Math-3-A
Lesson 7-4

Review the Exponential Function

The "Parent" Exponential Function

$$y = b^x$$

exponent

base

$$y = 2^x$$ **(base 2 exponential function)**

$$y = 3^x$$ **(base 3 exponential function)**

$$y = \left(\frac{1}{2}\right)^{\mathrm{x}}$$ **(base 1/2 exponential function)**

The base MUST BE <u>positive</u> and CANNOT equal 1.

$$b = (0,1) \cup (1,\infty)$$

Fill in the output values of the table and graph the points.

$$f(x) = 2^x$$

Growth Factor is the base of the exponential

x	$2^{(\ )}$	y
-2	2^{-2}	0.25
-1	2^{-1}	0.5
0	2^{0}	1
1	2^{1}	2
2	2^{2}	4

$* 2$

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$* 2$

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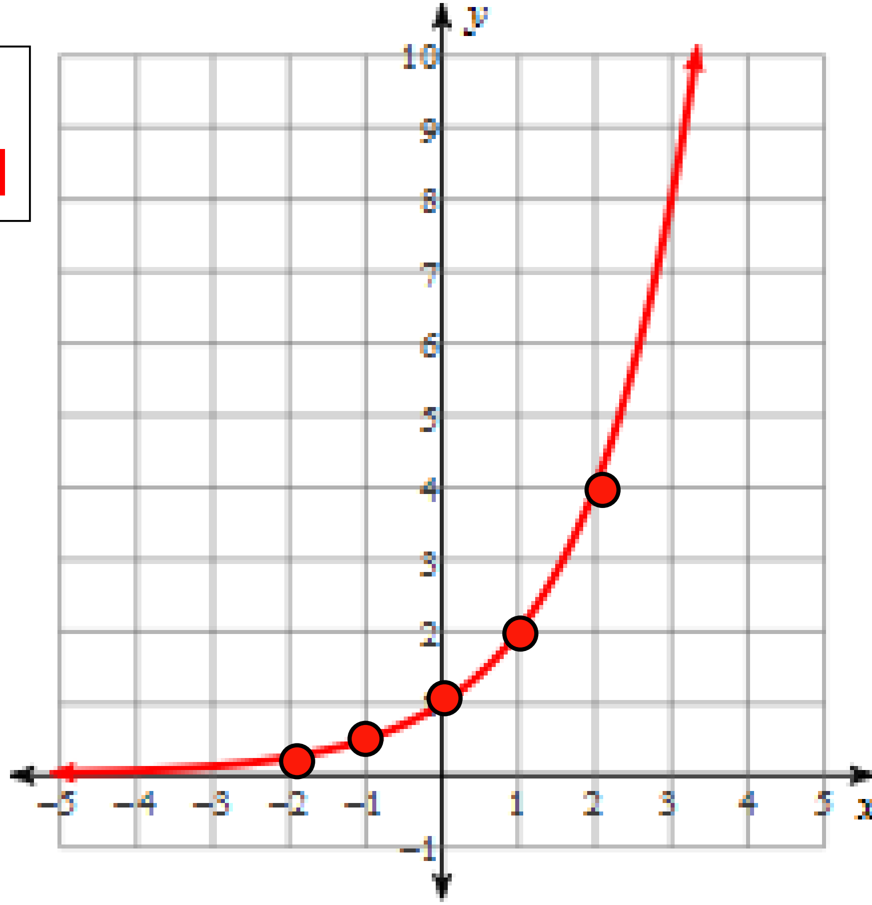


$$\left(\frac{2}{1}\right)^{-2} = \left(\frac{1}{2}\right)^{2} = \frac{1}{4} = 0.25$$

"negative exponent property"

$$2^0 = 1$$

"zero exponent property"

Exponential Function $f(x) = 2^x$

Will the 'y' value ever reach zero (on the left end of the graph)?

As the denominator gets bigger and bigger, the decimal version of the fraction gets smaller and smaller.

x	$2^{(\ )}$	y
-1	$2^{(-1)}$	$1/2$
-2	$2^{(-2)}$	$1/4$
-3	$2^{(-3)}$	$1/8$
-4	$2^{(-4)}$	$1/16$
-5	$2^{(-5)}$	$1/32$

'y' gets closer and closer to zero but never reaches zero.

$$f(-1) = 1/2$$
$$f(-2) = 1/4$$
$$f(-3) = 1/8$$
$$f(-4) = 1/16$$
$$f(-5) = 1/32$$

<u>Horizontal Asymptote</u>: a horizontal line the graph approaches but never reaches.

