Math-3-A
Lesson 7-7
The Logarithm Function

<u>Finding the Inverse</u>: exchange the locations of ‘x’ and ‘y’ in the equation then solve for ‘y’.

$$f(x) = (x-2)^2$$

$$y = (x-2)^2$$

$$x = (y-2)^2$$

$$\sqrt{x} = \sqrt{(y-2)^2}$$

$$\pm\sqrt{x} = y-2$$

$$\pm\sqrt{x} + 2 = y$$

$$y = 2 \pm \sqrt{x}$$

Domain, Range, and Inverse Functions

Domain: The input values (that have corresponding outputs)

Range: The output values (that have corresponding inputs)

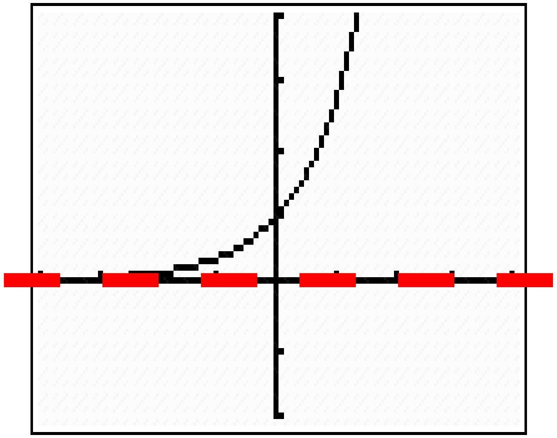
Inverse of a Function: A function resulting from an "exchange" of the inputs and outputs.

$$f(x): Domain, Range$$

$f^{-1}(x):$ Domain = range of f(x)

Range = domain of f(x)

Exponential Function



$$f(x) = 10^x$$

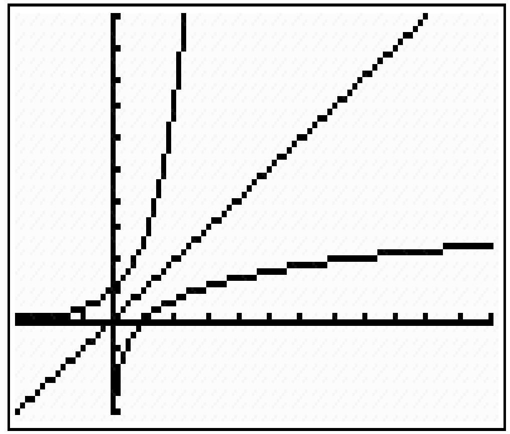
Domain = ? (-∞, ∞)

Range = ? (0, ∞)

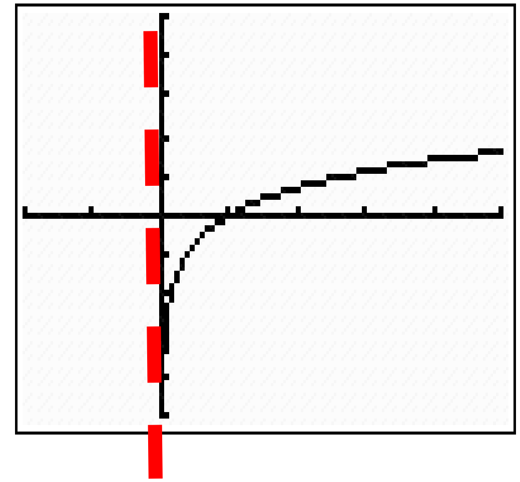
Horizontal asymptote = ?

