Math-3A

Lesson 7-8

Base ‘e’: Continuous Growth and Decay

Define the verb "to rent".

"A usually fixed periodical return made by a tenant or occupant of property to the owner for the possession and use thereof; especially **:**an agreed sum paid at fixed intervals by a tenant to the landlord." (Merriam-Webster Dictionary)

→ To pay for the possession and use of property.

If you rent an apartment, can the owner of the apartment use (live in) the apartment?

How often is rent usually paid?

How does the landlord determine what to charge for rent?

Rent

A rental property has a market value of $150,000. The owner rents out the property for $1100 per month.

What percentage of the market value of the house does the owner charge for rent each month?

$$\frac{part}{whole} = ? \quad \frac{1100}{150000} = 0.0073 \quad = 0.73\% \big/ month$$

What percentage of the market value of the house does the owner charge for rent for the whole year?

$$\frac{part}{whole} = ? \quad \frac{1100 * 12}{150000} = \quad \frac{13200}{150000} = \qquad = 8.8\% \big/ year$$

Rent

A landlord ends up charging a total of $18,000 for a tenant to rent a $200,000 house for a year (ouch).

What percentage of the market value of the house does the owner charge for rent for the year?

$$\frac{part}{whole} = ? \quad \frac{18000}{200000} = 0.09 = 9\%/yr$$

What percentage of the market value of the house does the owner charge for rent for a month?

$$\frac{part}{whole} = ? \quad \frac{9\%}{\cancel{year}} * \frac{1\,\cancel{year}}{12\,\text{months}} = \frac{0.75\%}{month} = \frac{0.0075}{month}$$

“Renting Money”

Can you rent money?

“Rent” → To pay for the possession and use of something.

Give an example of how money is “rented”.

1. Depositing money in a savings account.
2. Borrowing money to buy a car.

For each case, who is the “landlord” and who is the “tenant”?

1. savings account → you are the landlord.
2. Borrow money→ The bank is the landlord.

The interest rate for borrowing money is always given as an annual interest rate, but “rent” can be paid at the end of the year, the end of every 6 months, the end of each month, etc.

You deposit \$100 money into an account that pays 3.5% interest per year. The "rent" is "paid" yearly. How much money will be in the account at the end of the 1st year?

$$A(1) = 100(1 + 0.035)^{(1)} \quad A(1) = 100(1.035)^{(1)}$$

$$A(1) = \$103.5$$

How much will be in the account after the 2nd year?

$$A(2) = A(1)(1.035)^{(1)}$$

$$A(2) = 103.5(1.035)^{(1)}$$

$$A(2) = 100(1.035)^{(1)}(1.035)^{(1)}$$

$$A(t) = A_0(1 + r)^t$$

$$A(2) = 100(1.035)^{(2)}$$

$$A(2) = \$107.12$$

You deposit $100 money into an account that pays 3.5% interest per year. But the "rent" is paid "monthly." What is the interest rate that is paid each month?

$$\frac{3.5\%}{year} * \frac{1\text{ year}}{12\text{ months}} = \frac{0.29\%}{month} = \frac{0.0029}{month}$$

How much money will be in the account after 5 months?

$$A(t) = A_0(1+r)^t$$

$$A(5) = 100(1+0.0029)^5$$

Time uses units of <u>months</u>

$$A(5\text{ months}) = \$101.45$$

How much money will be in the account after 7 years?

$$A(7\text{ years}) = 100(1+0.0029)^{12(7)}$$

Time uses units of <u>years</u>.

$$A(t\text{ in yrs}) = A_0(1+ r/n)^{nt}$$

$$A(7) = \$127.54$$

"*n*": number of times "rent" is paid per year

The exponential growth equation for money in a bank for account where the bank pays you more frequently than at the end of the year is:

Annual interest rate

Amount of $$ in the account as a function of time

Initial value

Years after the deposit

$$\boxed{A(t) = A_0(1 + r/k)^{k*t}}$$

# of times the bank pays you each year

"Compounding period" → the number of times the bank pays you each year.

"A bank pays 3% per year compounded monthly."

$$\boxed{A(t) = A_0(1 + 0.03/12)^{12*t}}$$

Values of "k"	
Words to look for	K
Annually	1
Semi-annually	2
Quarterly	4
Monthly	12
Daily	365

Compound interest: the interest (rent) that is paid at the end of period of time.