$$f(x) = 2^x$$

$$y = 0$$

$$Domain = ?$$

$$x = (-\infty, \infty)$$

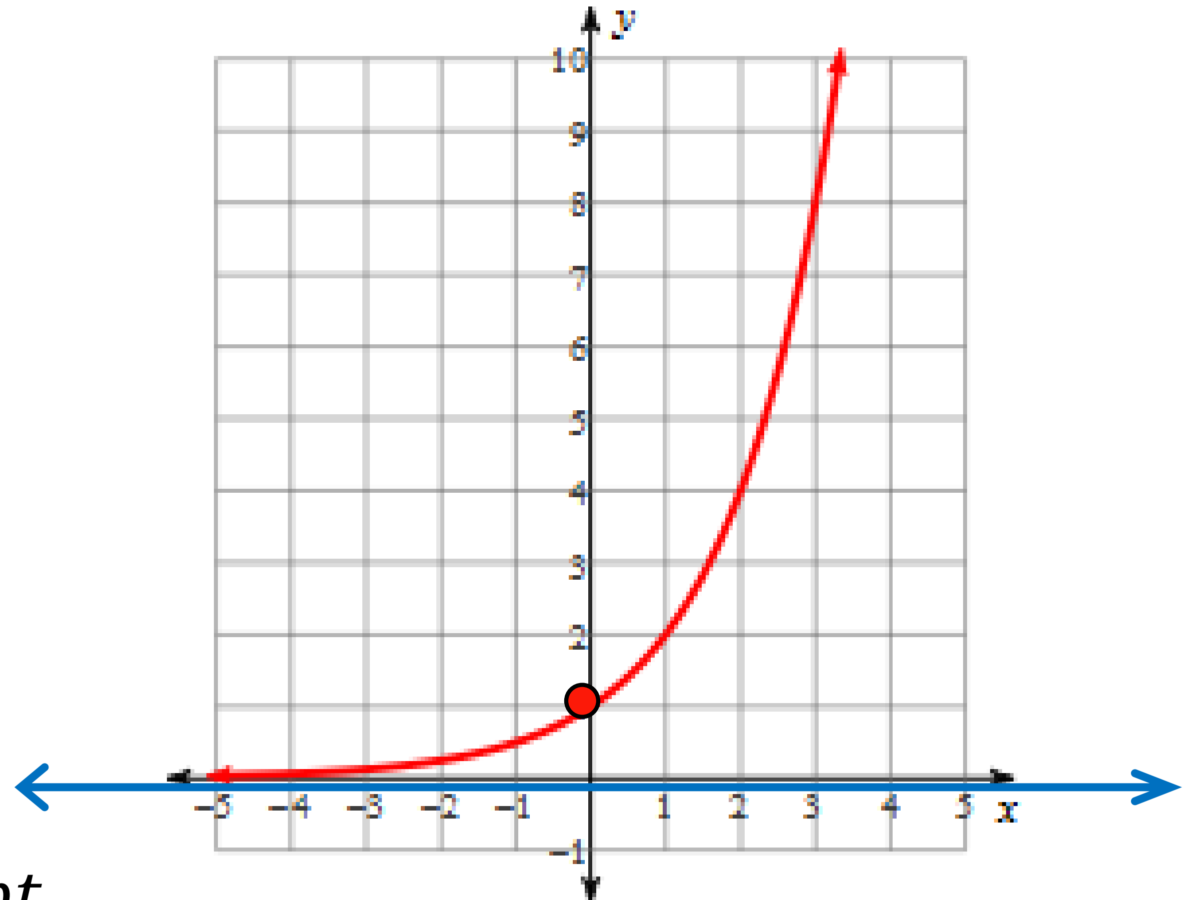
$$range = ?$$

$$y = (0, \infty)$$

y-intercept = ?

$$f(0) = y \;\; intercept$$

$$f(0) = 2^0 = 1$$



Exponential Growth: the graph is increasing. Growth occurs when the base of the exponential is greater than 1.

