Math-3A
Lesson 6-4

Inverse Functions

Exchanging the x and y values

x	y
2	1
0	2

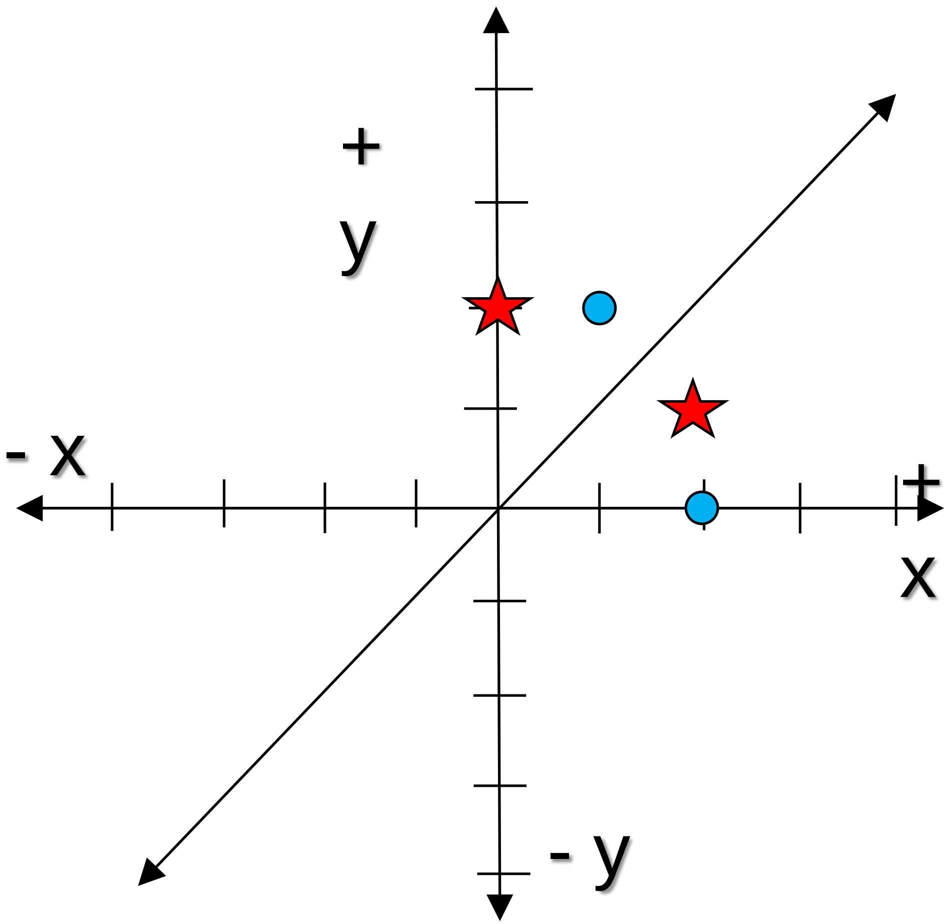
1. Plot the following points using stars

2. Fill in another table with the same values, but exchange the x and y values

x	y
1	2
2	0

3. Graph the new points using little circles.

4. What do you notice?



Vocabulary

Inverse Relation: A relation that interchanges the input and output values of the original relation.

Relation: (-2, 5), (5, 6), (-2, 6), (7, 6)

Inverse Relation: (5, -2), (6, 5), (6, -2), (6, 7)