$$A(t \text{ in yrs}) = A_0(1 + r/n)^{nt}$$

Compounded annually: *"n" = ?* $A(t \text{ in yrs}) = A_0(1 + r/1)^{1*t}$

Compounded semi-annually: *"n" = ?* $A(t \text{ in yrs}) = A_0(1 + r/2)^{2*t}$

Compounded quarterly: *"n" = ?* $A(t \text{ in yrs}) = A_0(1 + r/4)^{4*t}$

Compounded monthly: *"n" = ?* $A(t \text{ in yrs}) = A_0(1 + r/12)^{12*t}$

Compounded weekly: *"n" = ?* $A(t \text{ in yrs}) = A_0(1 + r/52)^{52*t}$

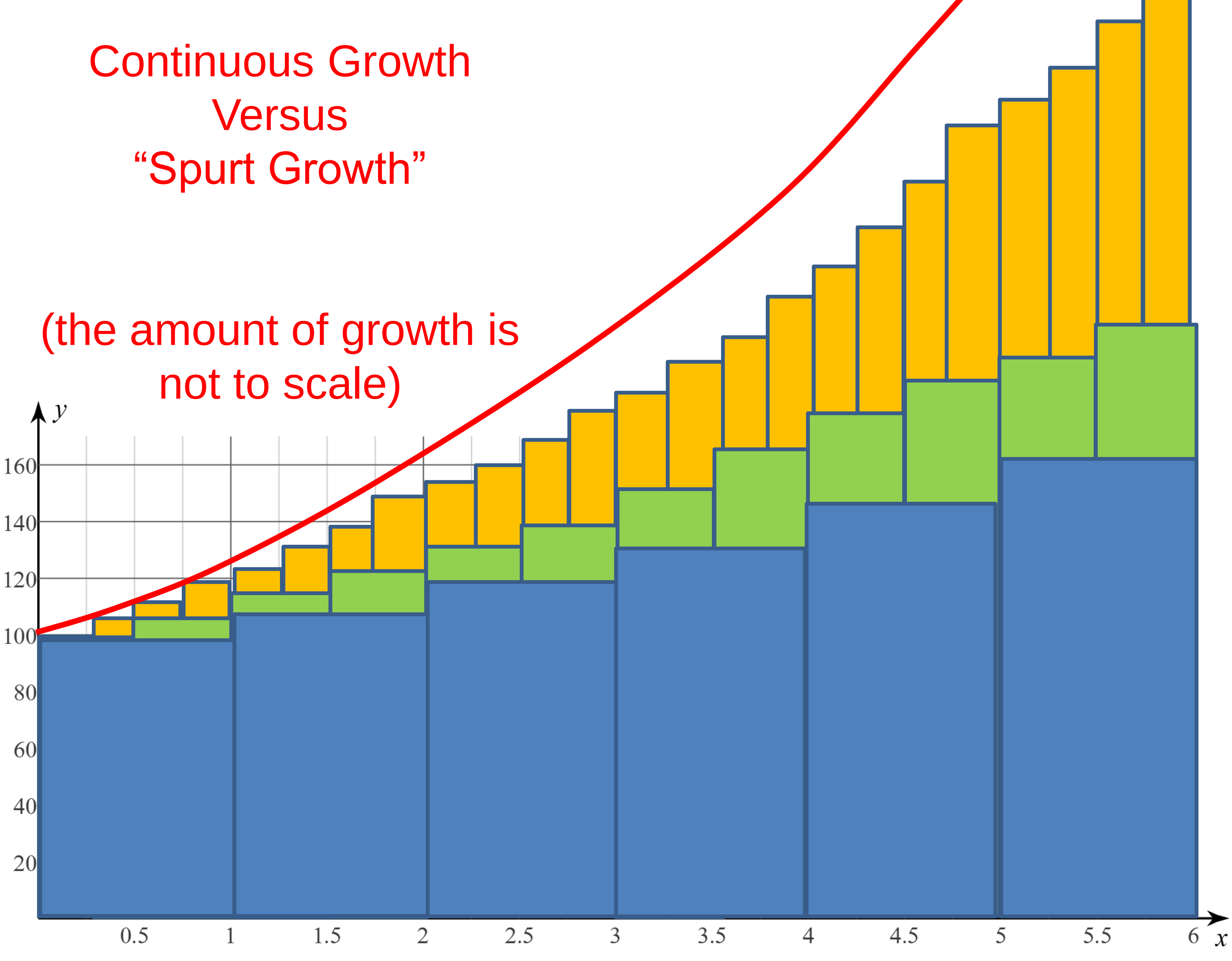
Compounded daily: *"n" = ?* $A(t \text{ in yrs}) = A_0(1 + r/365)^{365*t}$

Compounded hourly: *"n" = ?* $A(t \text{ in yrs}) = A_0(1 + r/8760)^{8760*t}$

Compounded minutely: *"n" = ?* $A(t \text{ in yrs}) = A_0(1 + r/525600)^{525600*t}$

Continuous Growth
Versus
"Spurt Growth"

(the amount of growth is
not to scale)



What is the number "e" ?

