Math-3A

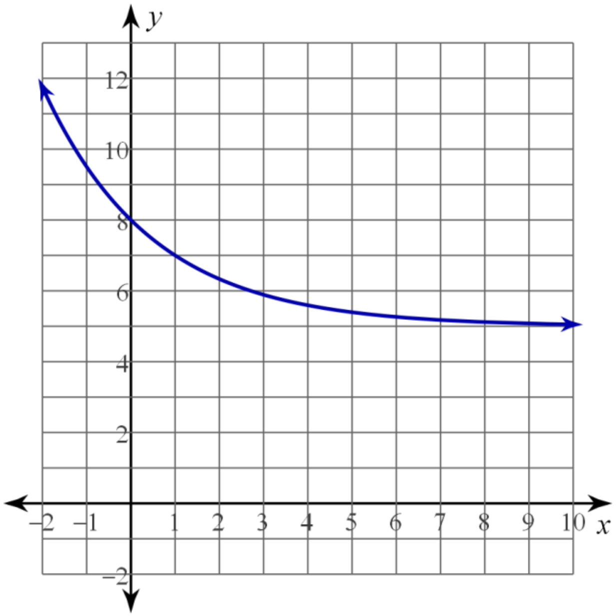
Lesson 7-6

More

Exponential Modeling

1. Money
2. Radioactive Decay
3. Carbon-14 Dating

Find the equation of the graph.



1. Horizontal asymptote: y = 5

$$y = AB^x + 5$$

2. Passes through: (x, y) = (0, 8)

$$8 = AB^0 + 5 \rightarrow 8 = A + 5 \rightarrow A = 3$$

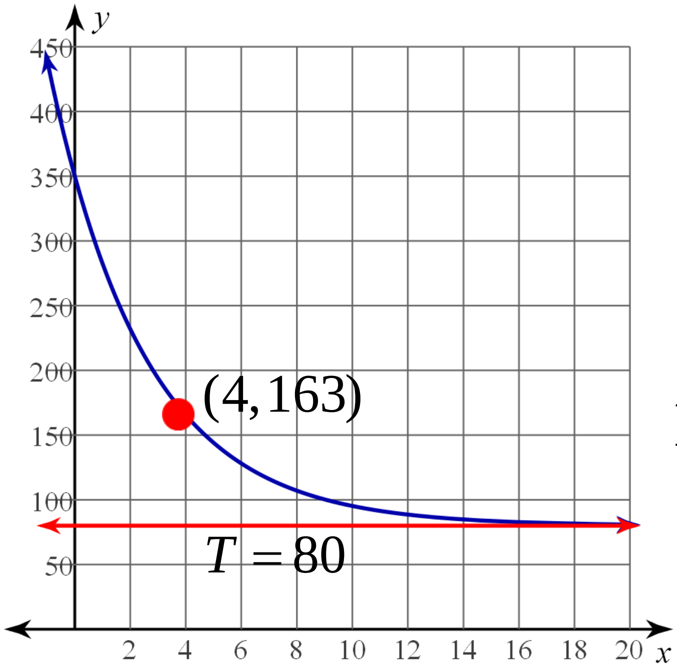
$$y = 3B^x + 5$$

3. Passes through: (x, y) = (1, 7)

$$7 = 3B^1 + 5 \rightarrow 2 = 3B \rightarrow B = 2/3$$

$$y = AB^x + k \rightarrow y = 3\left(2/3\right)^x + 5$$

Find the equation of the graph.



