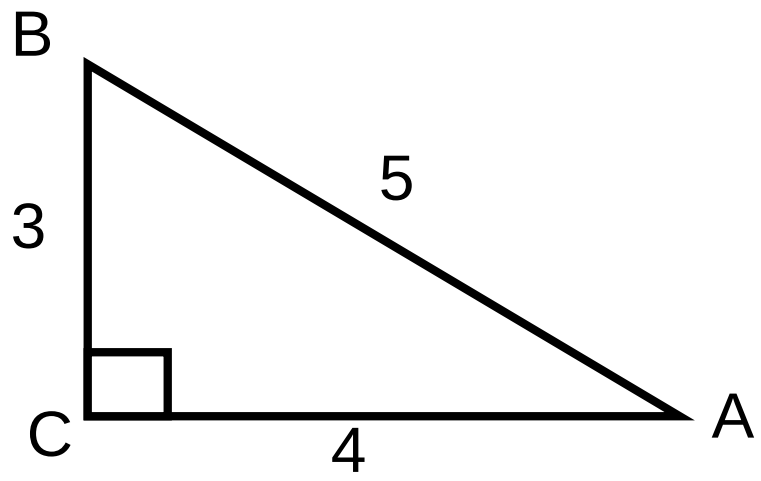
What patterns can you see?

$\cos A = ?$

$\cos A = \frac{4}{5}$

$\sec A = ?$

$\sec A = \frac{5}{4}$



$\sin B = ?$

$\sin B = \frac{4}{5}$

$\csc B = ?$

$\csc B = \frac{5}{4}$

Tan A= Cot B

Tan 30 = Cot 60

$\tan B = ?$

$\tan B = \frac{4}{3}$

$\cot A = ?$

$\cot A = \frac{4}{3}$

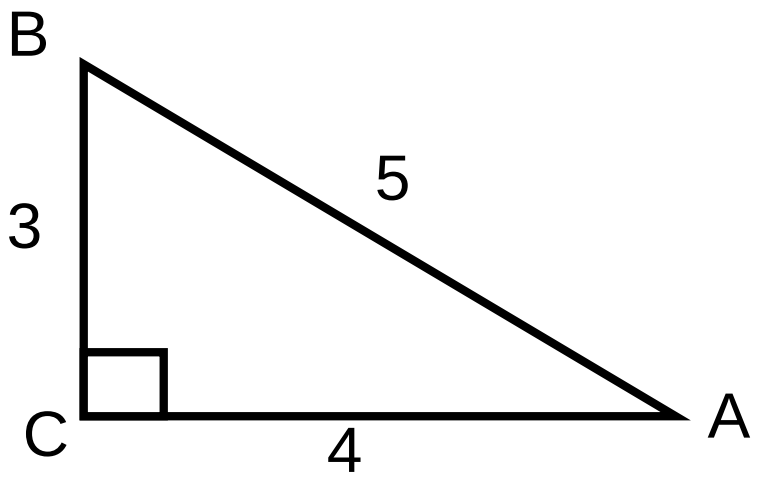
$\tan A = ?$

$\tan A = \frac{3}{4}$

What patterns can you see?

$\cos A = ?$ $\quad \cos A = \frac{4}{5}$

$\sec A = ?$ $\quad \sec A = \frac{5}{4}$



$\sin B = ?$ $\quad \sin B = \frac{4}{5}$

$\csc B = ?$ $\quad \csc B = \frac{5}{4}$

Tan A and Tan B are reciprocals.

$\tan B = ?$ $\quad \tan B = \frac{4}{3}$

$\cot A = ?$ $\quad \cot A = \frac{4}{3}$

$\tan A = ?$ $\quad \tan A = \frac{3}{4}$

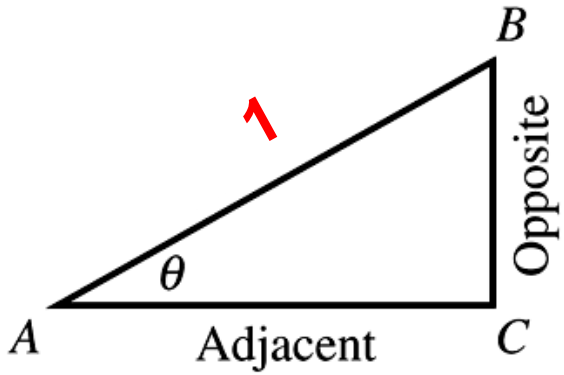
hypotenuse = 1

Why is it “nice” to have a hypotenuse whose length is ‘1’?