y = 0

Inverse Functions



Logarithm Function



$$f^{-1}(x) = log_{10}(x)$$

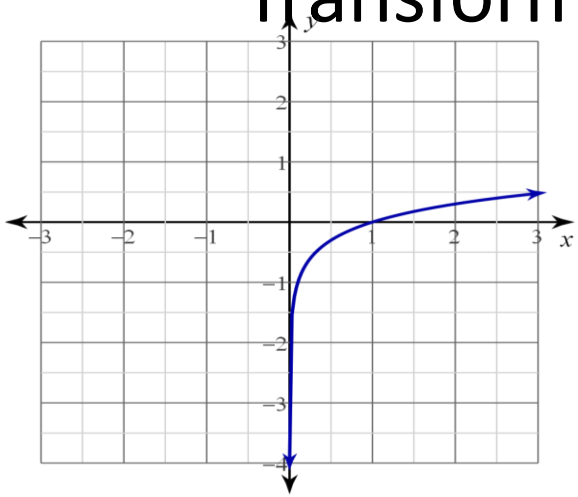
Domain = ? (0, ∞)

Range = ? (-∞, ∞)

Vertical asymptote = ?

x = 0

Transformations of the Log Function



$$f(x) = \log x$$

Domain = ? (0, ∞)

Range = ? (-∞, ∞)

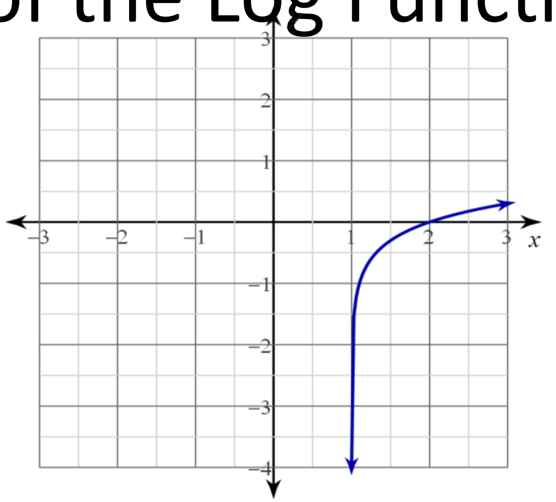
vertical asymptote = ?
x = 0

X-intercept = ?

x = 1

Where increasing = ?

(0, ∞)



$$g(x) = \log(x-1)$$

Right 1 shift

Domain = ? (1, ∞)

Range = ? (-∞, ∞)

asymptote = ? x = 1

Logarand

$$g(x) = 3\log(\underbrace{2x-1}_{\text{Logarand}}) + 5$$

Vertical Asymptote: The value of ‘x’ that makes the logarand equal to zero.

Vertical asymptote = ?

$2x - 1 = 0$

$x = \frac{1}{2}$

Evaluating Logs on your calculator

$$\log 8 = ?$$

Push buttons:

$\boxed{\text{log}}$ $\boxed{8)}$ $\boxed{=}$ 0.903089987

$$\ln 10 = ?$$

Push buttons:

$\boxed{\text{ln}}$ $\boxed{10)}$ $\boxed{=}$ 2.302585093

$$\boxed{\log 0 = ?}$$

$\boxed{\text{error}}$ Why?

Only input values x = (0, ∞) have corresponding outputs.

$$\boxed{\log(-3) = ?}$$

$\boxed{\text{error}}$ Why?

-3 is not in the “domain” of the function.

