Math-3A
Lesson 6-7
Quadratic Inequalities

<u>Property of Inequality</u>

If you perform the same mathematical operation to the left and right sides of the inequality ($<$, $>$, $\leq$, $\geq$) then the rewritten inequality is equivalent to the original inequality <u>Provided that</u> if you <u>multiply or divide by a negative number</u> you must switch the direction of the inequality. (Here's why:

$$x - 4 > 8$$

$$+4 \quad +4$$

$$x > 12$$

$$5 > 1$$

$$*(-1) \quad *(-1)$$

$$-5 > -1 \quad \text{(not true)}$$

$$-5 < -1 \quad \text{(true)}$$

<u>Three Ways to write the solution to an inequality:</u>

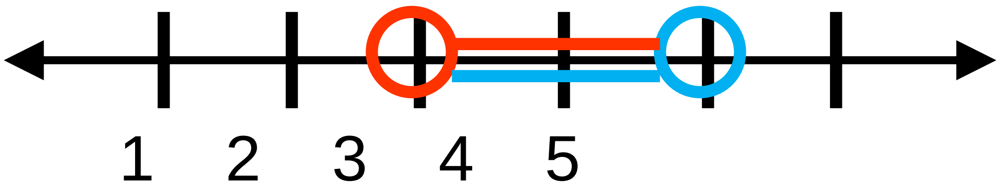
1. <u>Simplified inequality</u>

$x > 3$ and $x < 5$

Another way to write this is:

$3 < x < 5$

2. <u>Graph</u> (number line for a single variable inequality)



3. <u>Interval notation</u> (brackets/parentheses)

(3, 5)

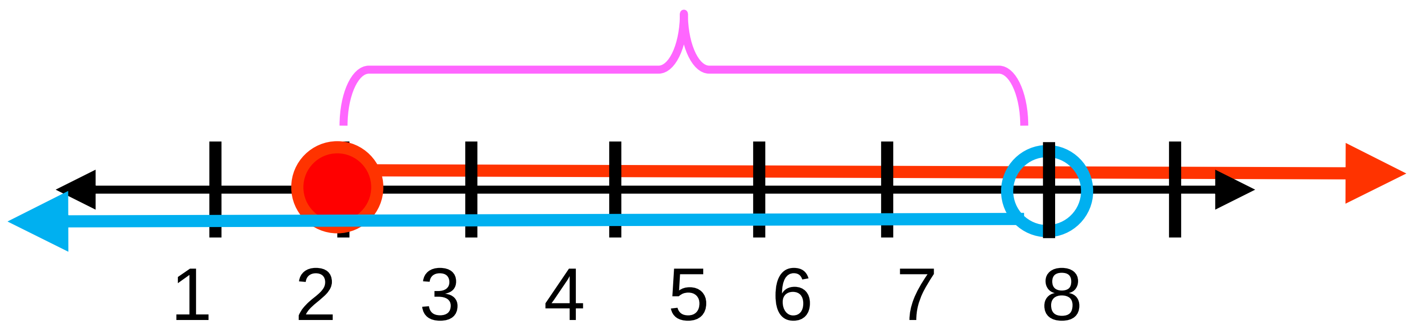
Solve: $x - 4 < 3$ and $x + 3 \geq 5$

To solve: find the values of the variable that make the both inequalities true (“and” inequality).