Sin Θ= opposite side

Cos Θ= adjacent side

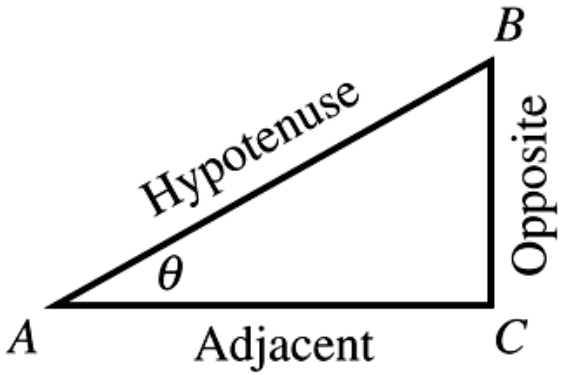
Tan Θ= opp/adj



The length of the hypotenuse is no longer in the ratio!

Trig Ratios of Acute Angles

What shape is used to define trig ratios? right triangle.



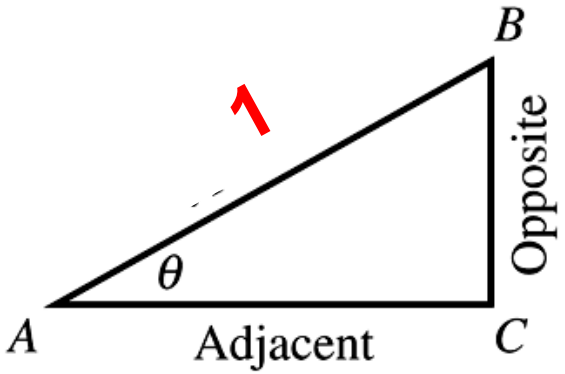
Using these definitions we can't have angles > 90!!!

Trig ratios for obtuse angles: we need acute angles!!