Transformations of the Log Function

$$f(x) = \log x$$

$$g(x) = 2\log(x+1) - 3$$

VSF = 2

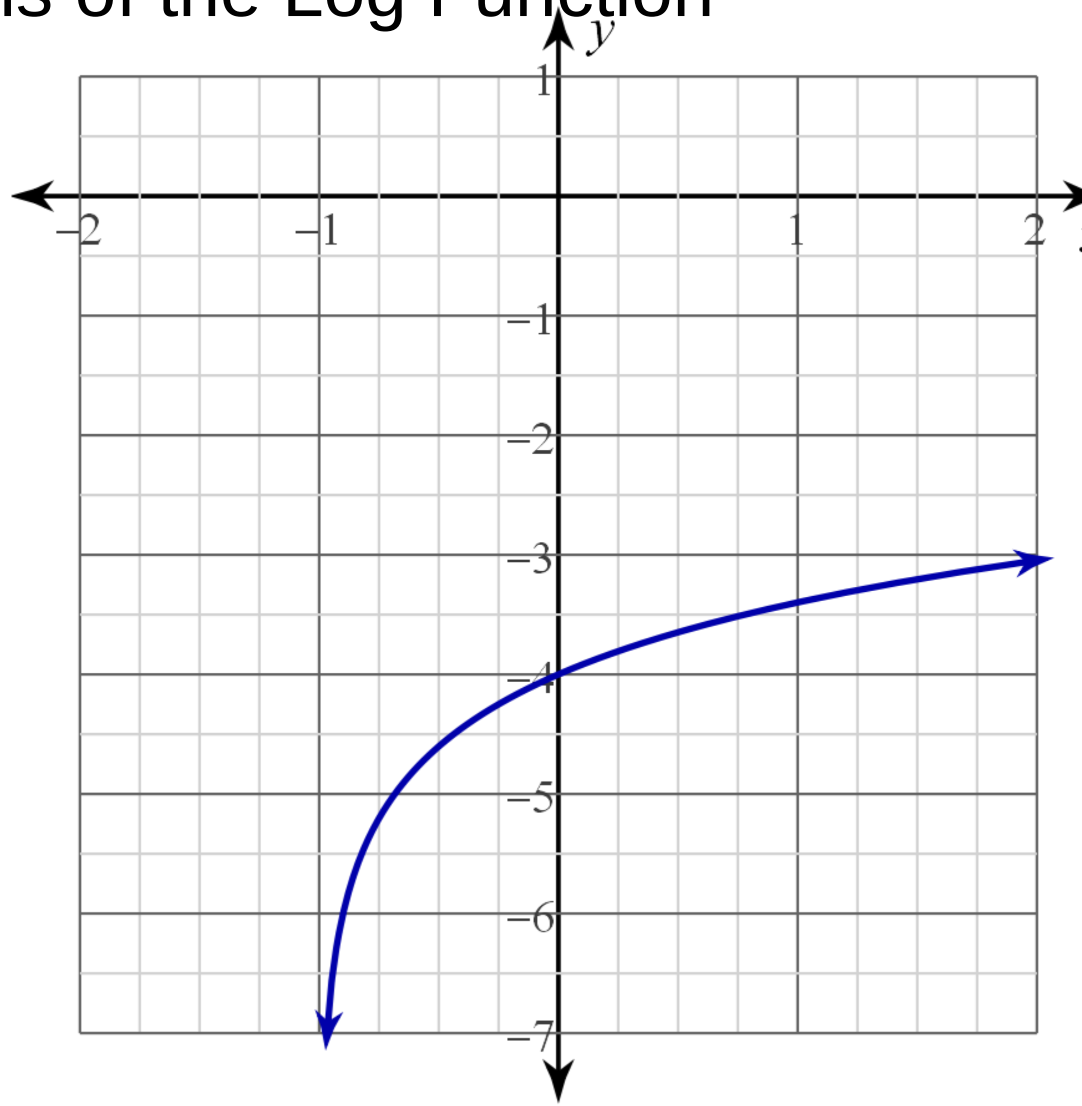
left 1 translation

Down 3 translation

Domain = ?   x = (-1, ∞)

