Math-3A Lesson 8-3

Solve Exponential And Logarithmic Equations Solving a Linear Equation:

"Isolate the variable"

What does "solve" a single variable equation mean?

Find the value of 'x' that makes the equation true.

$$3x + 2 = 4x - 5$$
 subtract 3x from both sides
 $2 = x - 5$ add 5 to both sides

7 = x

Radical Equation:

- $\sqrt{3x-2} 1 = 3$ +1 + 1
 - $\sqrt{3x-2} = 4$
 - 3x 2 = 16

"Isolate the radical" "Undo the radical"

add 1 to both sides

= 3

- square both sides
- Add 2 both sides
- 3x = 18 Divide by 3 both sides
- x = 6 check

$$\sqrt{3(6)} - 2 - 1 = 3$$

 $\sqrt{16} - 1 = 3$ $4 - 1$

<u>Your turn:</u>	"Isolate the radical"
Solve: $\sqrt{3x-5} = x-$	3 "Undo the radical"
$\left(\sqrt{3x-5}\right)^2 = (x-3)^2$	square both sides
$3x-5 = x^2 - 6x + 9$	Get into standard form!!!!!
$0 = x^2 - 9x + 14$	$\sqrt{3(7)-5} = (7)-3$
0 = (x - 7)(x - 2)	$\sqrt{16} = 4$
x = 7, 2	$\sqrt{3(2) - 5} = (2) - 3$
	$\sqrt{1} \neq -1$

Solving an Exponential Equation: The easiest problem

 $2^{x} = 2^{4-x}$ Exponents have to be equal to each other! x = 4 - x x = 4 - x x = 2 x = 4 2x = 4 x = 2x = 4 - 2

$$2^2 = 2^{4-2}$$

$$7^{2x+1} = 7^{13-4x}$$

÷X ÷X

$$x=2$$

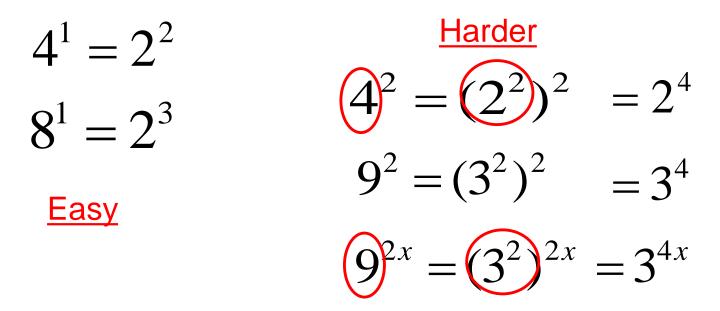
$$2x + 1 = 13 - 4x$$

+4x
$$6x + 1 = 13$$

-1 -1

6x = 12

Equivalent Powers with different bases.



Change the base of the power as indicated:

$$27^{1} = 3^{?}$$

 $16^{2} = 4^{?}$
 $25^{2x} = 5^{?}$

Solving using "convert to same base"

 $2^{4x-1} = 8^{x-1}$ $2^{4x-1} = (2^3)^{x-1}$ $2^{4x-1} = 2^{3x-3}$ 4x - 1 = 3x - 3-3x - 3x

$$x - 1 = -3$$

+1 +1

"convert to same base"

Exponent of a power Exponent Property

$$x = -2$$

Check your answer!

$$2^{4(-2)-1} = 8^{-2-1}$$

$$2^{-9} = 8^{-3}$$

($2^{-9} = 8^{-3}$)⁻¹
 $2^9 = 8^3$

512 = 512

Solving using "convert to same base"

 $9^{2x} = 27^{x-1}$ $(3^2)^{2x} = (3^3)^{x-1}$

- "convert to same base"
 - Power of a power Exponent Property
- $3^{2^{*2x}} = 3^{3(x-1)}$
 - $3^{4x} = 3^{3x-3}$
 - 4x = 3x 3

$$x = -3$$

- Check your answer!
- $9^{2(-3)} = 27^{-3-1}$ $9^{-6} = 27^{-4}$ $(9^{-6} = 27^{-4})^{-1}$ $9^{6} = 27^{4}$ 531441 = 531441

Solving using "log of a power" property

 $9^{2x} = 27^{x-1}$ Take natural log of both side $ln 9^{2x} = ln 27^{x-1}$ "log of pwr property" 2x ln 9 = (x-1) ln 27 $\div ln 9$ $\begin{array}{c} 0.5x = -1.5 \\ *2 \\ x = -3 \end{array}$

$$2x = (x-1)\frac{\ln 27}{\ln 9}$$
 simplify
$$2x = (x-1)(1.5)$$
 simplify
$$2x = 1.5x - 1.5$$