$$T(t) = AB^t + m$$

1. Horizontal asymptote: y = 80

$$T(t) = AB^t + 80$$

2. Passes through: (t, T) = (0, 350)

$$350 = AB^0 + 80 \rightarrow 270 = A$$

$$T(t) = 270B^t + 80$$

3. Passes through: (t, T) = (4, 163)

$$163 = 270B^4 + 80 \rightarrow \frac{163-80}{270} = B^4 = 0.3074$$

$$\rightarrow B = \sqrt[4]{0.3074} = 0.7446$$

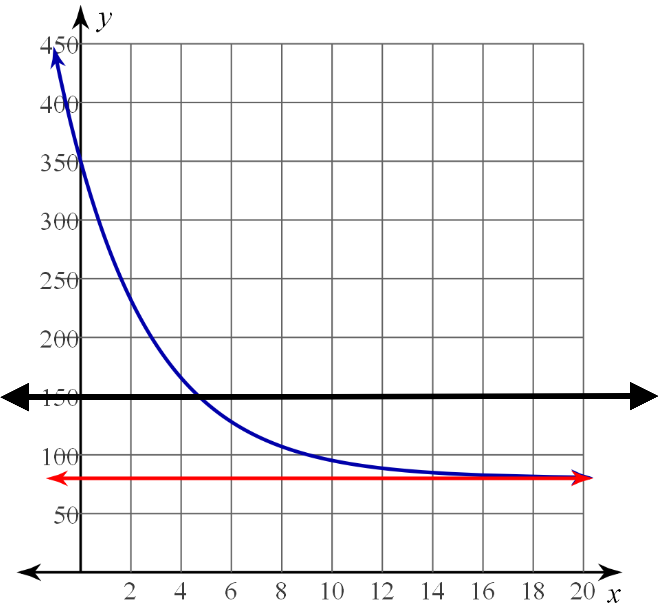
$$\rightarrow T(t) = 270(0.7446)^t + 80$$

2. What will be the temperature in 10 minutes?

$$T(10) = 270(0.7446)^{10} + 80$$

$$T(10) = 94.1\ F$$

Find the equation of the graph.



1. Find ‘t’ to reach 150 F

$$T(t) = 270(0.7446)^t + 80$$

$$150 = 270(0.7446)^t + 80$$

$$\frac{150 - 80}{270} = (0.7446)^t$$

→ $0.2593 = (0.7446)^t$

How do we solve for ‘t’?

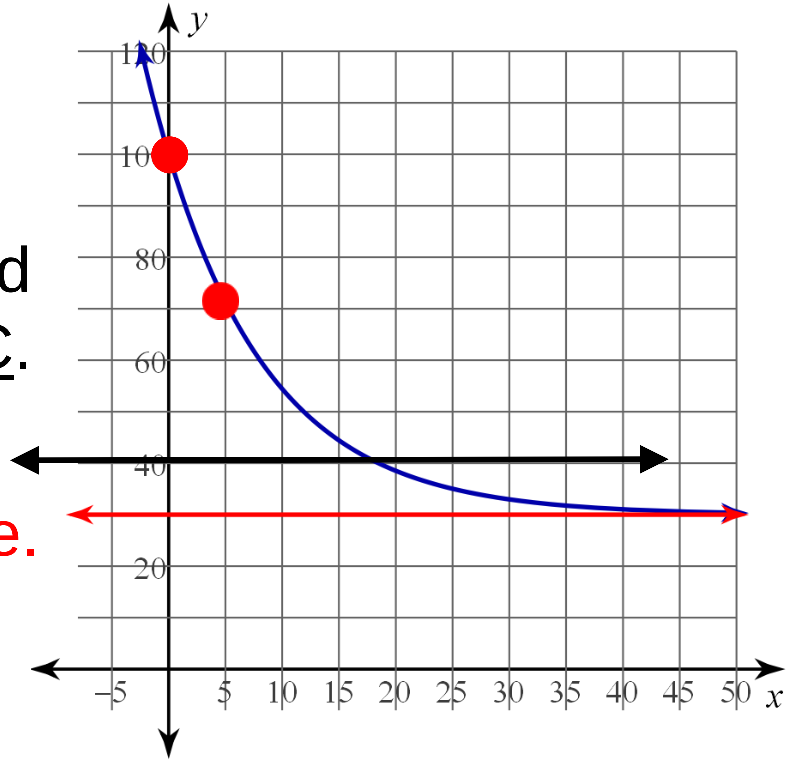
1. “Guess and check” → build a table and try some values for ‘t’

t	4	4.2	4.4	4.5	4.6
T	0.307	0.290	0.273	0.265	0.258

2. Solve by graphing: $y_1 = 270(0.7446)^t + 80$

$$y_2 = 150$$

A cup of hot water is taken out of the microwave oven. Its initial temperature is 100 C.
It is placed on the counter in a room whose temperature is 30 C. In 5 minutes it has cooled to 72 C. When will it reach 40 C.