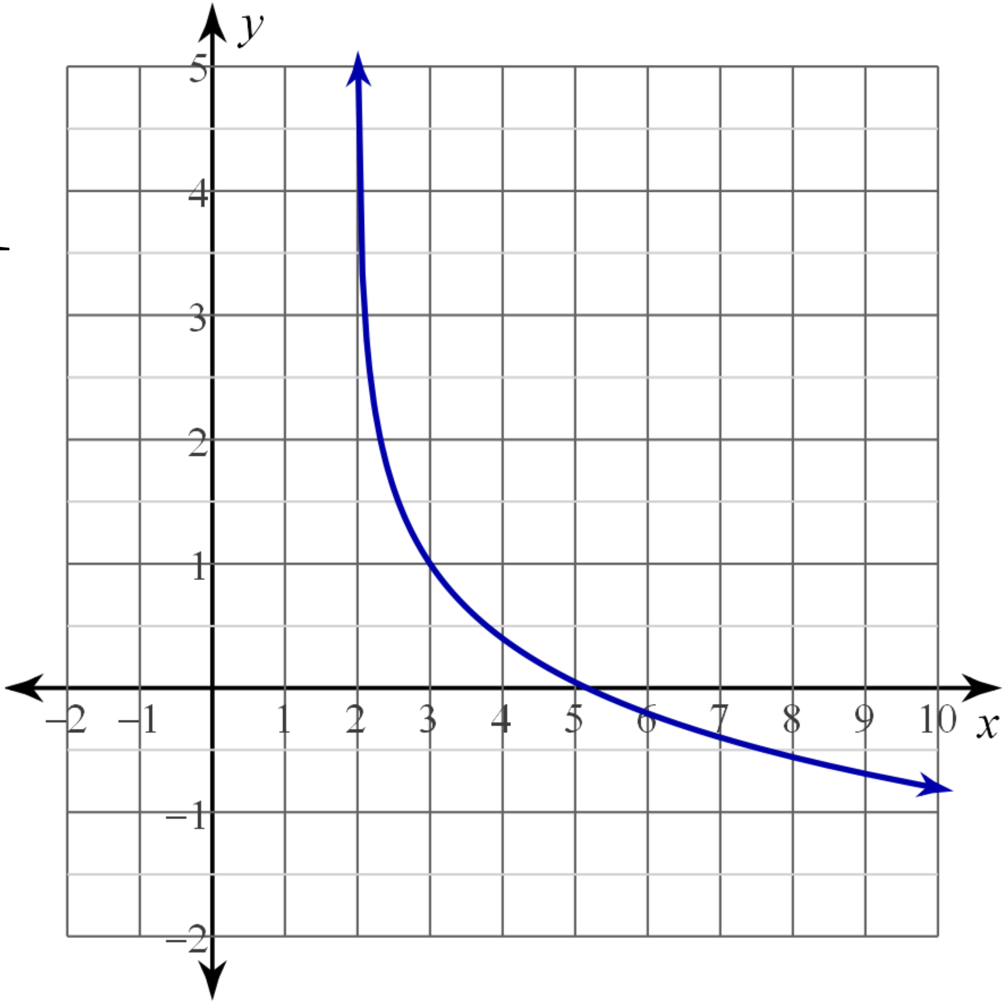
Range = ?   (-∞, ∞)

Asymptote = ?   x = -1



$$f(x) = \log x$$

$$g(x) = -3\log(x-2)+1$$



Reflected → x-axis
VSF = 3
Right 2 translation
Up 1 translation

Domain = ? x = (2, ∞)

Range = ? (-∞, ∞)

Asymptote = ? x = 2

NOT exponential (has a vertical asymptote, does NOT have a horizontal asymptote.

What is a logarithm?

A logarithm is another way of writing an exponent.

$$2^x = 8$$

x is the exponent

$$\log_2 8 = x$$

Log = exponent

Both of these equations are saying the same thing:
"2 raised to what power is 8?"

Exponential Form

$$2^3 = 8$$

base

"base 2 raised to the 3^{rd} is 8"

$$3^x = 9$$

What exponent of 3 equals 9?