-1.5x -1.5x

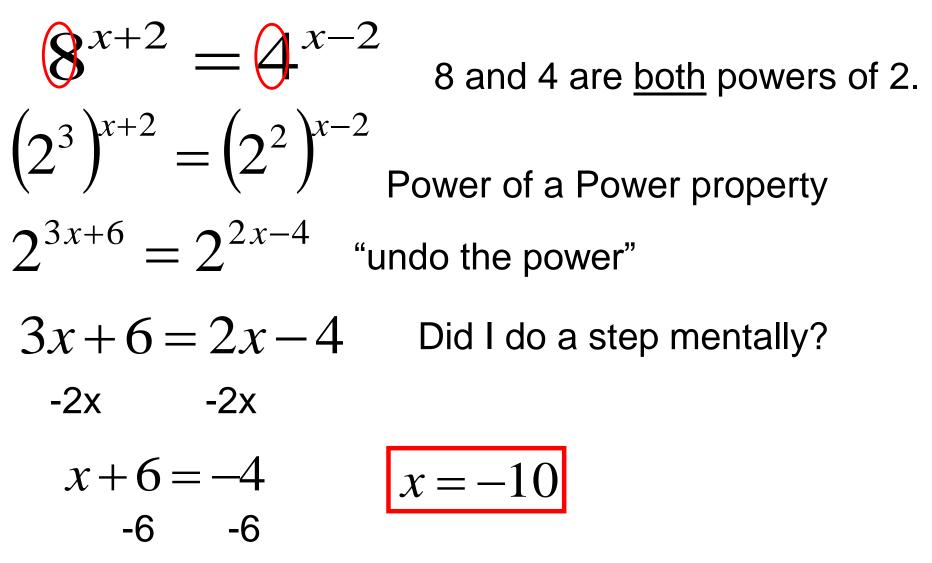
Same solution as previous method!

$$(3^2)^{2x} = (3^3)^{x-1}$$
$$3^{4x} = 3^{3x-3}$$

$$4x = 3x - 3$$

$$x = -3$$

Solve using "convert to same base"



Solving using "log of a power" property

$$8^{x+2} = 4^{x-2}$$
 Take natural log of both side

$$\ln 8^{x+2} = \ln 4^{x-2}$$
 "log of pwr property"

$$(x+2) \ln 8 = (x-2) \ln 4$$

$$3x+6=2x-4$$

$$-2x-6$$

$$-2x-6$$

$$(x+2) = (x-2) \frac{\ln 4}{\ln 8}$$
 simplify $x = -10$

$$x + 2 = (x - 2)(0.66666666) \text{ simplify}$$
$$x + 2 = (x - 2)\left(\frac{2}{3}\right)$$
$$3(x + 2) = (x - 2)\left(\frac{2}{3}\right)(3)$$

Sometimes you can't rewrite the exponentials with the same bases so you have no choice. Use log of a power property.

$$5^{x} = 7^{2x-1}$$

$$\ln 5^{x} = \ln 7^{2x-1}$$

$$x \ln 5 = (2x-1) \ln 7$$

$$\div \ln 5 \qquad \div \ln 5$$

$$x = (2x - 1)\frac{\ln 7}{\ln 5}$$

$$x = (2x - 1)(1.21)$$

$$x = 2.42x - 1.21 + 1.21 + 1.21 + 1.21$$
$$1.21 + x = 2.42x - x - x$$

1.21 = 1.42x $\div 1.42 \div 1.42$

$$x = 0.85$$

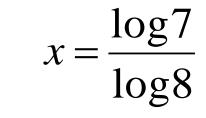
Solve using the "log of a power property"

 $5^{x+2} = 4^{x-2}$ Take natural log of both sides $\ln 5^{x+2} = \ln 4^{x-2}$ Power property $2 = -0.1387 \times -1.7227$ $(x+2)\ln 5 = (x-2)\ln 4$ ÷ ln 5 ln 5 +1.7227 +1.7227 $x+2 = (x-2)\frac{\ln 4}{\ln 5}$ 3.7227 = -0.1387x $\div -0.1387 \div -0.1387$ x + 2 = (x - 2)(0.8614)x = -26.8399x + 2 = 0.8614x - 1.7227**-X -X**

Solve for 'x' (how do you get the exponent 'x' all by itself?