$y = b^x$

'b' = 1 → no growth

'b' > 1 → growth

$f(x) = 1^x$

x	f(x)
-1	1
0	1
1	1

$g(x) = (1.1)^x$

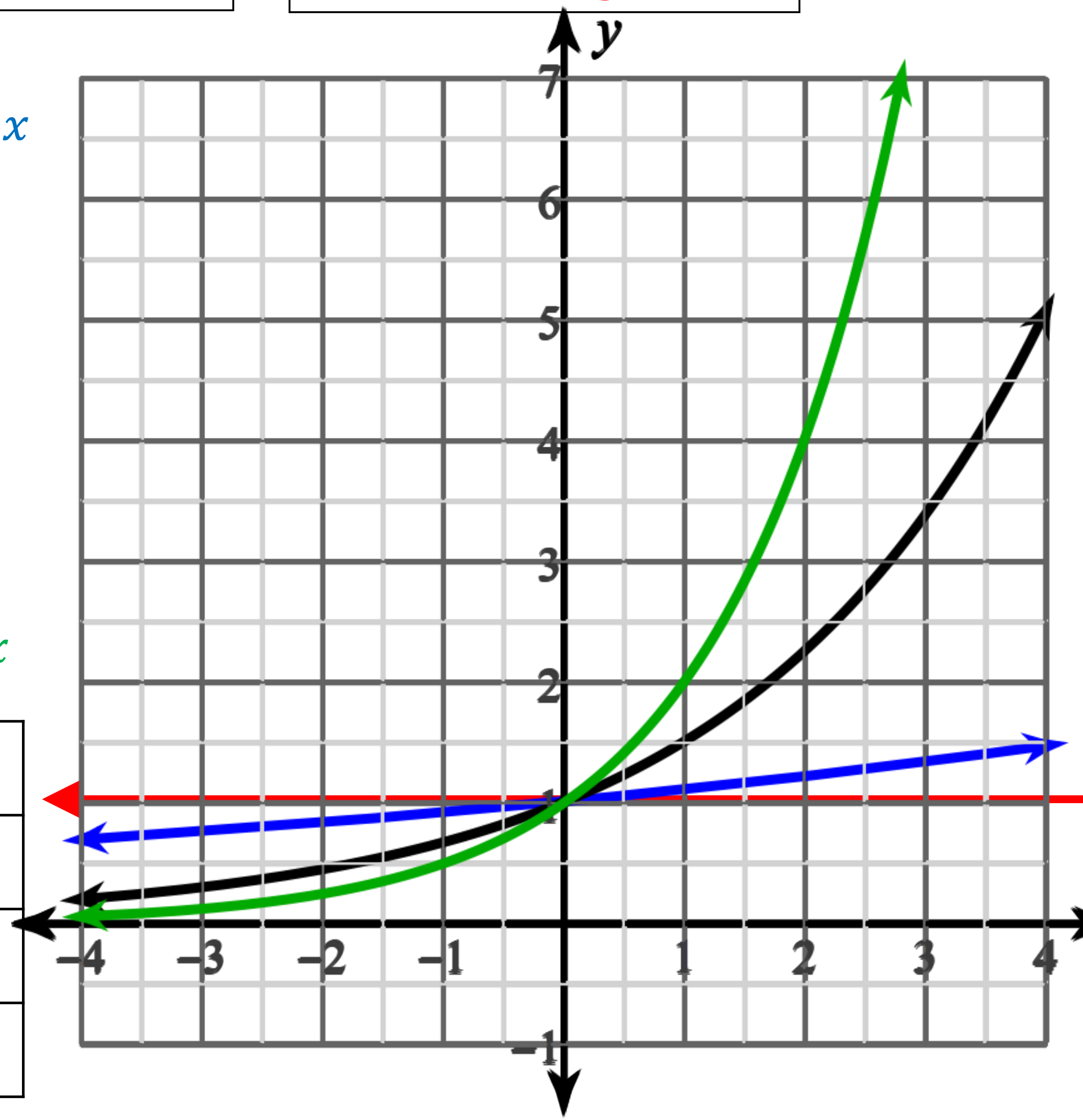
x	g(x)
-1	0.91
0	1
1	1.1

$h(x) = (1.5)^x$

x	h(x)
-1	0.67
0	1
1	1.5

$k(x) = (2)^x$

x	k(x)
-1	0.5
0	1
1	2



<u>Exponential Decay</u>: the graph is decreasing. decay occurs when the base of the exponential is between 0 and 1.