Sin Θ= opposite side

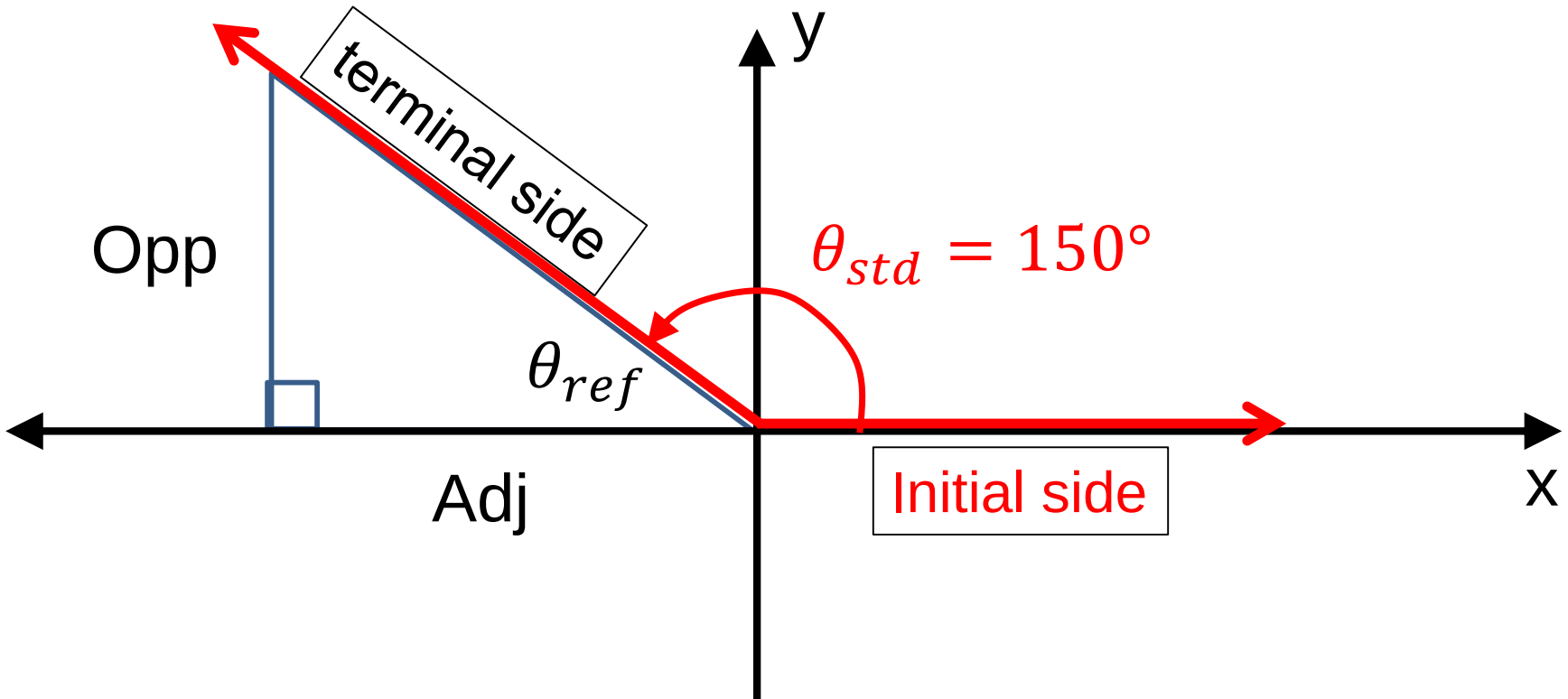
Cos Θ= adjacent side

Tan Θ= opp/adj



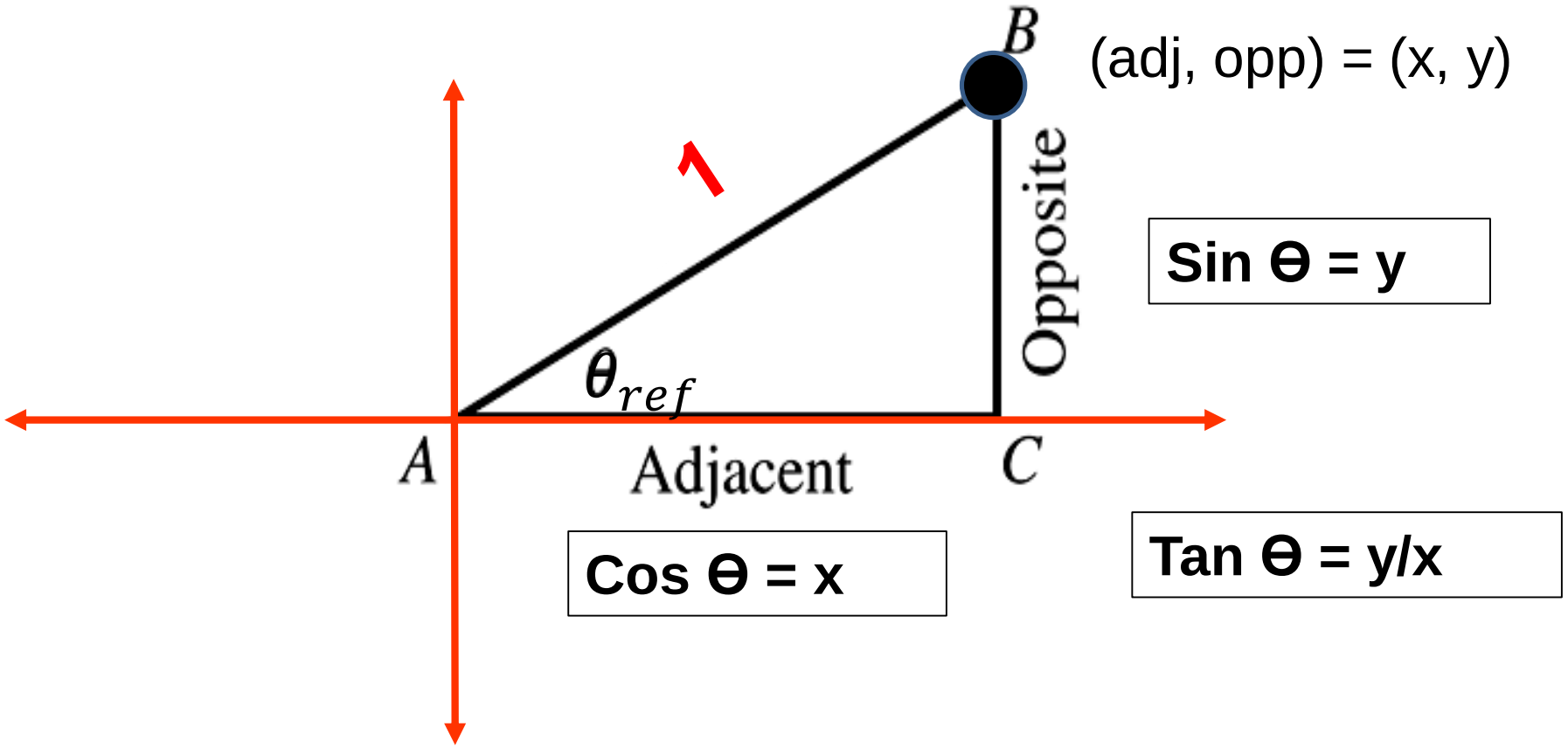
Trig ratios *only work for right triangles!* *If angle is greater than 90…*