 $8^x - 2 = 5$ "Isolate the exponential" +2 +2 $8^x = 7$ "convert to a log" $x = \log_8 7$ Change of base formula





$$x = 0.9358$$

Solve using "undo the exponential" $3^{2x-1} + 5 = 7$ "Isolate the exponential" -5 -5 $3^{2x-1} = 2$ "Undo the exponential" $2x-1 = \log_3 2 \rightarrow \text{Change of} \quad 2x-1 = \frac{\ln 2}{2x}$ base formula ln32x - 1 = 0.630932x-1=0.63093+1 +1 +1 +1 2x = 1.630932x = 1.63093÷2 ÷2 ÷2 ÷2 x = 0.815x = 0.815

The easiest log equation.

$$log(x + 3) = log(2x - 1)$$
$$x + 3 = 2x - 1 \quad \rightarrow x = 2$$

Why does this work?

y = y

$$y = \log(x + 3)$$
 $10^{y} = x + 3$
 $y = \log(2x - 1)$ $10^{y} = 2x - 1$

Substitution Property

$$x + 3 = 10^{y} = 2x - 1$$

 $x + 3 = 2x - 1$

Some functions don't have domain of all real numbers \rightarrow equations of these types <u>may</u> have extraneous solutions

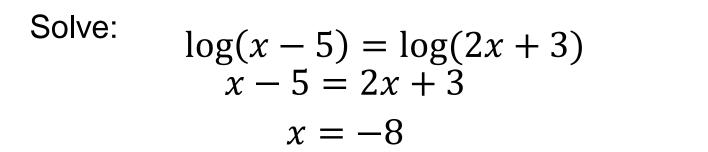
Extraneous solution: an apparent solution that does not work when plugged back into the original equation.

You <u>MUST</u> check the solutions in the original equation for any equation this is of the function type that has a restricted domain.

Square root equationsRadicands cannot be negativeSquare roots do not equal negative numbers

Rational equations Denominators cannot equal zero

Log equations Logarands cannot be zero or negative



Remember to check for extraneous solutions by plugging the solution for 'x' back into the original equation.

$$log(-8 - 5) = log(2(-8) + 3)$$
$$log(-13) = log(-13)$$

Can you have a negative logarand?

$\log_2 5^x = 5$ Power property of logarithms $x \frac{\ln 5}{2} = 5$ $x \log_2 5 = 5 \rightarrow$ Change of base ln 22.32192x = 5 $\div 2.32192 \div 2.32192$ x = 2.1534

Use inverse property of multiplication

$$x = 5\frac{\ln 2}{\ln 5}$$

x = 2.1534

Solve:

$$\log_2 5^x = 4$$

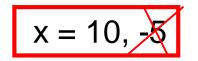
$\log_3 4^{5x} = 6$

Solving Logs requiring condensing the product.

 $\log 2x + \log(x-5) = 2$ "condense the product" log 2x(x-5) = 2"undo the logarithm" $10^2 = 2x(x-5)$ Quadratic \rightarrow put in standard form $100 = 2x^2 - 10x$ $2x^2 - 10x - 100 = 0$ Divide both sides by '2' $x^2 - 5x - 50 = 0$ factor

(x-10)(x+5) = 0 Zero product property

Check for extraneous solutions:



log 2x + log(x-5) = 2 log (2 * 10) + log (10 - 5) = 2 log (20) + log (5) = 2 All logarands are positive © log 100 = 2 "Condense the product" $10^2 = 100$ Checks log (2)(-5) + log (-5 - 5) = 2

$$\log(2)(-5) + \log(-5 - 5) = 2$$

log(-10) + log(-10) = 2 Negative logarands \otimes

x = 5 is an extraneous solution.

$\ln 5^{x+2} + \ln 5^2 = 2$

$\log_2 4x + \log_2 3 = 6$

More complicated Logarithmic Equations

- $2 + \log_2 5^{x-2} = 7$ "Isolate the logarithm" -2 -2 $\log_2 5^{x-2} = 5$ "undo the logarithm"
- $(x-2)\log_2 5 = 5$ $\div \log_2 5 \quad \div \log_2 5$
 - x 2 = 2.15338+2 +2 Add '2' to both sides.
 - x = 4.1524

$3 + \log_4 3^{2x-1} = 6$

$-7 + 2\ln 4^{x-3} = 5$

$\log_4(5x-1) = 3$	"Isolate the logarithm"
$5x - 1 = 4^3$	"undo the logarithm"
5x - 1 = 64	Add '1' to both sides
5x = 65	Divide both sides by '5'
x = 13	Plug back in to check!
$\log_4(5*13-1) = 3$	

$$\log_4 64 = 3$$
 Checks