$x - 4 < 3$ and $x + 3 \geq 5$

$+4 \quad +4$ $\qquad$ $-3 \quad -3$

$x < 7$ and $x \geq 2$



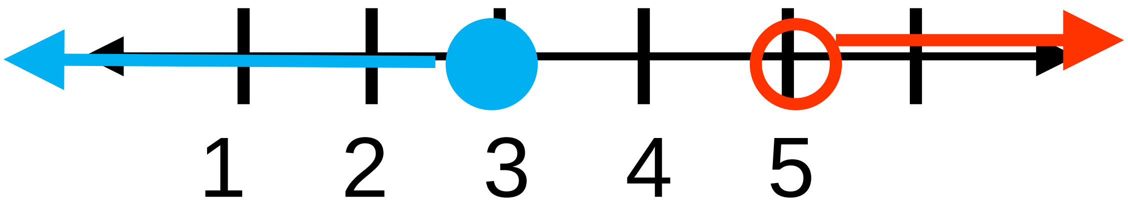
Solve: $2x - 3 \le 3$ or $2x - 5 > 5$

$x \le 3$ or $x > 5$

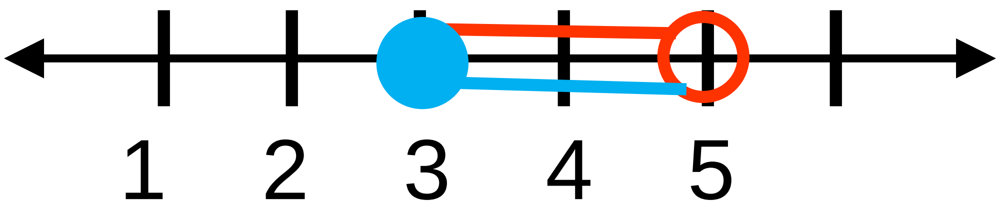
The "boundary numbers" $x = 3$ $x = 5$
separate the solution from the non-solution.

The solution is usually either:

1) Between the boundary numbers or
2) Outside of the boundary numbers



$x \le 3$ or $x > 5$



$3 \le x < 5$

The shaded part of the graph is the solution.

1. Find the boundary numbers: (Solve the equation)

$$0 > x^2 - x - 12$$

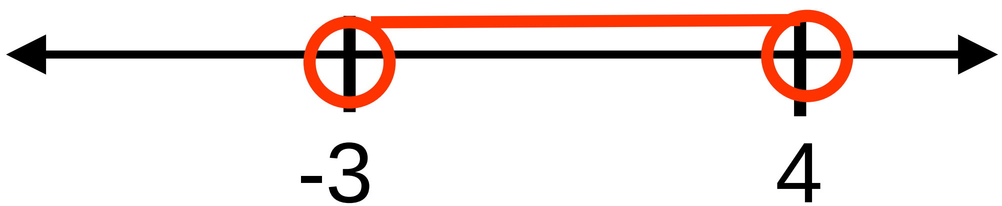
$$0 = (x - 4)(x + 3)$$

$$x = 4, \quad -3$$

$$0 = x^2 - x - 12$$

2. The solution is usually either:

1) Between the boundary numbers or

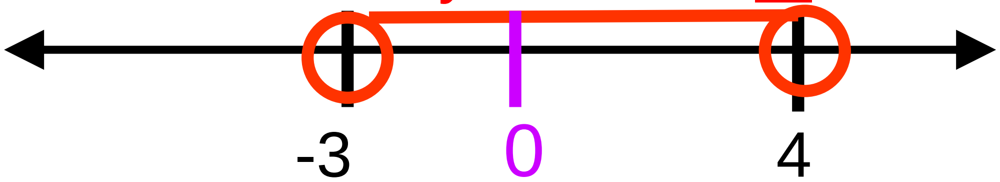


2) Outside of the boundary numbers

$$0 > x^2 - x - 12$$

The solution is <u>usually</u> either:

1) Between the boundary numbers <u>or</u>



2) Outside of the boundary numbers

3. Test a value to see if it is a solution. Zero is <u>often</u> the best number to test.

$$0 > (0)^2 - (0) - 12$$

$0 > -12$ Is “0” a solution? (does it make the inequality true?

The <u>shaded part</u> of the graph is the <u>solution</u>
→ we must pick the option that “shades” the number “0”.

$$-3 < x < 4$$

Steps to solve the Inequality $0 > x^2 - x - 12$

1. Find the boundary numbers: (Solve the equation)

2. The solution is usually either:
   a) Between the boundary numbers or
   b) Outside the boundary numbers

3. Test a number to see if it is a solution of the inequality:
   If a solution, pick the number line that shades this number
   If not a solution, pick the number line that doesn't shade

4. Answer the question
   a) Graph (if asked)
   b) Write solution in simplified inequality form (if asked)
   c) Write solution in interval form (if asked).