$y = b^x$

‘b’ = 1 → no growth

0 < ‘b’ < 1 → decay

$f(x) = 1^x$

x	f(x)
-1	1
0	1
1	1

$g(x) = (0.9)^x$

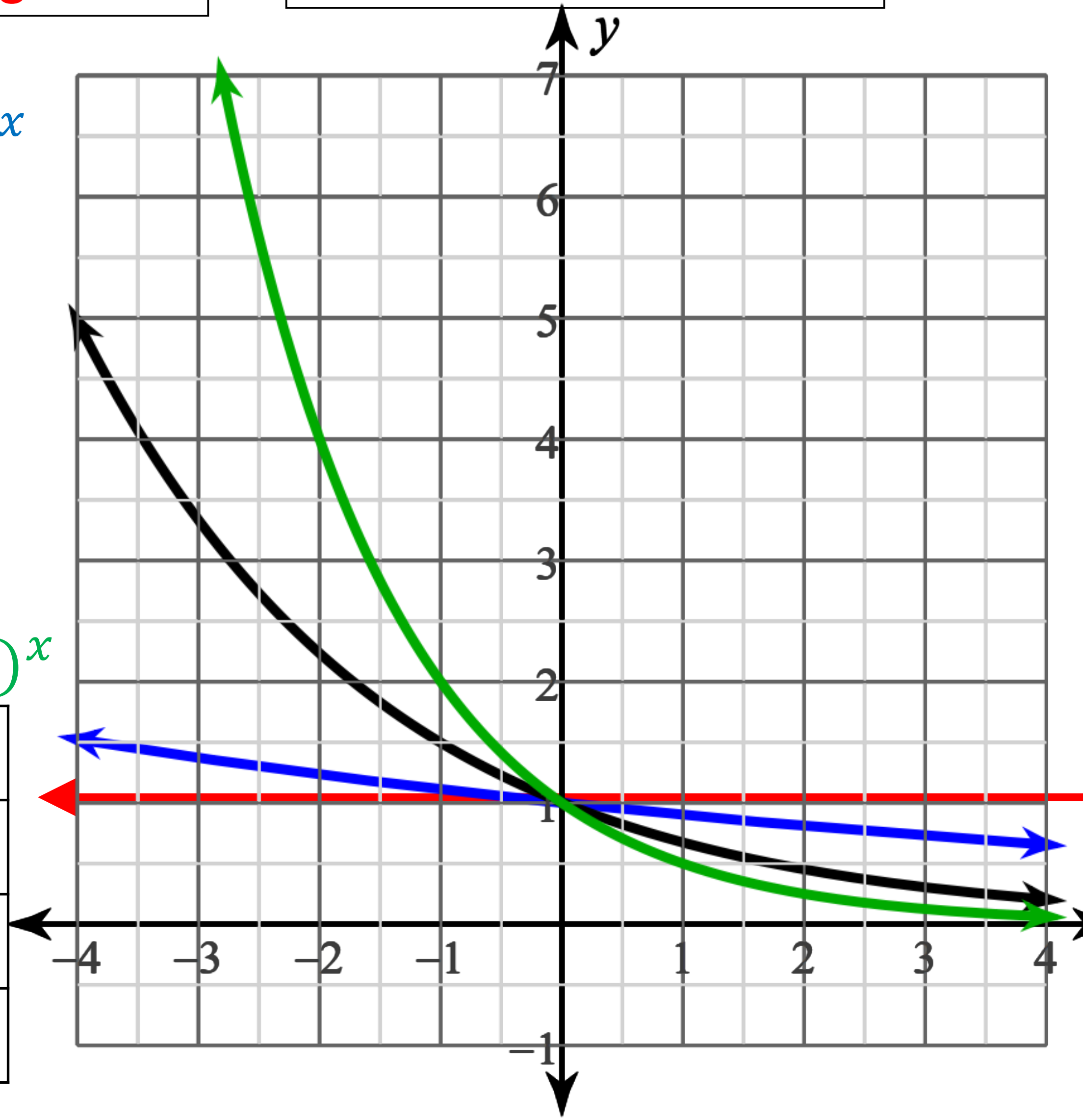
x	g(x)
-1	1.1
0	1
1	0.9

$\mathrm{h}(x) = (0.67)^x$

x	h(x)
-1	1.5
0	1
1	0.67

$k(x) = (0.5)^x$

x	k(x)
-1	5
0	1
1	0.2

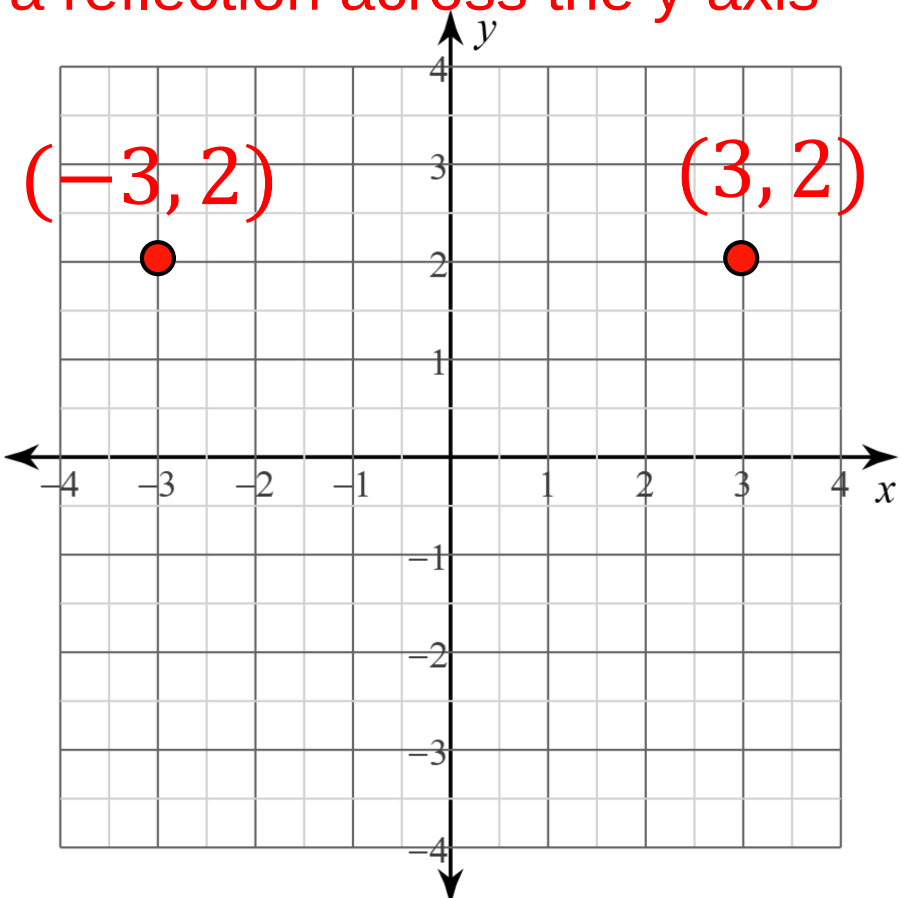


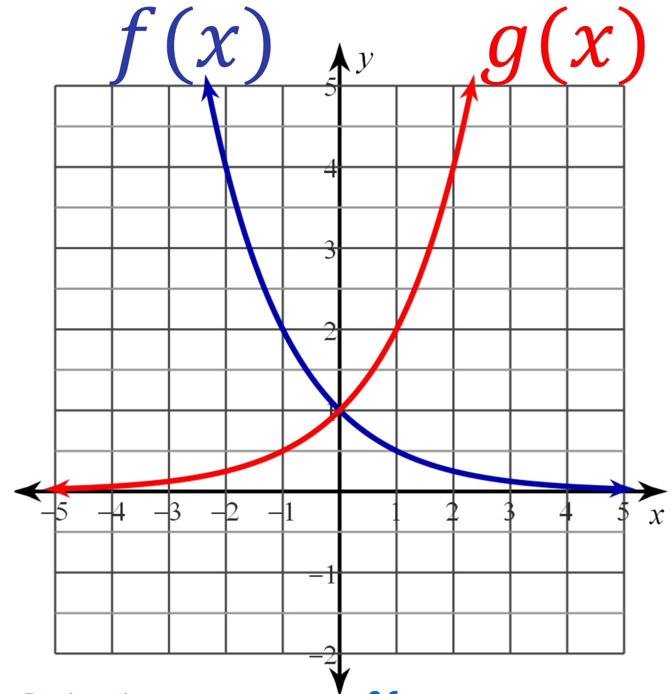
$$g(x) = 2^x \qquad f(x) = \left(\frac{1}{2}\right)^x$$

→ Reflection across the y-axis

If (3, 2) is reflected across the y-axis, where would it be?

