SM3-A Lesson 5-4 (Applications of Rational Functions) → Rates

Examples:

Quantity	Unit of Measure
Height	Inches, feet, miles, etc.
Weight	Pounds, ounces, kilograms, grams
Temperature	Degrees F, Degrees C, Degrees K

Sometimes ratios of quantities become new quantities. We call this quantity a "rate"

When you see the word "per" it is a ratio

Quantity	Ratio of:	Unit of Measure
Speed	Distance/time	Mile/hr (mile "per" hr) Ft/sec (ft "per" sec)
"unit price"	Cost/weight	$/lbm (dollar "per" pound)
Fuel efficiency	Distance/volume gas used	Miles/gallon (mph) (miles "per" gallon)

Jose takes 3 hours to clean a house
(time rate of cleaning: one house per 3 hours → $Rate_{Jose} = \frac{1\ house}{3\ hours}$

George takes 4 hours to clean a house
(time rate of cleaning: one Job per 4 hours → $\boxed{Rate_{George} = \frac{1\ house}{4\ hours}}$

How Long for both to clean one house by working together?

Rate George + Rate Jose = Combined Rate (George & Jose) $\boxed{Rate_G + Rate_J = Rate_{G+J}}$

$$\frac{1\,\text{house}}{4\,\text{hrs}} + \frac{1\,\text{house}}{3\,\text{hours}} = \frac{1\,\text{house}}{t\,\text{hours}} \qquad \frac{1}{4} + \frac{1}{3} = \frac{1}{t}$$

Multiply by the common denominator

$$\frac{12t}{4} + \frac{12t}{3} = \frac{12t}{t} \quad \text{simplify} \quad 3t + 4t = 12 \qquad 7t = 12 \qquad t = 12/7$$

$$t = 1.7\ \text{hrs}$$

James, Adam and Paul can paint a room together in 2 hours. If Adam does the job alone he can paint the room in 5 hours. If Paul works alone, he can paint the room in 6 hours. If James works alone, how long would it take him to paint the room?

$$\text{rate}_{\text{Pria, Jamie, Paul}} = \frac{room}{2\,\text{hrs}}$$

$$\text{rate}_{\text{Pria}} = \frac{room}{5\,\text{hrs}}$$

$$\text{rate}_{\text{Paul}} = \frac{room}{6\,\text{hrs}}$$

$$t = 7.5\,\text{hrs}$$

Jamie, Pria and Saul can paint a room together in 2 hours. If Pria does the job alone she can paint the room in 5 hours. If Paul works alone, he can paint the room in 6 hours. If Jamie works alone, how long would it take her to paint the room?

$$Rate_{J+P+S} = Rate_J + Rate_P + Rate_S$$

$$t = 7.5 \text{ hrs}$$

Tanya and Cam can each wash a car and vacuum its interior in 2 hours. Pat needs 3 hours to do this same job by himself. If Pat, Cam and Tanya work together, how long will it take them to clean a car?

$$Rate_{T+C+P} = Rate_T + Rate_C + Rate_P$$

$$t = 0.75 \text{ hrs}$$

Mixture Problem: mixtures of various concentrations of solutions, allows, items, etc.

30 ml. of a 20% saline (salt/water) solution is mixed with 50 ml. of a 75% saline solution. What is the concentration of the mixture?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{weight_{\text{pure}}}{weight_{\text{total}}}$$

$$20\% = 0.2 = \frac{x}{30ml} \qquad 75\% = 0.75 = \frac{x}{50ml}$$

$$x = 6\ \text{ml (salt)} \qquad x = 37.5\ \text{ml (salt)}$$

	A	B	A & B
Part	**6 ml**	**37.5 ml**	**43.5 ml**
Whole	30 ml	50 ml	80 ml
%	0.2	0.75	0.544

$$\%_{\text{pure}} = \frac{43.5}{80} = 0.544 = 54.4\%$$

Mixture Problem: mixtures of various concentrations of solutions, allows, items, etc.