1. Build a standard position angle on the x-y plane, with vertex at (0, 0)
2. Initial side of the angle: always points along the positive x-axis.
3. Terminal side of the angle: points outward from (0, 0).
4. We build a right triangle, hypotenuse is the terminal side.
5. We use the reference angle for our trig ratios.



Reference angle: the acute angle between the terminal side of a standard position angle and the x-axis.

With a *right triangle* with a hypotenuse =1 on the x-y plane, by using the *reference angle* for our trig ratios, the x-y pair at the end of the hypotenuse gives us *adjacent side length* and *opposite side length* of the right triangle.



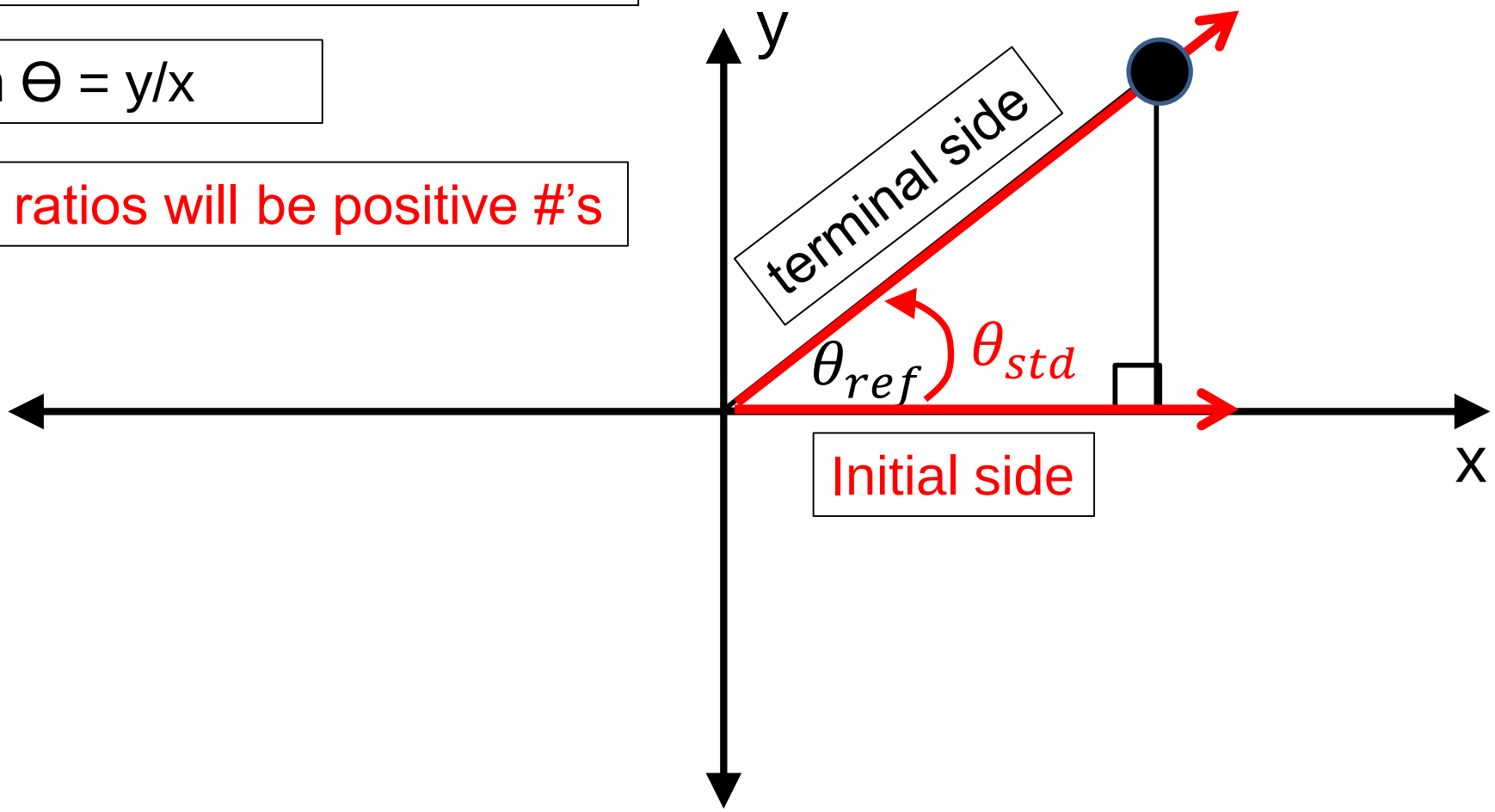
For Quadrant I $\theta_{ref} = \theta_{std}$