How to find the inverse relation

Relation: $y = 1/2x + 2$

1. Exchange ‘x’ and ‘y’ in the original relation.

2. Solve for ‘y’ (get ‘y’ all by itself).

$x = 1/2y + 2$

$-2 \qquad -2$

$x - 2 = 1/2y$

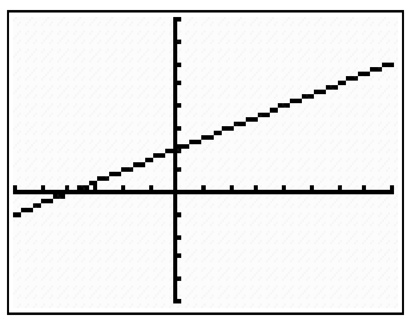
$*2 \qquad *2$

$$2(x-2) = \frac{2}{1} * \frac{1}{2} y$$

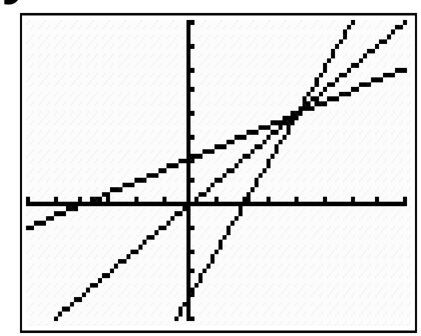
$2x - 4 = y$

$y = 2x - 4$

Graph of
$y = 2x - 4$
is a <u>reflection</u> of the graph of the line
$y = 1/2x + 2$
across the line
$y = x$.



$y = ½\,x + 2$



$y = 2x - 4$

Find the inverse of: f(x) = 4x + 2 Exchange ‘x’ and ‘y’

$$x = 4y + 2$$

This <u>IS</u> the inverse function (written as: “x as a function of y”)

Rewrite it so that it is written as: “y as a function of x”)

subtract ‘2’ (left and right)

$$x - 2 = 4y$$

Divide (all of the) left and right by 4

$$\frac{x}{4} - \frac{2}{4} = \frac{4y}{4}$$

Reduce the fractions

$$\frac{x}{4} - \frac{1}{2} = y$$

Rearrange into “slope intercept form”

$$y = \frac{x}{4} - \frac{1}{2}$$

This is the inverse of: y = 4x + 2

<u>Function Notation</u>: “the inverse of f(x)”

$$f(x) \qquad\qquad f^{-1}(x)$$

$f^{-1}(x)$ means “the inverse of f(x)”

<u>Do not confuse</u> this <u>notation</u> with the negative exponent property:

$$x^{-1} = \frac{1}{x^1}$$

Negative exponent on a <u>number</u> or an <u>expression</u> means “flip the number” (the reciprocal of the number)”

The inverse of a <u>function</u> means “exchange ‘x’ and ‘y’ (then solve for ‘y’).”

f(x) = x – 2 $f^{-1}(x) = ?$ Exchange 'x' and 'y'

There's no "y" !!! Remember: y = f(x) → y = x – 2

$$x = y - 2$$

This IS the inverse function
(written as: "x as a function of y")

Rewrite it so that it is written as: "y as a function of x")

Add '2' (left and right)

$$x + 2 = y$$

Rearrange into "slope intercept form"

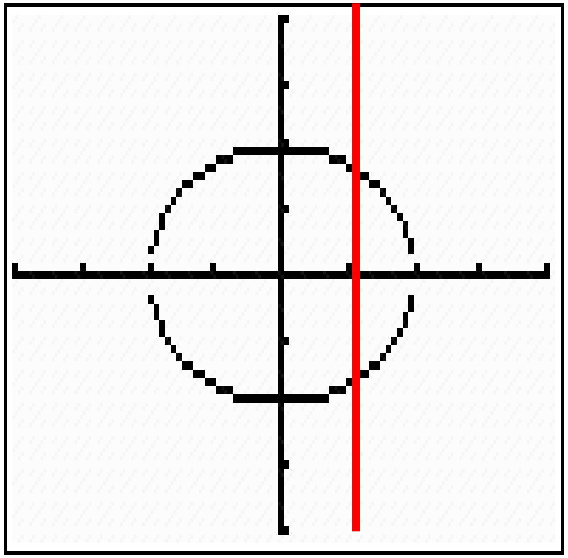
$$y = x + 2$$

This is the inverse of: y = x - 2

Graph both equations on your calculator.
Also graph the line y=x.
Push "zoom" option 5 (zoom square)

Are the two equations inverses of each other?

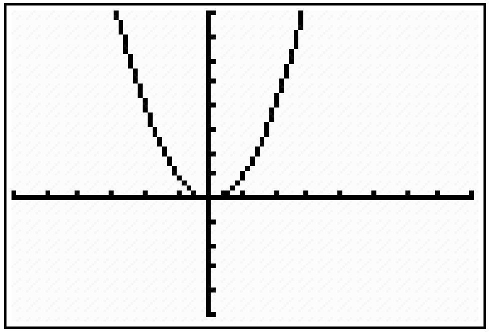
If you have the graph of a relation; <u>how can you tell</u> if the relation is a function?