→ Replacing ‘x’ with ‘(-x)’ causes a reflection across the y-axis





$$f(x) = 2^{-x}$$

$$f(x) = (2^{-1})^x$$

Exponent of a Power Property of Exponents

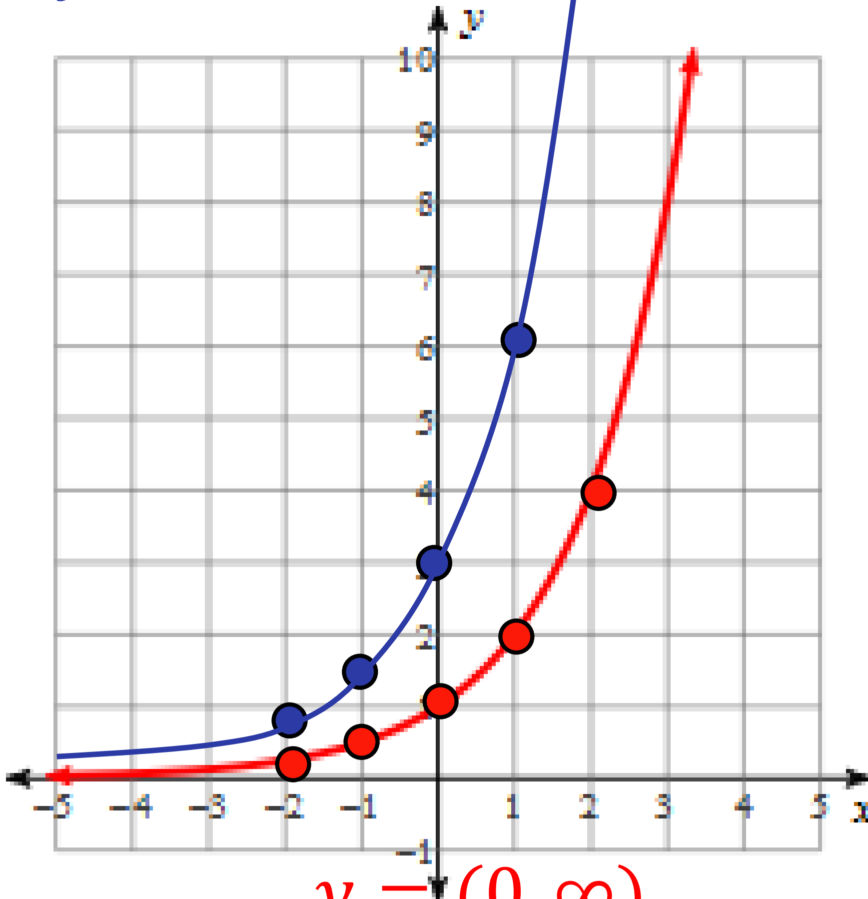
$$f(x) = \left(\frac{1}{2}\right)^x$$

Negative Exponent Property

$f(x) = 2^x$ $\quad g(x) = 3(2)^x$

Vertically stretched by a factor of 3

x	$2^{(\ )}$	f(x)	$g(x)$
-2	2^{-2}	0.25	0.75
-1	2^{-1}	0.5	1.5
0	2^0	1	3
1	2^1	2	6
2	2^2	4	12