1. Draw a graph that shows temperature as a function of time.

2. What is the equation of the graph? (use the following equation).

$$T(t) = AB^t + k$$

3. Draw a horizontal line for T = 40

4. Solve by graphing

$$y_1 = 70(0.903)^t + 30$$

$$y_2 = 40$$

A cake taken out of the oven at temperature of 450º F. It is placed on in a room with an ambient temperature of 75ºF to cool. 10 minutes later the temperature of the cake is 180ºF. When will the cake be cool enough to put the frosting on (90ºF)? (t=?, 90ºF)

Start with either:

$$T(t) = AB^t + k$$

$$T(t) = 375(0.8805)^t + 75$$

$$T(t) = 90$$

Solve by graphing

You deposit $100 money into an account that pays 3.5% interest per year. The interest is “compounded” monthly. How much money will be in the account at the end of the 5th year?

$$A(t) = A_0(1 + r/k)^{kt} \qquad A(5) = 100(1 + 0.035/12)^{12(5)}$$

$$A(5) = \$119.09$$

What is the doubling time for this account?

$$200 = 100(1 + 0.035/12)^{12t}$$

$$2 = (1.0029)^{12t}$$

$$y_1 = (1.0029)^t$$
$$y_2 = 2$$

Solve by graphing

You deposit $200 money into an account that pays 5.5% interest per year. The interest is "compounded" quarterly. How long will it take for your money to triple?

$$A(t) = A_0(1 + r/k)^{kt}$$

$$600 = 200(1 + 0.055/4)^{4(t)}$$

$$3 = (1.0138)^{4t}$$

$$y_1 = (1.0138)^t$$
$$y_2 = 3$$

Solve by graphing

Radioactive decay (also known as **nuclear decay** or **radioactivity**) is the process by which the nucleus of an unstable atom loses energy by emitting radiation, including alpha particles, beta particles, gamma rays, and conversion electrons. A material that spontaneously emits such radiation is considered **radioactive**.

The process of radioactive decay results in the nucleus of the atom becoming smaller → it turns into a new element.

We measure the time it takes to turn into another element using "half-life" (the time it takes to reach half of its original amount).

- Uranium-238 decays with a half-life of 4.5 billion years to thorium-234
- which decays with a half-life of 24 days to protactinium-234
- which decays with a half-life of 1.2 minutes to uranium-234
- which decays with a half-life of 240 thousand years to thorium-230
- which decays with a half-life of 77 thousand years to radium-226
- which decays with a half-life of 1.6 thousand years to radon-222
- which decays with a half-life of 3.8 days to polonium-218
- which decays with a half-life of 3.1 minutes to lead-214
- which decays with a half-life of 27 minutes to bismuth-214
- which decays with a half-life of 20 minutes to polonium-214
- which decays with a half-life of 160 microseconds to lead-210
- which decays with a half-life of 22 years to bismuth-210
- which decays with a half-life of 5 days to polonium-210
- which decays with a half-life of 140 days *to lead-206, which is a stable n*uclide.

The "half life" of Carbon-14 (a radioactive isotope of carbon), is 5730 years.

1. What is the base of the exponential?

$$t_{\frac{1}{2}} = 5730$$

$$A(t) = A_0(b)^t$$

$$0.5A_0 = A_0(b)^{5730}$$

$$0.5 = (b)^{5730}$$

$$0.5^{\left(\frac{1}{5730}\right)} = b$$

$$0.999879 = b$$

2. What is the exponential equation?

$$A(t) = A_0(0.999879)^t$$

3. If there was originally 10 gm of C-14, how many graphs would be left after 2000 years?

$$A(2000) = 10(0.999879)^{2000}$$

$$A(2000) = 7.85gm$$