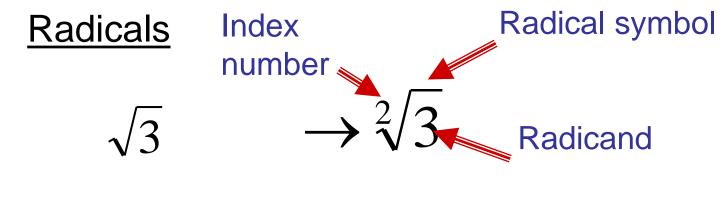
Math-3A Lesson 7-3 Radicals and Rational Exponents

 $\sqrt{3}$ What number is equivalent to the square root of 3? $x = \sqrt{3}$ Square both sides of the equation $(x)^2 = (\sqrt{3})^2$ $x^2 = 3$ $x = \sqrt{3}$ is an equivalent statement to $x^2 = 3$

 $\sqrt{3}$ ≈ 1.732 ≈ 1.7321 ≈ 1.73205 ≈ 1.732051 ≈ 1.7320508...



 $x = \sqrt[2]{3}$ The "square root of 3" means: $x^2 = 3$ "what number squared equals 3?"

$$x = \sqrt[3]{4}$$
$$x^3 = 4$$

The "<u>3rd root of 4</u>" means: "what number cubed equals 4?"

Adding and subtracting radicals

Can these two terms be combined using addition? 3x + 2xWrite 3x as repeated addition x + x + xWrite 2x as repeated addition x + x $3x + 2x \rightarrow x + x + x + x + x \rightarrow 5x$

When <u>multiplication</u> is written as <u>repeated addition</u>, "like terms" look <u>exactly alike.</u>

$$3\sqrt{x} + 2\sqrt{x} \quad \rightarrow \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} + \sqrt{x} \rightarrow 5\sqrt{x}$$
$$3\sqrt{6} + 2\sqrt{6} \quad \rightarrow \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} + \sqrt{6} \rightarrow 5\sqrt{6}$$

Define "like powers" "Same base, same exponent". $3x^4 + 2x^4 \rightarrow 5x^4$

Define "like radicals" "Same radicand, same index number". $3\sqrt{6} + 2\sqrt{6} \rightarrow 5\sqrt{6}$

Which of the following are "like radicals" that can be added?

- $\sqrt{2} + \sqrt{3}$ no $\sqrt[4]{5} + \sqrt[4]{5}$ yes
- $2\sqrt{3} + 3\sqrt{2}$ no $3\sqrt[5]{2} + 4\sqrt[5]{2}$ yes
 - $\sqrt[4]{2} + \sqrt[3]{2}$ no $6\sqrt[3]{4} + 6\sqrt[4]{4}$ yes

$$\sqrt{3} + \sqrt{2} \rightarrow \sqrt{3 + 2} = \sqrt{5}$$
 Are they equivalent?

If this is a property of radicals, it must work for every combination of numbers.

 $\sqrt{4} + \sqrt{9} \rightarrow \sqrt{13}$

 $\sqrt{4} + \sqrt{9} \rightarrow 2 + 3 \rightarrow 5 \neq \sqrt{13}$

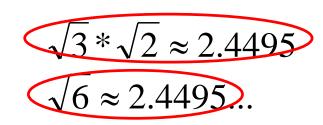


 $\sqrt{3} * \sqrt{2}$

 $\sqrt{3} \approx 1.7321... \sqrt{2} \approx 1.4142...$

 $\sqrt{3*2} \rightarrow \sqrt{6}$

Will this work?



Product of Radicals Property $\sqrt{a} * \sqrt{b} \to \sqrt{a * b}$

$$\sqrt{5} * \sqrt{2} = \sqrt{10}$$

 $\sqrt{4} * \sqrt{9} \rightarrow \sqrt{4*9}$

Are these equivalent? $\sqrt{a} * \sqrt{b} = \sqrt{ab}$

 $2*3 \rightarrow \sqrt{36}$

 $2*3 \rightarrow 6$

6 = 6

 $\sqrt{a} * \sqrt{b} = \sqrt{ab}$

Simplify the following:

$3\sqrt{8}*5\sqrt{2}$	$2\sqrt{3}*3\sqrt{5}$	$\rightarrow 6\sqrt{15}$
$3*\sqrt{8}*5*\sqrt{2}$	$7\sqrt{6} * 2\sqrt{5}$	$\rightarrow 14\sqrt{30}$
$3*5*\sqrt{8}*\sqrt{2}$ $15*\sqrt{8}*\sqrt{2}$	$\sqrt{5} + 3\sqrt{5}$	$\rightarrow 4\sqrt{5}$
$15 * \sqrt{16}$	$7\sqrt{6} + 2\sqrt{6}$	$\rightarrow 9\sqrt{6}$
15*4 = 60		