Logarithm Form

$$\text{Log}_{\boxed{}}\boxed{} = \boxed{}$$

$$\log_2 8 = 3$$

base

"log base 2 of 8 is 3"

$$\log_3 9 = x$$

What exponent of 3 equals 9?

x = 2

Convert to logarithm form $\log_{\square} \square = \square$

What is the solution?

x = 2	$5^x = 25$	$\log_5 25 = x$
x = 3	$4^x = 64$	$\log_4 64 = x$
x = ???	$b^x = y$	$\log_b y = x$
x = 2	$9^x = 81$	$\log_9 81 = x$
x = 3	$10^x = 1000$	$\log_{10} 1000 = x$

Convert to exponential form $\mathrm{Log}_{\square} \square = \square$

What is the solution?

x = 2	$\log_{10} 100 = x$	$10^x = 100$
x = 3	$\log_3 27 = x$	$3^x = 27$
x = 0	$\log_9 1 = x$	$9^x = 1$
x = 16	$\log_4 x = 2$	$4^2 = x$
x = 32	$\log_2 x = 5$	$2^5 = x$

<u>Common Logarithm</u>: has a base of <u>10.</u>

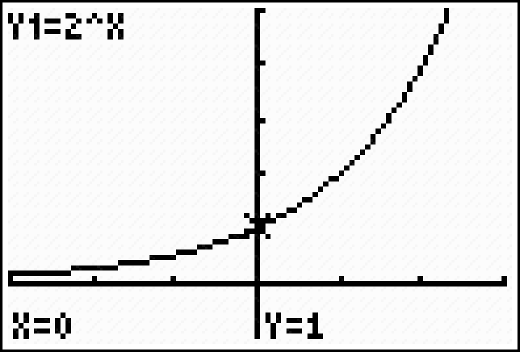
$$\log_{10} 100 = x$$

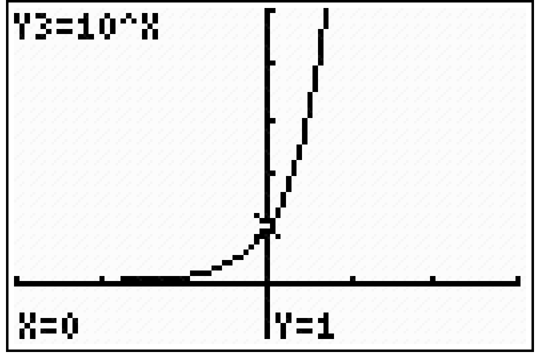
We <u>usually</u> write it in this form: $\log 100 = x$

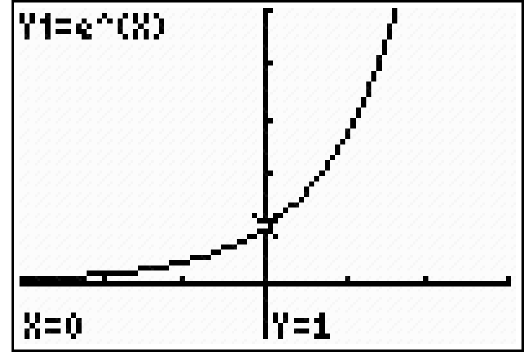
<u>Natural Logarithm</u>: has a base of <u>e.</u>

$$\log_{e} 2.718 = 1$$

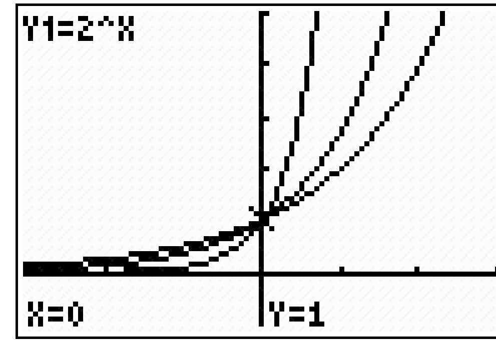
We <u>always</u> write it in this form: $\ln 2.718 = 1$