75 ml. of a 30% saline (salt/water) solution is mixed with 65 ml. of a 45% saline solution. What is the concentration of the mixture?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{weight_{\text{pure}}}{weight_{\text{total}}}$$

	A	B	A & B
Part	**22.5 ml**	**29.25 ml**	**51.75 ml**
Whole	75 ml	65 ml	140 ml
%	0.3	0.45	0.3693

5 gallons of a 20% acid mixture was added to 10 gallons of an unknown mixture. The resulting mixture concentration was 26.7%. What was the concentration of the 10 gallon mixture?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{Vol_{\text{pure}}}{Vol_{\text{mixture}}}$$

	A	B	A & B
Part	1 gal	10x gal	1+10x
Whole	5 gal	10 gal	15 gal
%	0.2	x	0.267

$$26.7\% = 0.267$$

$$15 * 0.267 = \frac{1 + 10x}{15} \quad * 15$$

$$4.005 = 1 + 10x$$

$$-1 \qquad -1$$

$$3.005 = 10x$$

$$0.3005 = x$$

$$30.1\% = x$$

Mixture Problem: mixtures of various concentrations of solutions, allows, items, etc.

How much of a 40% saline (salt/water) solution must be mixed with 35 ml. of a 25% saline solution to get a solution with 30% concentration?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{weight_{\text{pure}}}{weight_{\text{total}}}$$

	A	B	A & B
Part	**0.4x ml**	**8.75 ml**	**0.4x + 8.75 ml**
Whole	x ml	35 ml	x + 35 ml
%	0.4	0.25	0.3

$$0.3 = \frac{0.4x + 8.75}{x + 35}$$

$$0.3(x + 35) = 0.4x + 8.75$$

$$0.3x + 10.5 = 0.4x + 8.75$$

$$1.75 = 0.1x$$

$$17.5 = x$$

Mixture Problem: mixtures of various concentrations of solutions, allows, items, etc.

How much of a ***pure*** (100%) grape juice must be added to 2 quarts of 35% grape juice mixture to yield 65% grape juice mixture?

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{weight_{\text{pure}}}{weight_{\text{total}}}$$

	A	B	A & B
Part	**x qt**	**0.7 qt**	**x + 0.7 qt**
Whole	x qt	2 qt	x + 2 qt
%	1	0.35	0.65

$$0.65 = \frac{x + 0.7}{x + 2}$$

$$0.65(x + 2) = x + 0.7$$

$$0.65x + 1.3 = x + 0.7$$

$$0.6 = 0.35x$$

$$1.71 = x$$

$$\%\ \text{concentration}_{\text{pure}} = \frac{part}{whole} = \frac{Vol_{\text{pure}}}{Vol_{\text{mixture}}}$$

3 gallons of an unknown mixture concentration was added to 4 gallons of a 15% acid mixture. The resulting mixture concentration was 20.5%. What was the concentration of the 3 gallon mixture?

	A	B	A & B
Part	**3x gal**	**0.6 gal**	**3x + 0.6 gal**
Whole	3 gal	4 gal	7 gal
%	x	0.15	0.205

$$0.205 = \frac{3x + 0.6}{7}$$

$$0.8353 = 3x$$

$$1.4353 = 3x + 0.6$$

$$0.2783 = x$$

Metal Alloy: a mixture of different metals. For example “rose gold” is a mixture of copper (reddish color) with gold (yellow color). “Yellow gold” is a mixture of silver and gold. The purity of gold alloy is measured in “carats”.

The a pure substance is mixed with a “filler” we call the ratio of the pure substance to the total amount the concentration.

carats	% Gold
24	100
18	75
12	50
6	25

$$\%\,\text{concentration}_{\text{gold}} = \frac{part_{gold}}{whole_{mixture}} = \frac{weight_{\text{gold}}}{weight_{\text{total}}}$$