<u>Simplify radicals</u>: use the Product of Radicals Property to factor ("break apart") the radical into a "perfect square" times a number. $\sqrt{a} * \sqrt{b} = \sqrt{ab}$

$$\sqrt{18} \rightarrow \sqrt{9} * \sqrt{2} \rightarrow 3 * \sqrt{2} \rightarrow 3\sqrt{2}$$

<u>Simplify</u> $\sqrt{24} \rightarrow \sqrt{4} * \sqrt{6} \rightarrow 2\sqrt{6}$

 $3\sqrt{32x^2} \rightarrow 3^*\sqrt{16}^*\sqrt{x^2} + \sqrt{2} \rightarrow 3^*4^*x^*\sqrt{2} \rightarrow 12x\sqrt{2}$

$$\sqrt[3]{x^4} \rightarrow \sqrt[3]{x^3} * \sqrt[3]{x} \rightarrow x\sqrt[3]{x}$$

$$\sqrt[4]{3x^5y} \rightarrow \sqrt[4]{x^4} * \sqrt[4]{3xy} \rightarrow x\sqrt[4]{3xy}$$

Can we add "unlike" radicals?

$$\sqrt{a} * \sqrt{b} = \sqrt{ab}$$
Simplify $7\sqrt{6} + 2\sqrt{24} \rightarrow 7\sqrt{6} + (2*\sqrt{4}*\sqrt{6})$
 $\rightarrow 7\sqrt{6} + (2*2*\sqrt{6})$
 $\rightarrow 7\sqrt{6} + (2*2*\sqrt{6})$
 $\rightarrow 7\sqrt{6} + 4\sqrt{6}$
 $\rightarrow 11\sqrt{6}$
 $-3\sqrt{32} + 2\sqrt{8} \rightarrow (-3*\sqrt{16}*\sqrt{2}) + (2*\sqrt{4}*\sqrt{2})$
 $\rightarrow (-3*4*\sqrt{2}) + (2*2*\sqrt{2})$
 $\rightarrow -12\sqrt{2} + 4\sqrt{2}$
 $\rightarrow -8\sqrt{2}$

Another way to Simplify Radicals Factor, factor, factor!!!

$$\sqrt{54} \rightarrow \sqrt[2]{54} \rightarrow \sqrt[2]{2*27} \rightarrow \sqrt[2]{2*3*9} \rightarrow \sqrt[2]{2*3*3}$$

What is the factor that is used (Index number) '2' times under the radical?

Bring the out factor that is used 2 times.

$$\rightarrow 3\sqrt[2]{2*3} \rightarrow 3\sqrt{6}$$

Using Properties of Exponents to reduce the writing:

$$\sqrt[4]{32x^6} \rightarrow \sqrt[4]{32*x^4*x^2}
\rightarrow x\sqrt[4]{32*x^2}
\rightarrow x\sqrt[4]{32*x^2}
\rightarrow x\sqrt[4]{2^4*2^1*x^2}
\rightarrow 2x\sqrt[4]{2x^2}$$

<u>Rationalizing the denominator</u>: using mathematical properties to change an irrational number (or imaginary number) in the denominator into a rational number.

We take advantage of the idea:

$$\sqrt{2} * \sqrt{2} = \sqrt{2 * 2} = \sqrt{4} = 2$$

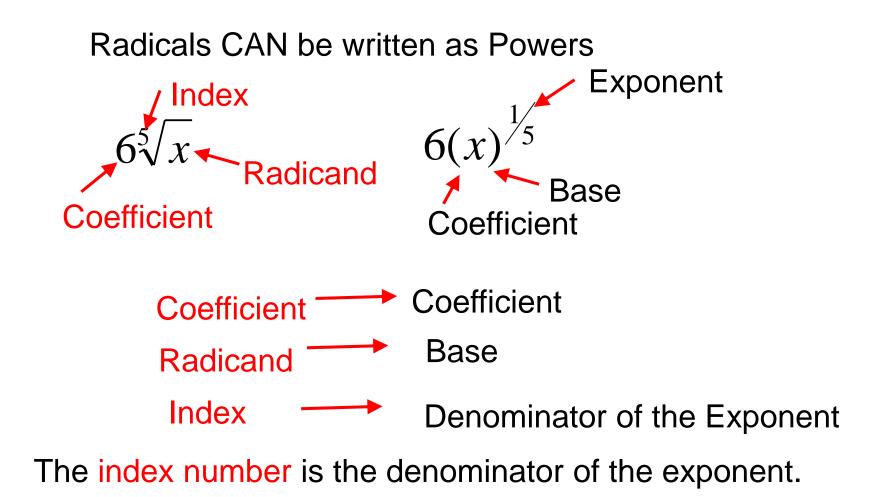
$$\sqrt{3} * \sqrt{3} = \sqrt{3 * 3} = \sqrt{9} = 3$$

$$\frac{1}{\sqrt{2}} * \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$
Identity
Property of
Multiplication
multiplying by '1' doesn't change
the number.

