

Math-3A

Lesson 8-2

Log of a Quotient Property of Logarithms
The Change of Base formula,
Simplifying Logarithms

1. $f(x) = 3(4)^{x-1} + 5$ Find $f^{-1}(x)$

2. $\log_3 x^4 y$

Expand the log:

3. $3\log 2 + 2\log x + \log 3 + 5\log y$

Condense the log:

4. $7 = 2(3)^x$

Convert to a logarithm:

5. $3 \log_5 (x - 6) = 6$

Convert to an exponential;

6. $f(x) = 3 \log(x + 2) - 5$

What is the
Domain and range?

7. $f(x) = 2\log(2x - 4) - 6$
What is the logarand?

8. What is the asymptote?

9. What do I mean when I say: "A log is an exponent"?

Log of a Product Property of Logarithms:

$$\log_b (RS) = \log_b R + \log_b S$$

$$\log_2 15 = \log_2 3 + \log_2 5$$

log of a product = sum of the logs of the factors.

Log of Power Property of Logarithms

$$c \log_b R^c \rightarrow c \log_b R$$

$$\log_2 3^4 = 4 \log_2 3$$

$$\log 32 = \log 2^5 = 5 \log 2$$

$\frac{x^2}{y^3}$ Using properties of exponents: rewrite this so the 'y' term is NOT in the denominator. $x^2 y^{-3}$

$$\log_3 \left(\frac{5}{2} \right) = \log_3 (5 * 2^{-1}) \quad \text{Negative Exponent Property}$$

$$= \log_3 (5) + \log_3 (2)^{-1} \quad \text{Log of a Product Property}$$

$$= \log_3 5 + (-1) \log_3 2 \quad \text{Log of a Power Property}$$

$$= \log_3 5 - \log_3 2 \quad \text{Definition of Subtraction: (adding a negative is subtraction)}$$

Log of a Quotient Property

$$\log_b \left(\frac{R}{S} \right) = \log_b R - \log_b S$$

$$\log_3 \left(\frac{5}{2} \right) \text{ “expand the quotient”} \quad \log_3 5 - \log_3 2$$

$$\ln 8 - \ln 3 \text{ “condense the quotient”} \quad \ln \frac{8}{3}$$

“Negative Log” → denominator of the logarand

Expand the Quotient

$$\begin{aligned}\log \frac{4}{5} &= \log 4 - \log 5 = \log 2 + \log 2 - \log 5 \\ &= 2 \log 2 - \log 5\end{aligned}$$

$$\ln \frac{3}{7} = \ln 3 - \ln 7$$

Condense the quotient

$$\log_4 5 - \log_4 2 = \log_4 \frac{5}{2}$$

$$\log_5 8 - \log_5 16 = \log_5 \frac{8}{16} = \log_5 \frac{1}{2}$$

Expand the Logarithm

$$\log\left(\frac{2x}{3y^5}\right) = \log 2x - \log 3y^5$$

The denominator is a product!

$$= \log 2x - (\log 3 + 5 \log y)$$

Distributive property!

$$= \log 2x - \log 3 - 5 \log y$$

$$= \log 2 + \log x - \log 3 - 5 \log y$$

Logs of factors in the numerator will be positive.

Logs of factors in the denominator will be negative.

Expand the quotient

$$\begin{aligned}\ln \frac{zx^2}{5y^3} &= \ln zx^2 - \ln 5y^3 \\ &= \ln z + \ln x^2 - (\ln 5 + \ln y^3) \\ &= \ln z + 2\ln x - (\ln 5 + 3\ln y) \\ &= \ln z + 2\ln x - \ln 5 - 3\ln y\end{aligned}$$

$$\begin{aligned}&= \log_4 \left(\frac{w^5}{x^7} \right)^2 = \log_4 \frac{w^{10}}{x^{14}} \\ &= \log_4 w^{10} - \log_4 x^{14} \\ &= 10\log_4 w - 14\log_4 x\end{aligned}$$

$$\log_4 \frac{2\sqrt{x}}{4yz} = \log_4 2\sqrt{x} - \log_4 4yz$$

$$= \log_4 2 + \log_4 \sqrt{x} - \log_4 4 - \log_4 y - \log_4 z$$

$$= \log_4 2 + \frac{1}{2} \log_4 x - \log_4 4 - \log_4 y - \log_4 z$$

Change-of-Base Formula for Logarithms

$$\log_{\textcircled{c}} \textcircled{a} = \frac{\log_b \textcircled{a}}{\log_b \textcircled{c}}$$

Change to log base 10 or base 'e'
(your calculator can do these).

Convert to base 10.

$$\log_{\textcircled{4}} \textcircled{5} = \frac{\log_{10} \textcircled{5}}{\log_{10} \textcircled{4}} = \frac{0.699}{0.6021} = 1.161$$

$$\log_4 5 = \frac{\ln \textcircled{5}}{\ln \textcircled{4}} = \frac{1.609}{1.386} = 1.161$$

Simplify

$$\log_2 2$$

$$\log_2 2 = x$$

$$2^x = 2$$

$$x = 1$$

Using Change of base:

$$\log_2 2 = \frac{\log 2}{\log 2} = 1$$

Simplify: $\log_4 16$ “4 raised to what power equals 16?”

$$\log_4 4^2 \quad 2\log_4 4 \quad 2(1) = 2$$

$$\log_2 \sqrt{2}$$

“2 raised to what power equals the square root of 2?”

$$\log_2 2^{1/2} \quad \frac{1}{2}\log_2 2 \quad \frac{1}{2}(1) \quad \frac{1}{2}$$

Simplify: $5 \log_3 27$

$$5 \log_3 3^3 \quad (3 * 5) \log_3 3 \quad 15$$

$$6 \log_2 (16) - 4$$

$$6 \log_2 (2^4) - 4$$

$$(4 * 6) \log_2 (2) - 4$$

$$(24)(1) - 4$$

$$20$$

Simplify:

$$8\log_5(125) + 3 = 27$$

$$2\log_9(81) - 5 = -1$$

$$-6\log_3(\sqrt[2]{3}) + 4 = 1$$