Solve the inequality

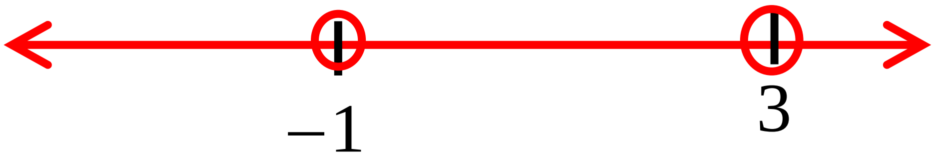
$$0 < -9x^2 + 18x + 27 \qquad 0 < -9(x^2 - 2x - 3)$$

1. Find the "real" zeroes of the polynomial <u>equation</u>.

$$0 = -9(x+1)(x-3) \qquad 0 = (x + 1)(x - 3)$$

2. What are the boundary numbers?

3. Is it shaded between or outside of the boundary numbers.?



$$28 \le x^2 - 12x$$

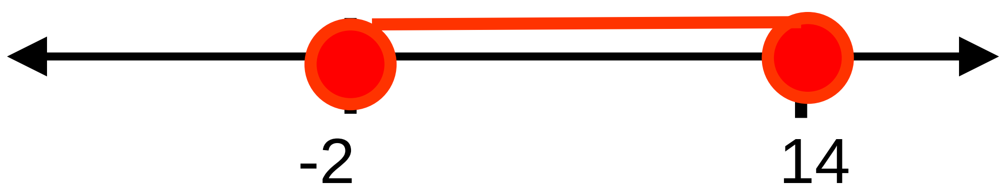
$$0 = x^2 - 12x - 28$$

$$0 = (x-14)(x+2)$$

x = 14, -2

The solution is either:

1) Between the boundary numbers <u>or</u>

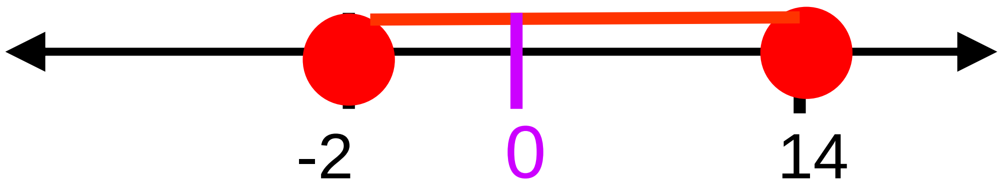


2) Outside of the boundary numbers

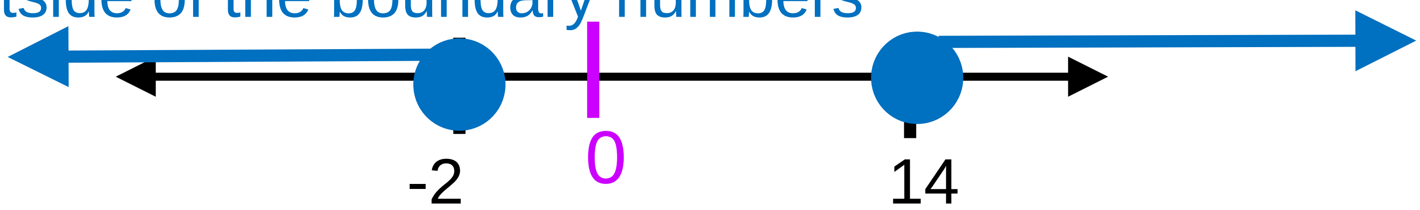
The solution is either:

$$0 \le x^2 - 12x - 28$$

1) Between the boundary numbers or



2) Outside of the boundary numbers



How do we decide?

Test a value to see if it is a solution. Zero is often the best number to test. $0 < (0)^2 - 12(0) - 28$

$0 \le -28$ Is "0" a solution? (does it make the inequality true?

The shaded part of the graph is the solution
→ we must pick the option that "shades" the number "0".

$$(-\infty,\ -2] \cup [14,\ \infty)$$

Solve $x^2 - 9 > 0$

1. <u>Find the boundary numbers</u>: (solve equation)

$0 = (x - 3)(x + 3)$ $\qquad$ $x = -3, 3$

2. <u>The solution is either</u>:

a) Between the boundary numbers <u>or</u>

b) Outside of the boundary numbers

3. <u>Test a number </u>(→ “0”) $\qquad$ $(0)^2 - 9 > 0$

“0” is not a solution.

4. <u>Solution is</u>: $\qquad$ (-∞, -3) U (3, ∞)