

Math-3A

Lesson 4-7

Holes in the Graph of the
Reciprocal Function

Divide

$$g(x) = \frac{2x}{x-3}$$

$$x-3 \overline{) 2x}$$

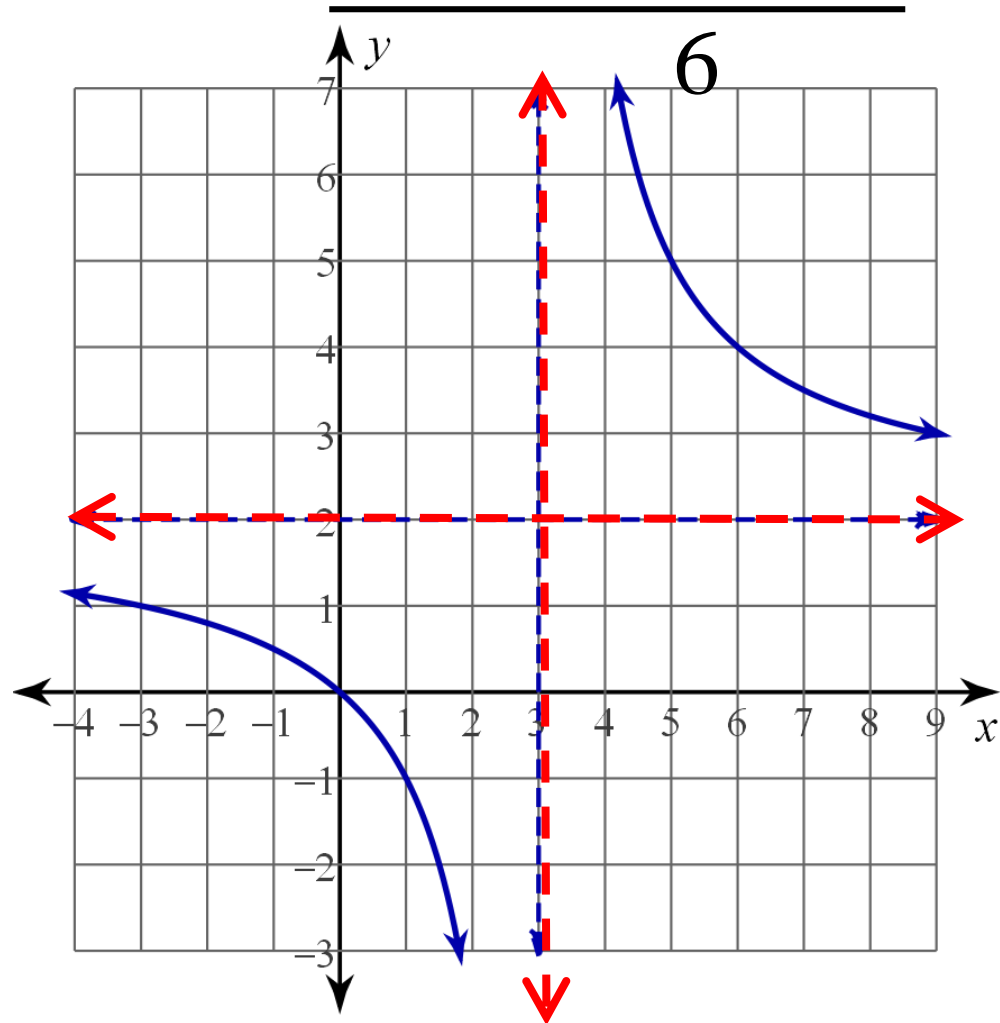
$$\begin{array}{r} 2 \\ x-3 \overline{) 2x} \\ \underline{-(2x-6)} \end{array}$$

$$g(x) = 2 + \frac{6}{x-3}$$

$$g(x) = 2 + \frac{6}{x-3}$$

Horizontal Asymptote: $y = 2$

Vertical Asymptote: $x = 3$



$$g(x) = \frac{2x}{x-3} \quad g(x) = 2 + \frac{6}{x-3}$$

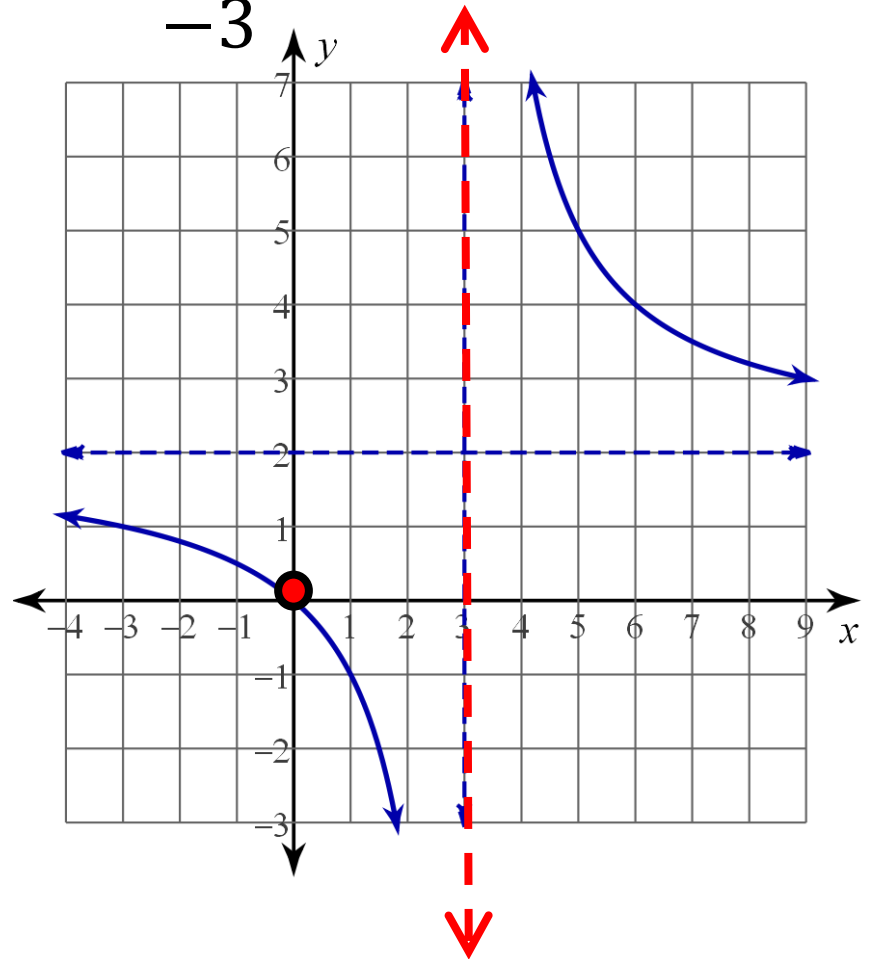
Zeroes of the numerator are x-intercepts (or are imaginary)