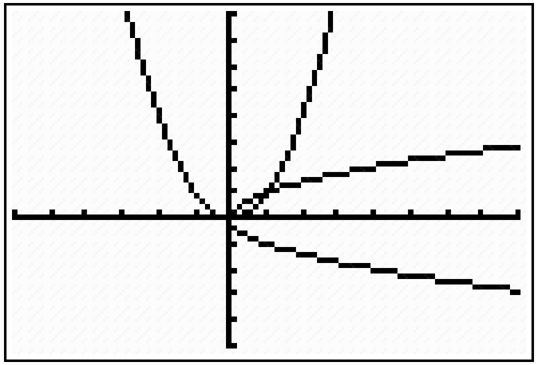


<u>Vertical Line Test</u> if the line intersects the graph more than once, it is <u>NOT</u> a function.

$$f(x) = x^2$$

$$f^{-1}(x) = ?$$





$$x = y^2 \qquad y = ??$$

$$y = \pm\sqrt{x}$$

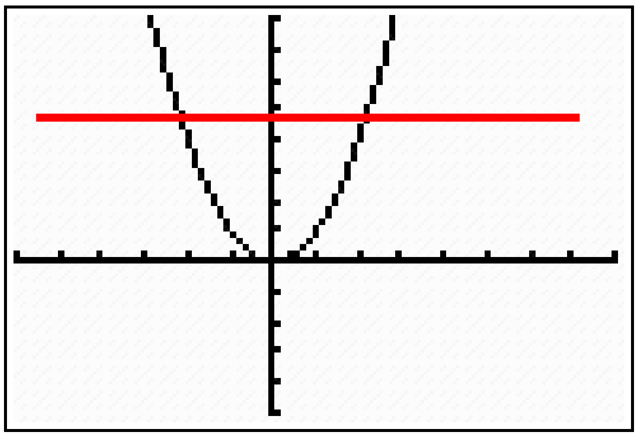
$$x = (??)^2$$

$$x = (\sqrt{x})^2 \qquad x = (-\sqrt{x})^2$$

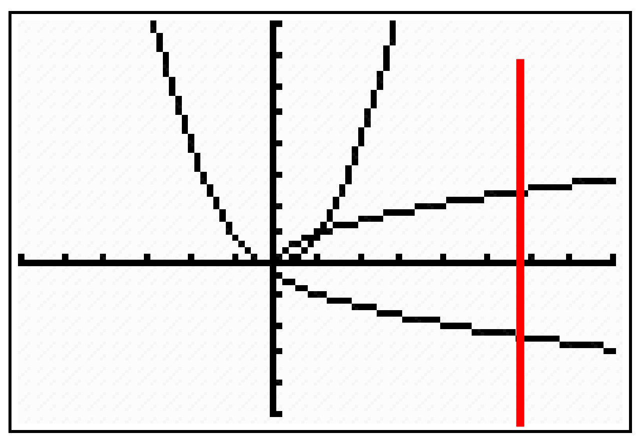
Is the inverse of f(x) a function?

If you have a graph; <u>how can you tell</u> if the inverse of the graphed function is also a function?

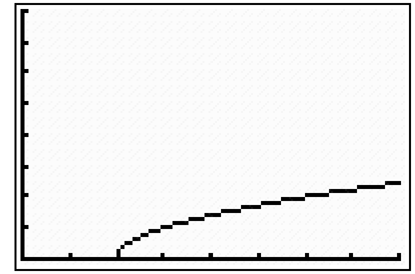
$$f(x) = x^2$$



<u>Horizontal Line Test</u>: if the line intersects the graph more than once, then the
<u>Inverse</u> of the function is <u>NOT</u> a function.



$$f(x) = \sqrt{x-2} \qquad f^{-1}(x) = ?$$



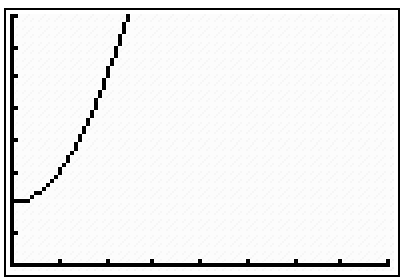
Exchange ‘x’ and ‘y’ in the original relation.