Horizontal asymptote: $y = 0$

$y = 0$

$range = ?$ $\quad y = (0, \infty)$

$y = (0, \infty)$

$Domain = ?$ $\quad x = (-\infty, \infty)$

$x = (-\infty, \infty)$

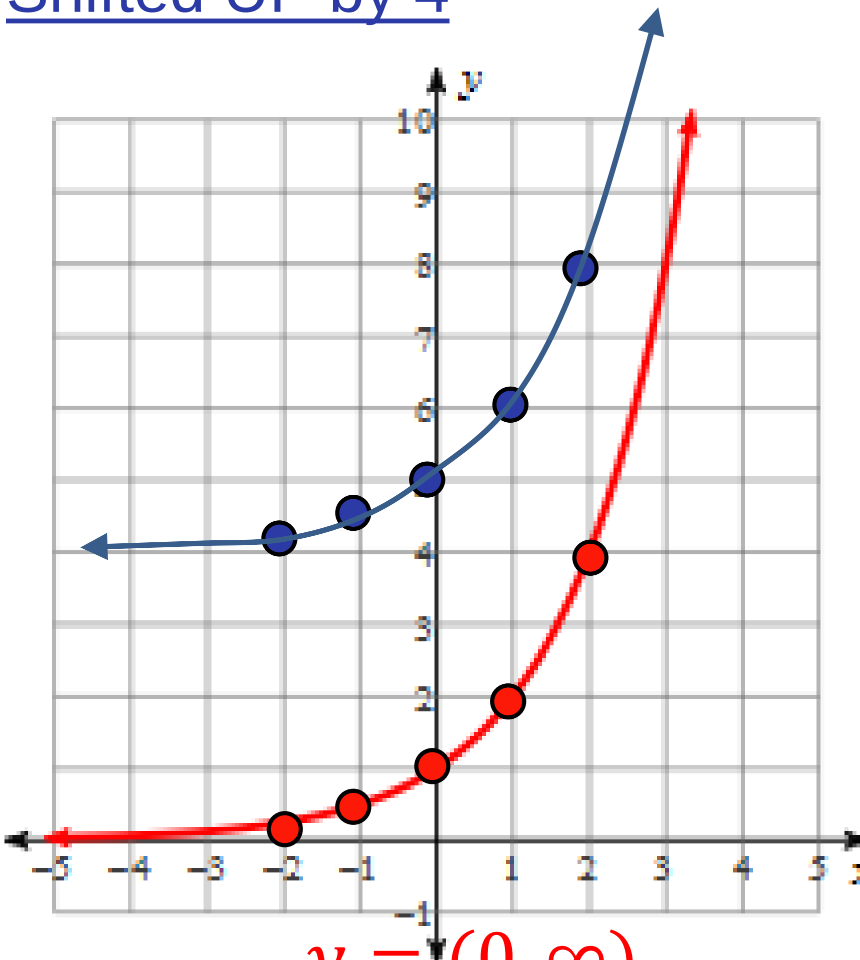
y-intercept = ? $\quad (0, 1)$

$(0, 3)$

$f(x) = 2^x$ $\quad$ $\text{k}(x) = 2^x + 4$

Shifted UP by 4

x	$2^{(\ )}$	f(x)	$k(x)$
-2	2^{-2}	0.25	4.25
-1	2^{-1}	0.5	4.5
0	2^0	1	5
1	2^1	2	6
2	2^2	4	8



Horizontal asymptote: $y = 0$

$y = 4$

$range = ?$ $\quad y = (0, \infty)$

$y = (4, \infty)$

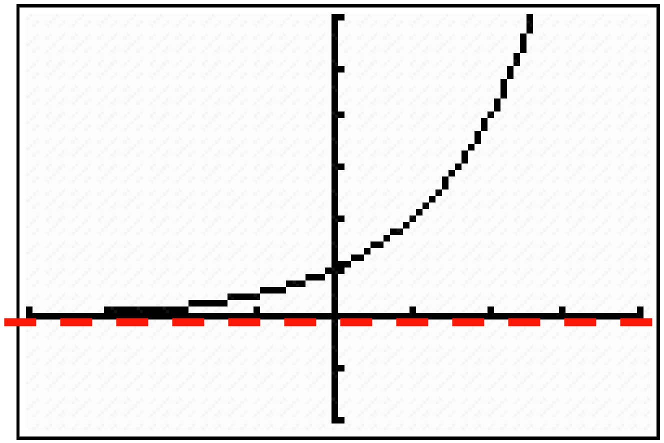
$Domain = ?$ $\quad x = (-\infty, \infty)$

$x = (-\infty, \infty)$

y-intercept = ? $\quad (0, 1)$

$(0, 5)$

Transformations of the Exponential Function

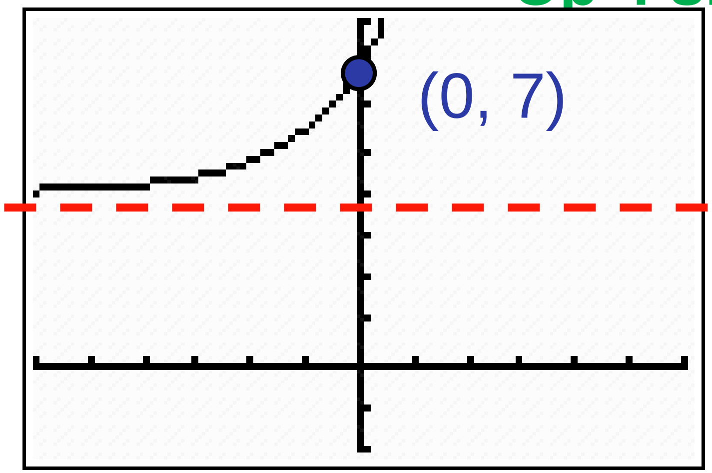


$f(x) = 2^x$ Base-2 Exponential Parent Function

VSF= 3

$$h(x) = 3(2)^x + 4$$

Up 4 shift



<u>Transformation Form of the Exponential Function</u>

$$y = ab^x + k$$

vertical shift and horizontal Asymptote

VSF:

y-intercept: (0, a + k)

<u>Growth Factor</u> (the base of the exponential)