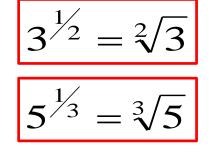
 $\frac{2}{\sqrt{6}} * \frac{\sqrt{6}}{\sqrt{6}} \to \frac{2\sqrt{6}}{6} \to \frac{2*\sqrt{6}}{2*3} \to \frac{\sqrt{6}}{3}$

 $\frac{25}{\sqrt{15}} * \frac{\sqrt{15}}{\sqrt{15}} \rightarrow \frac{25\sqrt{15}}{15} \rightarrow \frac{5*5**\sqrt{15}}{5*3} \rightarrow \frac{5\sqrt{15}}{3}$

 $\frac{14}{3\sqrt{21}} * \frac{\sqrt{21}}{\sqrt{21}} \to \frac{14\sqrt{21}}{3*21} \to \frac{2*7**\sqrt{21}}{3*7*3} \to \frac{2\sqrt{21}}{9}$



Are radicals related to powers?



$$3\sqrt[2]{y} = 3y^{\frac{1}{2}}$$

$$5\sqrt[3]{7} = 5(7)^{\frac{1}{3}}$$

$$\sqrt[2]{x} = x^{\frac{1}{2}}$$

 $\sqrt[3]{7} = 7^{\frac{1}{3}}$

None of these have coefficients! Multiplication (by a coefficient) is "repeated addition." This explains why coefficients of radicals become coefficients of powers.

$$\sqrt{y} = y^{\frac{1}{2}}$$

$$3\sqrt[2]{y} = \sqrt{y} + \sqrt{y} + \sqrt{y}$$
$$3y^{\frac{1}{2}} = y^{\frac{1}{2}} + y^{\frac{1}{2}} + y^{\frac{1}{2}}$$

What happens if there is a product under the radical?

$$2\sqrt{xy} = (xy)^{\frac{1}{2}}$$

$$5\sqrt[3]{3x} = 5(3x)^{\frac{1}{3}}$$

$$2\sqrt[4]{21mn} = 2(21mn)^{\frac{1}{4}}$$

How did we show that the index number applied to the entire product (radicand) when re-written in "power form"?

Power of a product \rightarrow product inside parentheses with an exponent.

$$\sqrt[5]{x^2 y} = (x^2 y)^{\frac{1}{5}} = x^{\frac{2}{5}} y^{\frac{1}{5}}$$

$$6\sqrt[3]{3m^2} = 6(3m^2)^{\frac{1}{3}} = 6(3^{\frac{1}{3}})m^{\frac{2}{3}}$$

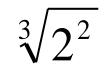
"Exponential Form" that has both a numerator and denominator

The exponent can be written as a rational number.





Numerator: Exponent of the base. Denominator: Root of the base.



Radical Form

 $=2^{2/3}$

Exponential Form

Re-write in power form.

 $\sqrt[2]{3m} \rightarrow (3m)^{\frac{1}{2}}$

Rewrite in "radical form"

 $m^{\frac{1}{5}} \rightarrow \sqrt[5]{m}$

 $4\sqrt[3]{5y} \rightarrow 4(5y)^{\frac{1}{3}}$

 $3nm^{\frac{1}{4}} \rightarrow 3n\sqrt[4]{m}$

 $\sqrt[5]{x^3y^2} \rightarrow (x^3y^2)^{\frac{1}{5}} \rightarrow x^{\frac{3}{5}}y^{\frac{2}{5}} \qquad 2(18n^2)^{\frac{1}{6}} \rightarrow 2\sqrt[6]{18n^2}$

Multiply Powers Property

$$y^2 * y^3 = ? = y^{2+3} = y^5$$

When multiplying "same based powers" add the exponents.

$$\chi^{\frac{2}{3}} * \chi^{\frac{3}{4}} \to \chi^{\frac{2}{3} + \frac{3}{4}} \to \chi^{\frac{17}{12}}$$

Yes, you must be able to add fractions

Exponent of a Power Property
$$(y^2)^3 = ? = y^{2*3} = y^6$$

When multiplying "same based powers" add the exponents.

$$\left(y^{\frac{1}{2}}\right)^{\frac{2}{3}} = y^{\frac{1}{2}\frac{2}{3}} = y^{\frac{1}{3}}$$