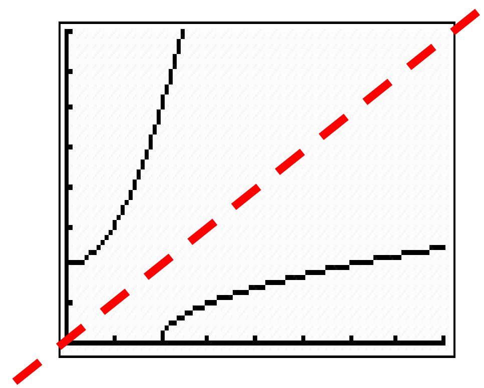
$$x = \sqrt{y-2}$$

This <u>IS</u> the inverse function (written as: “x as a function of y”)

Rewrite it so that it is written as: “y as a function of x”)

$$(x)^2 = \left(\sqrt{y-2}\right)^2 \qquad x^2 = y-2 \qquad y = x^2 + 2$$





Why do we only graph the right side of the parabola?

Since the x-y pairs of SQRT are all positive, then the x-y pairs of the inverse of the SQRT (square function) will be positive.

Find the inverse of: $f(x) = x^3 - 3$ Exchange 'x' and 'y'

$x = y^3 - 3$ This IS the inverse function, but it is written in the form "x as a function of y"

Rewrite it so that it is written as: "y as a function of x")

$x + 3 = y^3$ Add '3' (left and right)

$\sqrt[3]{x+3} = \sqrt[3]{y^3}$ cubed root both sides

$\sqrt[3]{x+3} = y$ Simplify

$f^{-1}(x) = \sqrt[3]{x+3}$ Is the inverse of: $f(x) = x^3 - 3$

Function A: heating by 10 degrees

What is the inverse of this function?

"Cooling something down by 10 degrees"

Function B: cooling by 10 degrees

The temperature of a bowl of soup is 100 degrees.

The temperature of a bowl of soup is 100 degrees. Apply function A then function B (in sequence) to the bowl of soup. What is the final temperature of the soup?

Temperature = 100 + 10 – 10

Composition of <u>*inverse functions*</u>

<u>Function A</u> and <u>Function B</u> are inverses of each other.

<u>Function A</u>: *"does something"* to the input.

<u>Function B</u>: *"<u>undoes whatever function A</u>* did to the input.

2

Function A "does something" to input value 2

Function B "undoes (whatever A did) to the input value 2

What is the output of function B? **2**

Composition of inverse functions.

$f(x) = \sqrt{x}$ $f^{-1}(x) = x^2$

2

$g(\ ) = (\ )^2$ $f(\ ) = \sqrt{(\ )}$

$g(2) = (2)^2$ $f(4) = \sqrt{(4)}$

4

2

<u>What just happened?</u>

"A Function "<u>un-does" whatever its inverse</u> "did to the input value".

$$f(x) = (x+1)^{2/3} \qquad f^{-1}(x) = ?$$

$$x = (y+1)^{2/3}$$

$$x^{3/2} = \left((y+1)^{2/3} \right)^{3/2}$$

$$x^{3/2} = y+1$$

$$y = x^{3/2} - 1$$

What function would "undo" a:

1. $f(x) = \{x^4,\ x = [0, \infty)\}$ $\qquad f^{-1}(x) = \sqrt[4]{x}$

2. $g(x) = x^{2/3}$ $\qquad g^{-1}(x) = x^{3/2}$

3. $h(x) = x^{4/5}$ $\qquad h^{-1}(x) = ? \quad = x^{5/4}$

4. $k(x) = x^5$ $\qquad k^{-1}(x) = ? \ = x^{1/5} = \sqrt[5]{x}$

Identify the pairs that cannot be inverses. Why can't they be inverses?