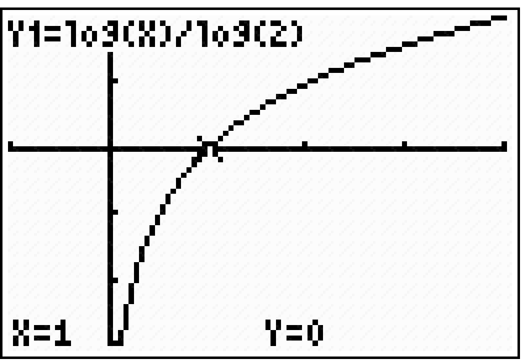


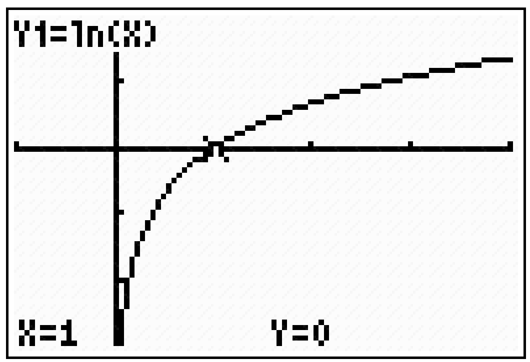


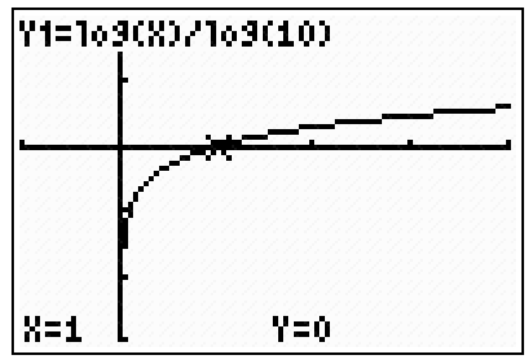


Base increasing: →

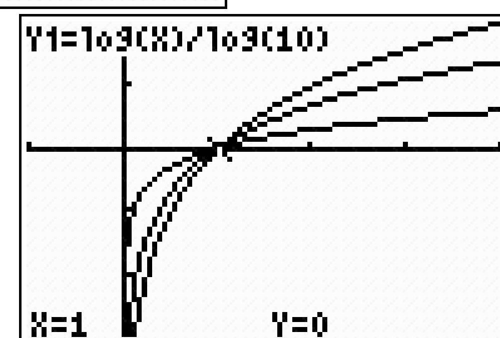








Base increasing: →



What is the base?

$$\log_2 8 = x \qquad \ln 5 = x \qquad \log 20 = x$$

What is the Solution?

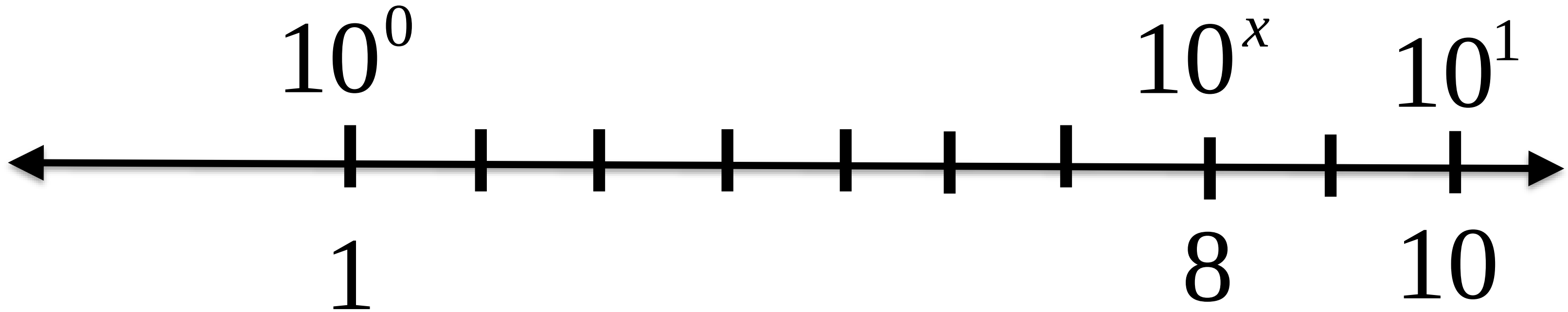
$$\frac{1}{100} = log_{10}(x) \qquad x = -2$$