Sin Θ = y-value of the point

Cos Θ = x-value of the point

Tan Θ = y/x

Trig ratios will be positive #'s



For Quadrant II $\theta_{ref} = 180 - \theta_{std}$

Sine ratio is a positive number

Cosine ratio is a negative number

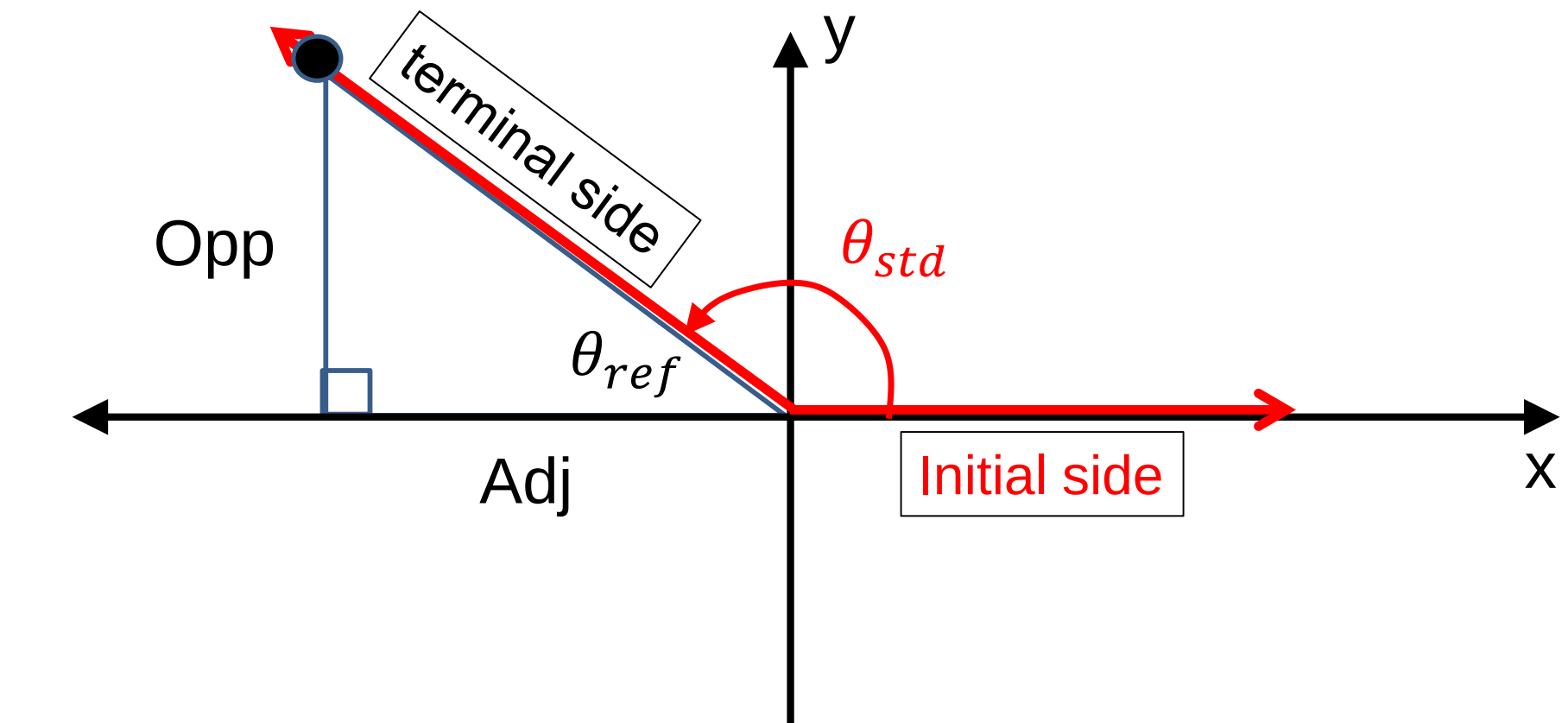
Tangent ratio is a negative number

Sin Θ = y

Cos Θ = x

Tan Θ = y/x

(adj, opp) = (x, y)



