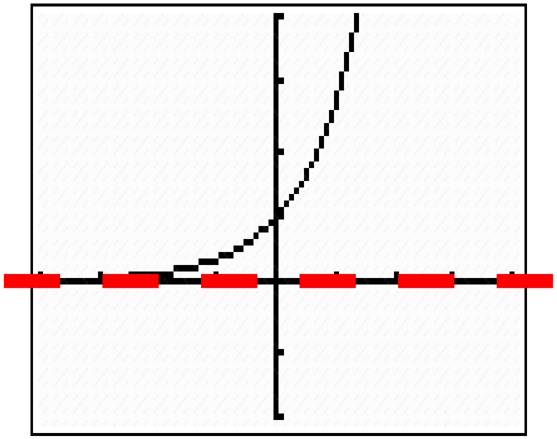
Math-3A
Lesson 8-1

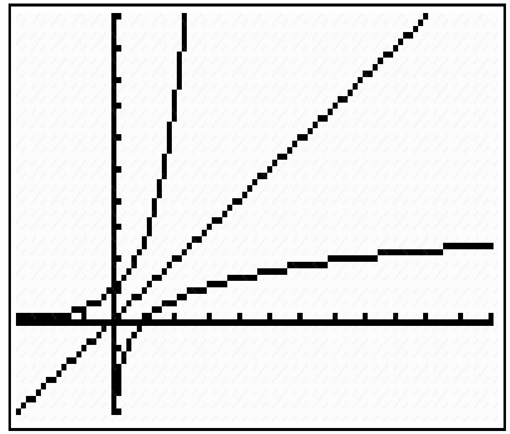
Properties of Logarithmic Functions
(Product of Logs
Log of a Power)

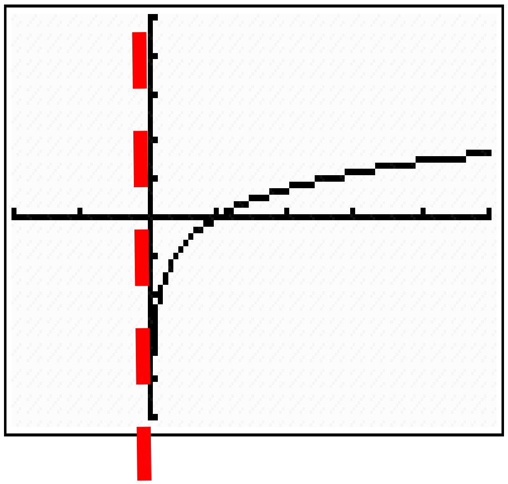
Exponential Function

Inverse Functions

Logarithm Function







$$f(x) = 10^x$$

$$f^{-1}(x) = log_{10}(x)$$

Domain = ? (-∞, ∞)

Range = ? (0, ∞)

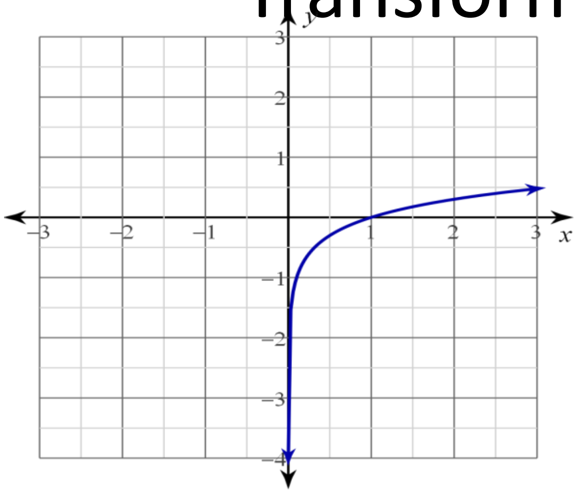
Horizontal asymptote = ?
y = 0

Domain = ? (0, ∞)

Range = ? (-∞, ∞)

Vertical asymptote = ?
x = 0

Transformations of the Log Function



$$f(x) = \log x$$

Domain = ? (0, ∞)

Range = ? (-∞, ∞)

vertical asymptote = ?