$$x = log_2\sqrt{2} \qquad x = \frac{1}{2}$$

$$x = log_5 \frac{1}{\sqrt[3]{5}} \qquad x = -\frac{1}{3}$$

Estimate the value of the log: $log(8)$

$$\log 8 = x \qquad 10^x = 8$$

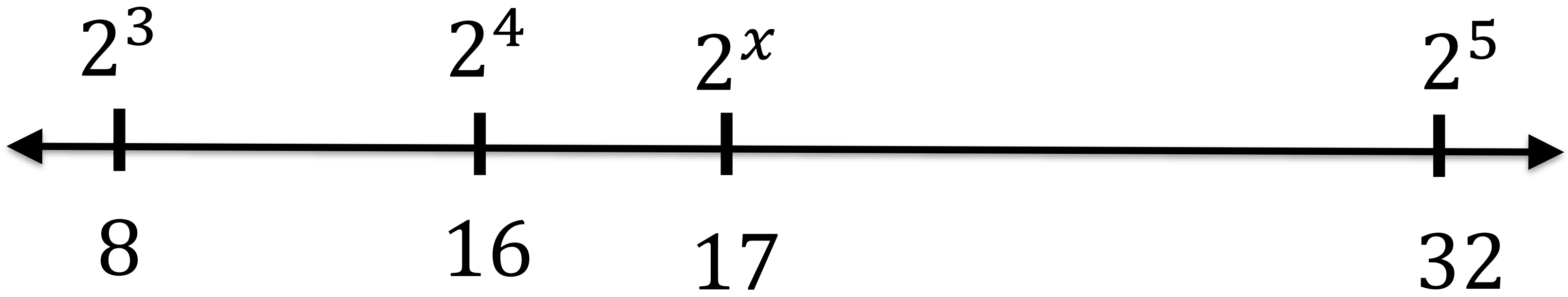


$$x \approx 0.8 \qquad x \approx 0.9$$

Find $\log 8$ on your calculator. $\log 8 = 0.903$

Estimate the value of the log: $\log_2 17$

$$\log_2 17 = x \quad \rightarrow 2^x = 17$$



$$x \approx 4.1 \ ?$$

Find $\log_2 17$ on your calculator. $\log_2 17 = 4.09$

Estimate the value of the log (without using your calculator)

$log_3 30$ $log_5 30$ $log_6 30$

Finding the Inverse $f^{-1}(x) = ?$

$f(x) = 3^x$ Shift ‘x’ and ‘y’

$x = 3^y$ “Undo the Exponential” (Convert it to a log)

“A log is an exponent”

$y = \log_3 x$ $\boxed{f^{-1}(x) = \log_3 x}$

Finding the Inverse $f^{-1}(x) = ?$

$$f(x) = (3)^{x-1} + 2$$ Shift ‘x’ and ‘y’

$$x = (3)^{y-1} + 2$$ “isolate” the exponential”

$$x - 2 = (3)^{y-1}$$ “Undo the Exponential” (Convert it to a log)

“A log is an exponent”

$$y - 1 = \log_3(x-2)$$

$$y = \log_3(x-2) + 1$$

$$f^{-1}(x) = \log_3(x-2) + 1$$

Finding the Inverse $f^{-1}(x) = ?$

$$f(x) = (3)^{x-1} + 2 \qquad f^{-1}(x) = \log_3(x-2) + 1$$

Right 1 → up 2

Right 2 → up 1

Finding the Inverse $f^{-1}(x) = ?$

$f(x) = 2\log_2(x+1)$ Shift ‘x’ and ‘y’

$x = 2\log_2(y+1)$ “Isolate the log”

$\frac{x}{2} = \log_2(y+1)$ “Undo the log” (Convert it to an exponential)

“A log is an exponent”

$$y+1 = 2^{x/2}$$

$$y = 2^{x/2} - 1$$

$$f^{-1}(x) = 2^{x/2} - 1$$