For Quadrant III

$\theta_{ref} = \theta_{std} - 180$

Sin Θ = y

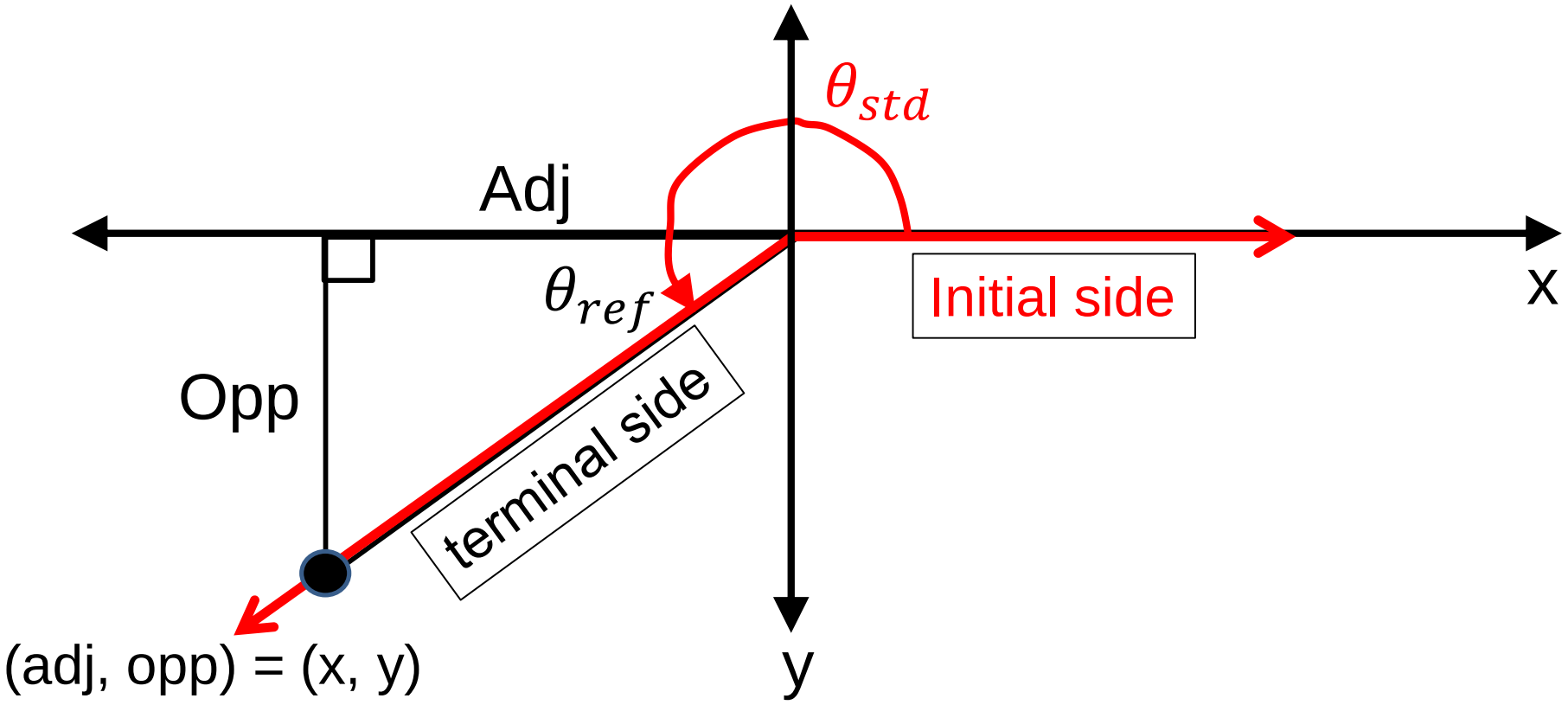
Cos Θ = x

Tan Θ = y/x

Sine ratio is a negative number

Cosine ratio is a negative number

Tangent ratio is a positive number



For Quadrant III