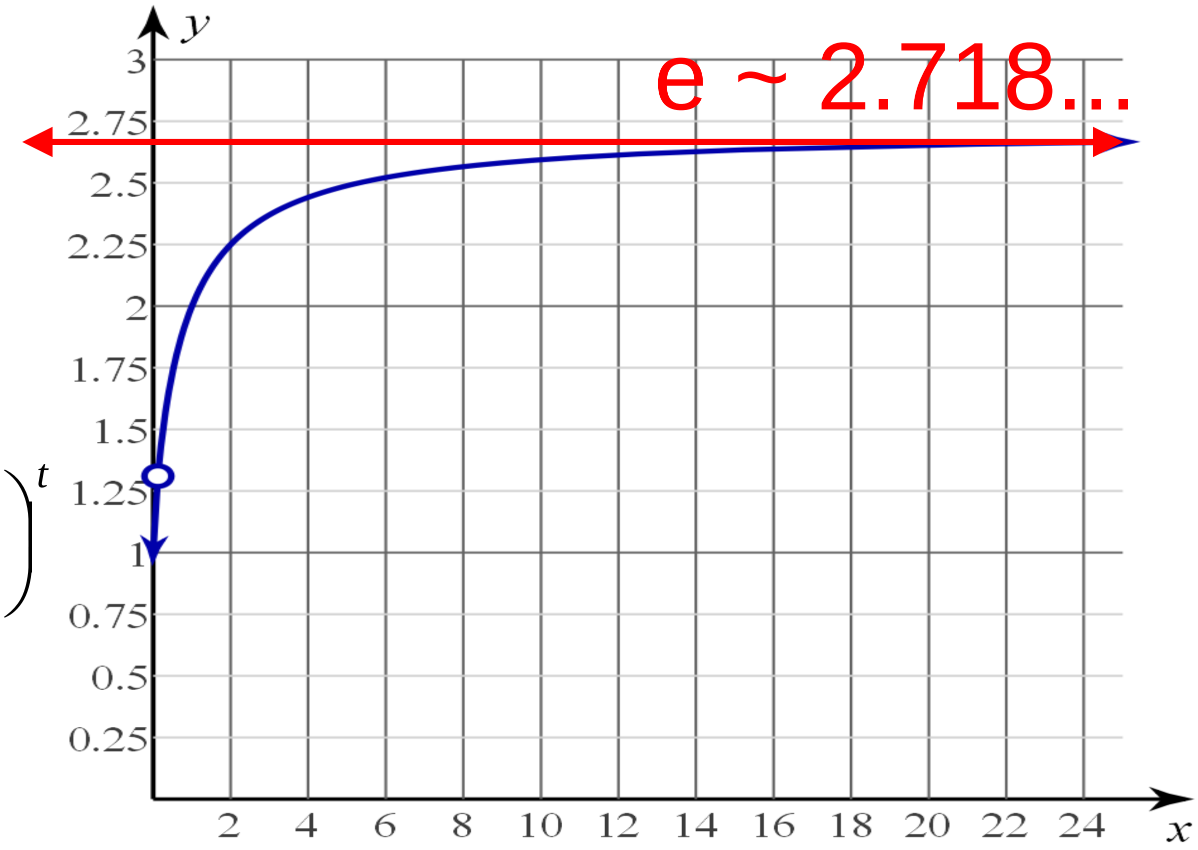
"e" is the horizontal asymptote of the function:

$$f(x) = \boxed{\left(1+\frac{1}{x}\right)^{x}} \Rightarrow e$$

$$A(t \text{ in yrs}) = A_0(1+{}^{r}\!/\!_{n})^{nt}$$

$$A(t \text{ in yrs}) = A_0\left(\boxed{\left(1+{}^{r}\!/\!_{n}\right)^{n}}\right)^{t}$$

$$A(t \text{ in yrs}) = A_0 e^{rt}$$



As the compounding period gets infinitely short, the base of the exponential becomes the number "e"

("<u>continuous compounding</u>")

$$y = AB^x$$

Growth (decay) factor

Initial Value

Initial Value is the *y*-intercept.

$$y = Ae^{kx} \qquad y = A(e^k)^x$$

Initial Value

Growth (decay) factor

$$B = e^k$$

$100 is placed into an account that is continuously compounded at a rate of 3% per year. How much money will be in the account at the end of the 1st year? $A(t \text{ in yrs}) = A_0(e)^{rt}$

$$A(1) = 100(e)^{0.03*1} \qquad A(1) = 103.05$$

What is the base of the exponential? $A(t \text{ in yrs}) = A_0(e^r)^t$

$$A(t \text{ in yrs}) = A_0(e^{0.03})^t \qquad A(t \text{ in yrs}) = A_0(1.0305)^t$$

$100 is placed into an account that is continuously compounded at a rate of 4% per year. How much money will be in the account at the end of the 1st year? $A(t \text{ in yrs}) = A_0(e)^{rt}$

$$A(1) = 100(e)^{0.04*1} \qquad A(1) = 104.08$$

What is the base of the exponential?

$$A(t) = 100(e^{0.04})^t \qquad A(t) = 100(1.0408)^t$$

$$y = 4^x \quad = e^{1.386x}$$

$$y = 1.1^x \quad = e^{0.095x}$$

$$y = 1.01^x \quad = e^{0.010x}$$

$$y = 1^x \quad = e^{(0)x}$$

$$y = 0.85^x \quad = e^{-0.163x}$$

$$y = 0.25^x \quad = e^{-1.386x}$$

$$y = B^x$$

Growth: B > 1 Decay: 0 < B < 1

look at the pattern of the exponents of ‘e’

$$y = e^{kx}$$

Growth: k > 0

Decay: k < 0