1. $y = x^4 - 3$ $\quad$ $y = \sqrt[4]{x+3}$

2. $y = (x-5)^{2/3}$ $\quad$ $y = x^{3/2} + 5$

3. $y = (x+6)^{4/5}$ $\quad$ $y = x^{5/4} + 5$

4. $y = (x+2)^5 - 3$ $\quad$ $y = (x+3)^{1/4} - 2$

Stop here

Your Turn:

Are f(x) and g(x) inverses of each other ?

$$g(x)=\frac{x+1}{4} \qquad f(x)=4x-1$$

Are f(x) and g(x) inverses of each other ?

$$g(x)=\frac{(x-1)^2}{5} \qquad f(x)=1+\sqrt{5x}$$

$$f(x) = \frac{2}{x-3} + 4$$

$$f^{-1}(x) = ?$$

$$x = \frac{2}{y-3} + 4$$

$$y - 3 = \frac{2}{(x-4)}$$

$$x - 4 = \frac{2}{y-3}$$

$$y = \frac{2}{(x-4)} + 3$$

$$(y-3)(x-4) = 2$$

$$f(x) = \frac{3x}{x+1} + 6$$

$$f^{-1}(x) = ?$$

$$x = \frac{3y}{y+1} + 6$$

$$x - 6 = \frac{3y}{y+1}$$

$$(y+1)(x-6) = 3y$$

multiply this out!

$$xy - 6y + x - 6 = 3y$$

$$xy - 6y - 3y = -x + 6$$

$$xy - 9y = -x + 6$$

$$y(x-9) = -x + 6$$

$$y = \frac{-x+6}{(x-9)}$$

Your Turn: $g(x)=5x^2-1$ $g^{-1}(x)=?$

$$x=5y^2-1$$

$$x+1=5y^2$$

$$\frac{x+1}{5}=y^2$$

$$y=\sqrt[2]{\frac{x+1}{5}}$$

$$y=\frac{\sqrt[2]{x+1}}{\sqrt[2]{5}}$$

$$y=\frac{\sqrt[2]{x+1}}{\sqrt[2]{5}}*\frac{\sqrt[2]{5}}{\sqrt[2]{5}}$$

$$y=\frac{\sqrt[2]{5(x+1)}}{5}$$

$$g(x) = 3 + \sqrt[3]{2x+1} \qquad g^{-1}(x) = ?$$

$$x = 3 + \sqrt[3]{2y+1} \qquad y = \frac{(x-3)^3 - 1}{2}$$

$$x - 3 = \sqrt[3]{2y+1}$$

$$(x-3)^3 = 2y+1$$

$$(x-3)^3 - 1 = 2y$$

We use compositions of inverse functions to solve equations.

$$(x-3)^2+4=40$$

"Isolate the square, undo the square".

-4 -4

$$(x-3)^2=36$$

"undo the square" means "inverse function" of the square

$$\sqrt[2]{(x-3)^2}=\sqrt{36}$$

$$x-3=\pm 6$$

$$x=3+6=9$$

$$x=3-6=-3$$

Solve the following equation

$$23 = 3x^3 - 1$$

Isolate the power:

$$24 = 3x^3$$

$$8 = x^3$$

undo the power

$$\sqrt[3]{8} = \sqrt[3]{x^3}$$

$$2 = x$$

Your Turn:

Solve $$13 = x^4 - 3$$

$$16 = x^4 \qquad \pm\sqrt[4]{16} = x \qquad x = \pm 2$$

Solve $$\sqrt{2x+1} = 3 \qquad \left(\sqrt{2x+1}\right)^2 = 3^2$$

$$2x+1 = 9$$

$$2x = 8$$

$$x = 4$$

Using an inverse function to solve an equation.

Ticket prices in the NFL can be modeled by:

$$P = 35t^{0.192}$$

where ‘t’ is the number of years since 1995.

(price as a function of time since 1995)

During what year was the price of a ticket $50.85 ?

$$P = 35t^{0.192} \qquad 50.85 = 35t^{0.192}$$

$$\frac{50.85}{35} = t^{0.192} \qquad \left(\frac{50.85}{35}\right)^{1/0.192} = t \qquad t = 6$$

$$1995 + 6 = 2001$$