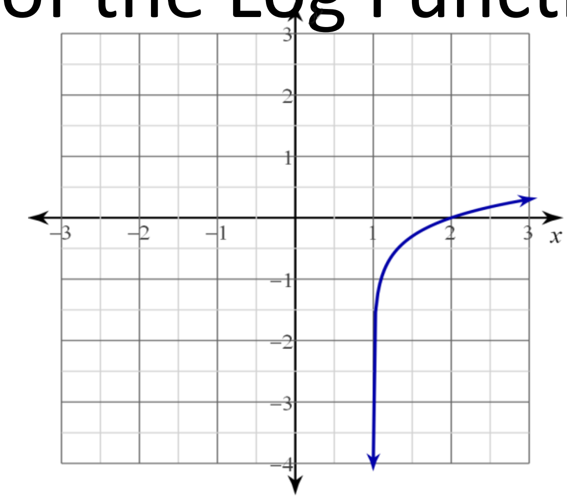
x = 0

X-intercept = ?

x = 1

Where increasing = ?

(0, ∞)



$$g(x) = \log(x-1)$$

Right 1 shift

Domain = ? (1, ∞)

Range = ? (-∞, ∞)

asymptote = ? x = 1

Logarand

$$g(x) = 3\log(2x-1)+5$$

Logarand

Vertical Asymptote: The value of 'x' that makes the logarand equal to zero.

Vertical asymptote = ?

$2x - 1 = 0$

$x = ½$

Convert to logarithm form $\text{Log}_{\square}\square = \square$

What is the solution?

x = 2	$5^x = 25$	$\log_5 25 = x$
x = 3	$4^x = 64$	$\log_4 64 = x$
x = ???	$b^x = y$	$\log_b y = x$
x = 2	$9^x = 81$	$\log_9 81 = x$
x = 3	$10^x = 1000$	$\log_{10} 1000 = x$

Convert to exponential form

$\text{Log}_{\square} \square = \square$

What is the solution?

x = 2 $\log_{10} 100 = x$ $10^x = 100$

x = 3 $\log_3 27 = x$ $3^x = 27$

x = 0 $\log_9 1 = x$ $9^x = 1$

x = 16 $\log_4 x = 2$ $4^2 = x$

x = 32 $\log_2 x = 5$ $2^5 = x$

$$f(x) = 5^{2x+4}$$ Find $f^{-1}(x)$

$$y = 5^{2x+4}$$ Replace f(x) with ‘y’

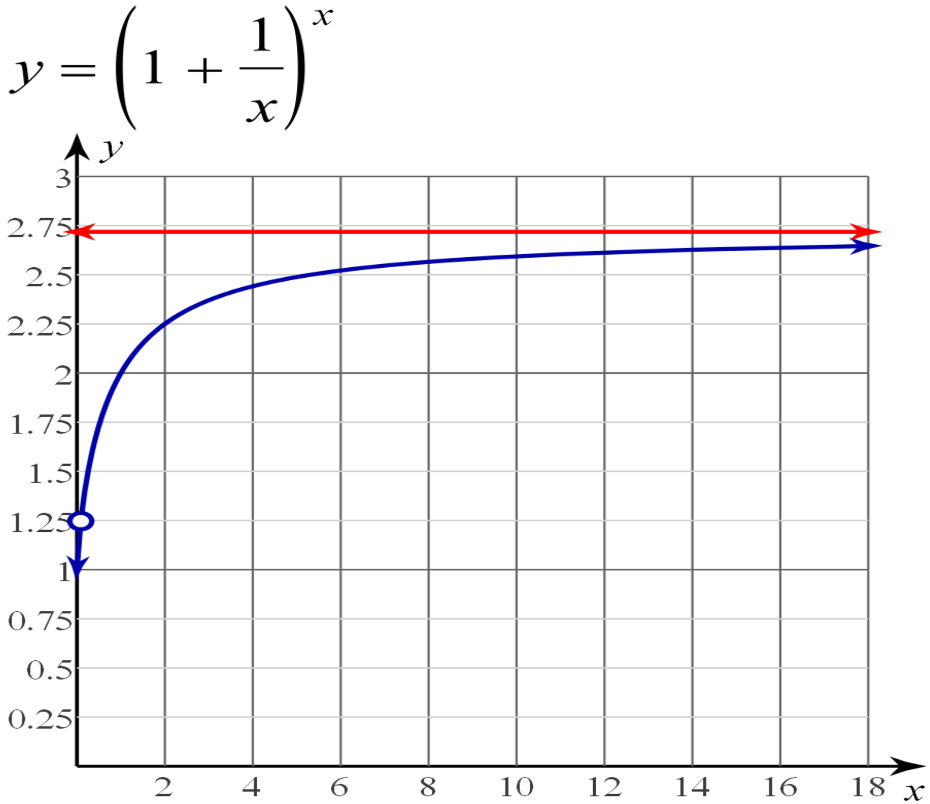
$$x = 5^{2y+4}$$ exchange ‘x’ and ‘y’

$$\log_5 x = 2y + 4$$ Log is the exponent (remember how to convert between the two?)