$$h(0) = 3(2)^0 + 4$$

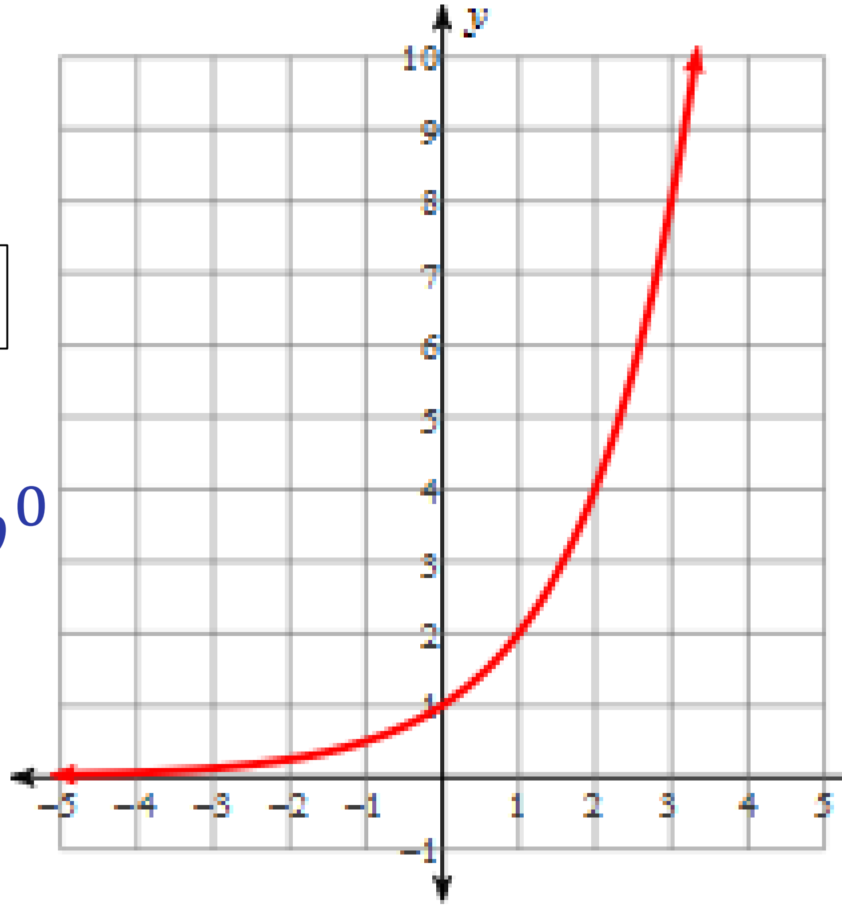
$$h(0) = 7$$

Summary

1) Start with $\boxed{g(x) = ab^x + k}$

2) Find the value of 'k' (horizontal asymptote). $k = 0$

$g(x) = ab^x + k$ → $\boxed{y = ab^x}$



3) Substitute the y-intercept

(0, 1) → $y = ab^x$ → $1 = ab^0$

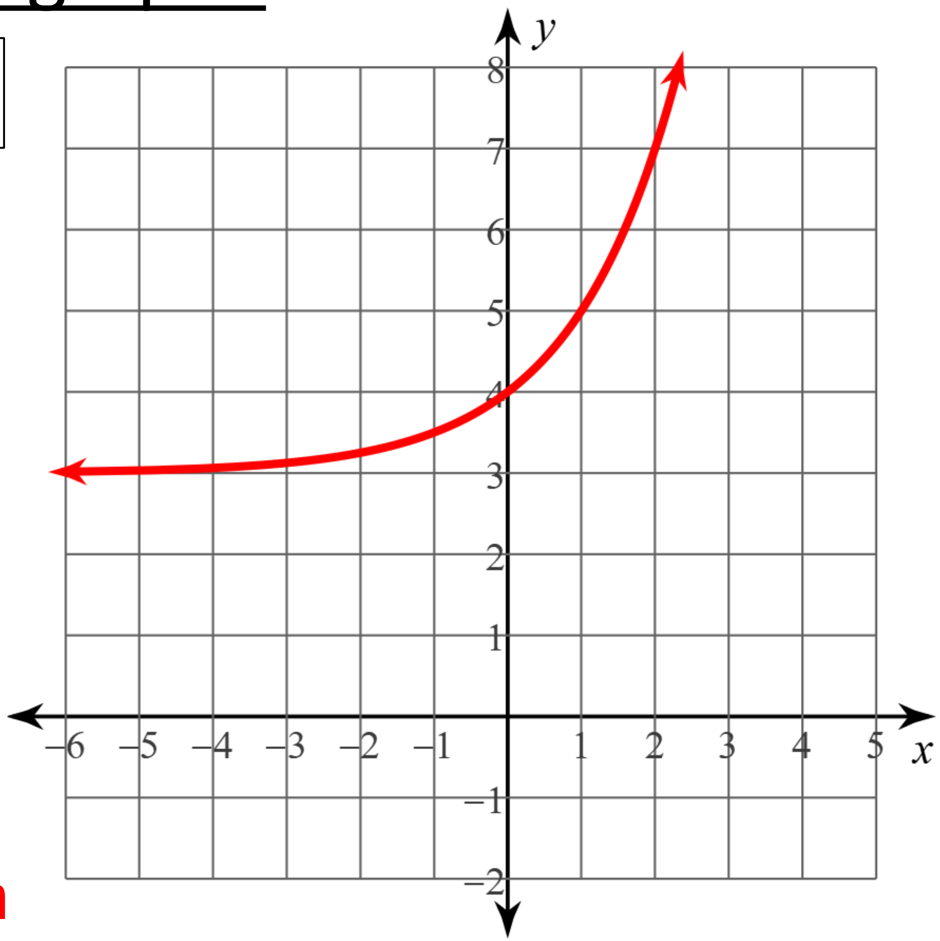
→ a = 1 → $\boxed{y = b^x}$

4) Substitute a "nice" x-y pair from the graph into the equation.

(1, 2) → $y = b^x$ → $2 = b^1$ → b = 2 → $\boxed{y = 2^x}$

What is the equation of the graph?