$\theta_{ref} = 360 - \theta_{std}$

Sin Θ = y

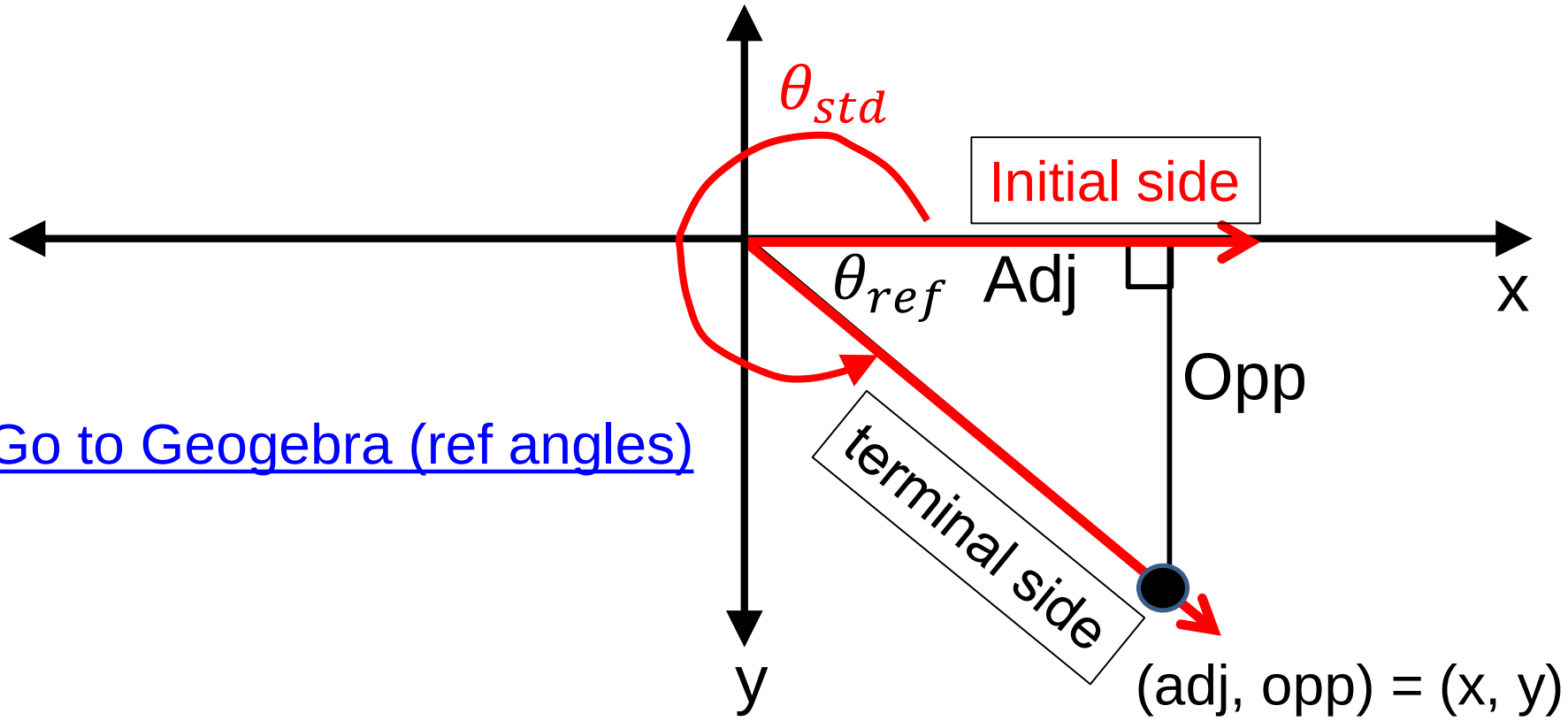
Cos Θ = x

Tan Θ = y/x

Sine ratio is a negative number

Cosine ratio is a positive number

Tangent ratio is a negative number



Go to Geogebra (ref angles)