$$-4 + \log_5 x = 2y$$ Solve for ‘y’

$$\frac{-4 + \log_5 x}{2} = y$$

$$\boxed{y = -2 + \tfrac{1}{2} \log_5 x}$$



e ~ 2.718...

$$A(t) = (1 + r/k)^{kt}$$

If we compound continuously (k → ∞) then

$$A(t) = e^{rt}$$

$$2^3 * 2^2 = 2^5$$

The <u>product of powers</u> → <u>add the exponents</u>

$$2^3 * 2^2 = 2^5$$

<u>Logarithm</u>: another way of writing the <u>exponent</u>

Convert each exponent above into a log:

$$\log_2 8 + \log_2 4 = \log_2 32$$

$$3 + 2 = 5$$

This is the logarithm equivalent of the multiply powers property of exponents.

Log of a Product Property

$$\log_2 15 = \log_2 (3 * 5)$$

$$\log_2 15 = \log_2 3 + \log_2 5$$

$$\log_b (RS) = \log_b R + \log_b S$$

log of a product = sum of the logs of the factors.

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

$$\log_3 xy \qquad \log_3 x + \log_3 y$$

$$\log_3 45 \qquad \log_3 3 + \log_3 3 + \log_3 5$$

$$45 = 3 * 3 * 5 \qquad 2\log_3 3 + \log_3 5$$

Expand the Logarithm: use properties of logs to rewrite a single log as an expression of separate logs.

$$\boxed{\log(3xy^2)} = \log 3 + \log x + \log y^2$$

$$= \log 3 + \log x + \log y + \log y$$

$$= \log 3 + \log x + 2\log y$$

$$\boxed{\log_4 6} = \log_4 3 + \log_4 2$$

$$\boxed{\ln 2xyw} = \ln 2 + \ln x + \ln y + \ln w$$

Condense the Logarithm: apply properties of logarithms to rewrite the log expression as a single log.

$$\boxed{\log_2 7 + \log_2 5} \quad = \log_2 (7 * 5) \quad = \log_2 35$$

$$\boxed{\log 5 + \log x} \quad = \log 5x$$

$$\log_7 5 + \log_5 7$$

"unlike logs" → can't condense

“Condense the Log”

$\log_5 2 + \log_5 7 = \log_5 14$

$\log 9 + \log 4 = \log 36$

$\log_5 6 + \log_8 4$ “unlike logs” → can’t condense

“Expand the Log”

$$\log_2 9 = \log_2 (3 * 3)$$

$$= \log_2 3 + \log_2 3$$

$$= 2 \log_2 3$$

Notice something <u>interesting</u>

$$\log_2 9 = \log_2 (3)^2 = 2 \log_2 3$$

"Expand the Product"

$$\log_3 16 = \log_3(4*4)$$

$$= \log_3 4 + \log_3 4$$

$$= 2\log_3 4$$

Notice something <u>interesting</u>

$$\log_3 16 = \log_3(4)^2 = 2\log_3 4$$

“Expand the Product”

$\log_5 10^2$

Log of a product is the sum of the logs of the factors.

$= \log_5 10 + \log_5 10$

Combine “like terms”

$= 2\log_5 10$

$\log_5 10^2 = 2\log_5 10$

New property: “log of a power”

Use Log of a Power simplify

$$\log x^3 = 3\log x$$

$$\ln 8 = \ln 2^3 = 3\ln 2$$

$$\log\sqrt{x} = \log x^{1/2} = \frac{1}{2}\log x$$

Gotcha'

$$\log 3y^5$$

$$= 5\log 3y$$

$$= \log 3 + \log y^5$$

Which one?

$$5\log 3y = \log(3y)^5 = \log 3^5 y^5$$

Log of a Power

$$c \leftarrow \log_b R^{c} \rightarrow c\log_b R$$

A potential error is this:

$$\log_2 6x^3 = \cancel{3\log_2 6x}$$

What is the error ? ‘3’ is an exponent of the base ‘x’ not ‘6x’

Correct the error.

$$\log_2 6x^3 = \log_2 6 + \log_2 x^3$$

$$= \log_2 3 + \log_2 2 + 3\log_2 x$$