1) Start with $\boxed{g(x) = ab^x + k}$



2) Find 'k'

Horizontal asymptote: y = 3

$k = 3$ $\boxed{y = ab^x + 3}$

3) Substitute the y-intercept

(0, 4) → $4 = ab^0 + 3$

a = 1 → $\boxed{y = b^x + 3}$

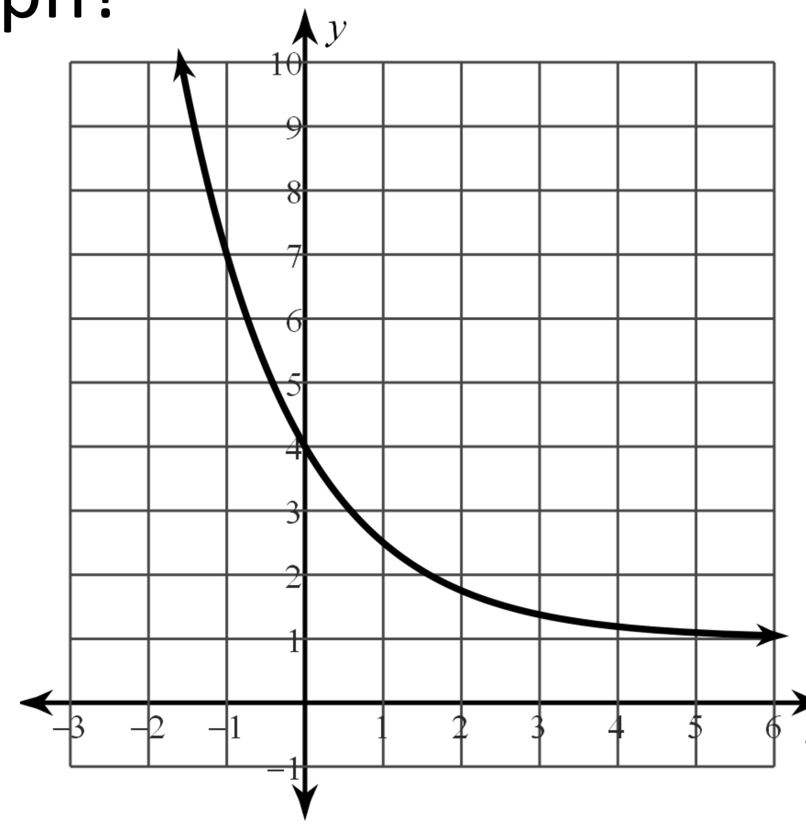
4) Substitute a "nice" x-y pair from the graph into the equation.

(1, 5) → $5 = b^1 + 3$ → b = 2

$\boxed{y = 2^x + 3}$

What is the equation of the graph?

1) Start with $\boxed{g(x) = ab^x + k}$



2) horizontal asymptote y = 1

$k = 1$ $\boxed{y = ab^x + 1}$

3) y-intercept $(0, 4)$ $4 = ab^0 + 1$

$a = 3$ $y = 3b^x + 1$

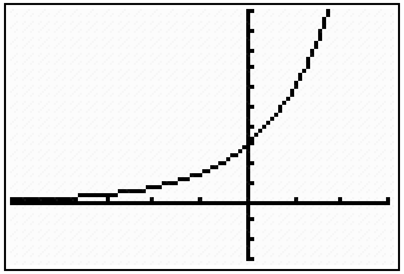
4) "Nice" x-y pair $(-1, 7)$

$$7 = 3b^{-1} + 1$$

$$6 = 3b^{-1}$$

$$2 = b^{-1} \qquad 2 = \frac{1}{b} \qquad b = \frac{1}{2} \qquad y = 3\left(\frac{1}{2}\right)^x + 1$$

Initial Value: (of the exponential) is the vertical stretch factor (for problems with no up/down shifts)

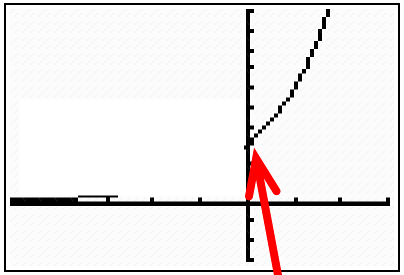


If in input is time ("stopwatch time") the initial value occurs when t = 0.

$$f(t) = 3(2)^t$$

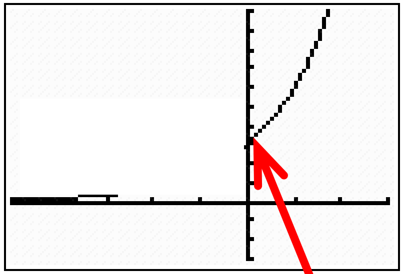
Domain: x = [0, ∞)

$$f(0) = 3(2)^0 = ?$$



$$f(0) = 3$$

Initial Value: (of the exponential) is the vertical stretch factor (for problems with no up/down shifts)



$f(0) = 3$

"Initial Value" is a term that is applicable to modeling of real world situations.

Population $P(t) = 500(1.03)^t$

Money in a bank account $A(t) = \$2500(1.032)^t$

Concentration of salt when adding fresh water to salt water

$$C(t) = 0.5\ gm/liter(0.73)^t$$

Decay of radioactive Carbon 14

$$A(t) = 10\ gm\ (0.999879)^t$$