$$g(0) = 2 + \frac{6}{0-3} \quad g(0) = 2 + \frac{6}{-3} = 0$$

Zeroes of the denominator
are “excluded values”

For the reciprocal function, the
zeroes of the denominator are
vertical asymptotes.

$$x = 3$$



What are the equations of:

a) Horizontal asymptote? Range = ?

b) Vertical asymptote? Domain = ?

$$g(x) = \frac{1}{x} + 7$$

a) $y = 7$, range: $y \neq 7$

b) $x = 0$, domain: $x \neq 0$

$$f(x) = \frac{-7}{(x + 3)} + 5$$

a) $y = 5$, range: $y \neq 5$

b) $x = -3$, domain: $x \neq -3$

$$h(x) = \frac{1}{3(x - 2)} - 4$$

a) $y = -4$, range: $y \neq -4$

b) $x = 2$, domain: $x \neq 2$

$$f(x) = \frac{x + 3}{x - 4}$$

Domain=? $x \neq 4$

x-intercept? → are “zeroes” of the numerator $(-3, 0)$

vertical asymptote? → are “zeroes” of the denominator $x = 4$

y-intercept? → $f(0) = ?$ $f(0) = \frac{0 + 3}{0 - 4} = (0, -3/4)$

Horizontal asymptote?

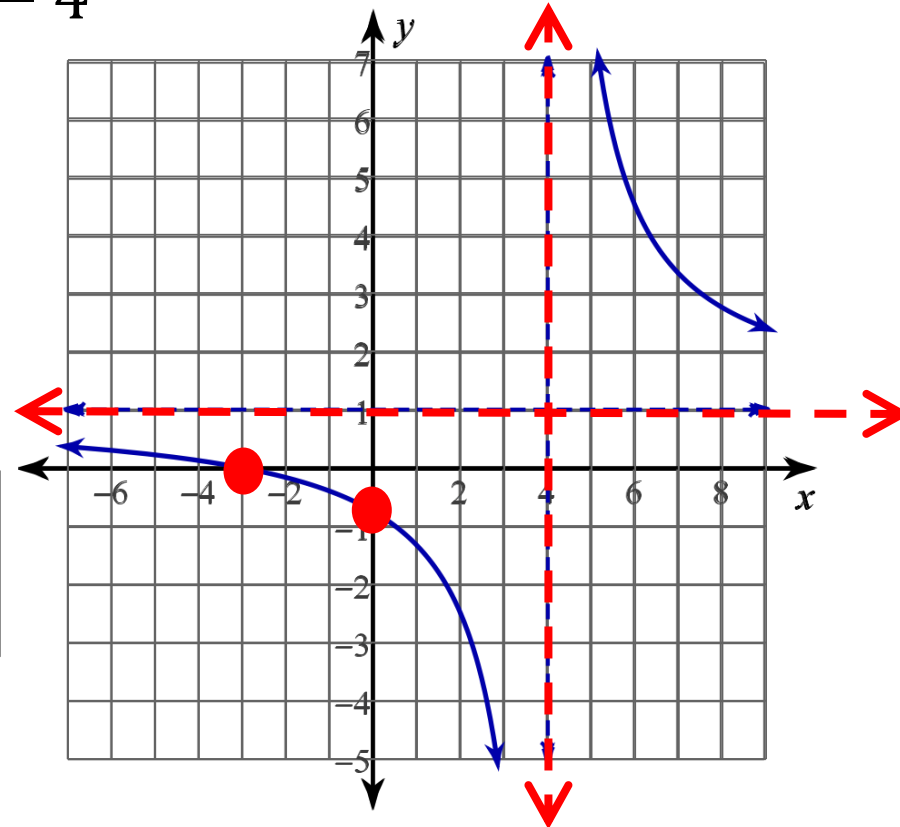
→ Is the “quotient” of division

	1	R
x	x	7
-4	-4	

$$f(x) = 1 + \frac{7}{x - 4}$$

$$f(x) = \frac{7}{x - 4} + 1$$

$$y = 1$$



$$y = \frac{x^2 + 5x + 6}{x^2 - 4} = \frac{(x+2)(x+3)}{(x+2)(x-2)}$$

Domain=?

$$x \neq -2, 2$$

$$y = \frac{(x+3)}{(x-2)}$$

x-intercept? → are “zeroes” of the numerator (that do not disappear due to simplification) (-3,0)

vertical asymptote? → are “zeroes” of the denominator (that do not disappear due to simplification) $x = 2$

Horizontal asymptote?

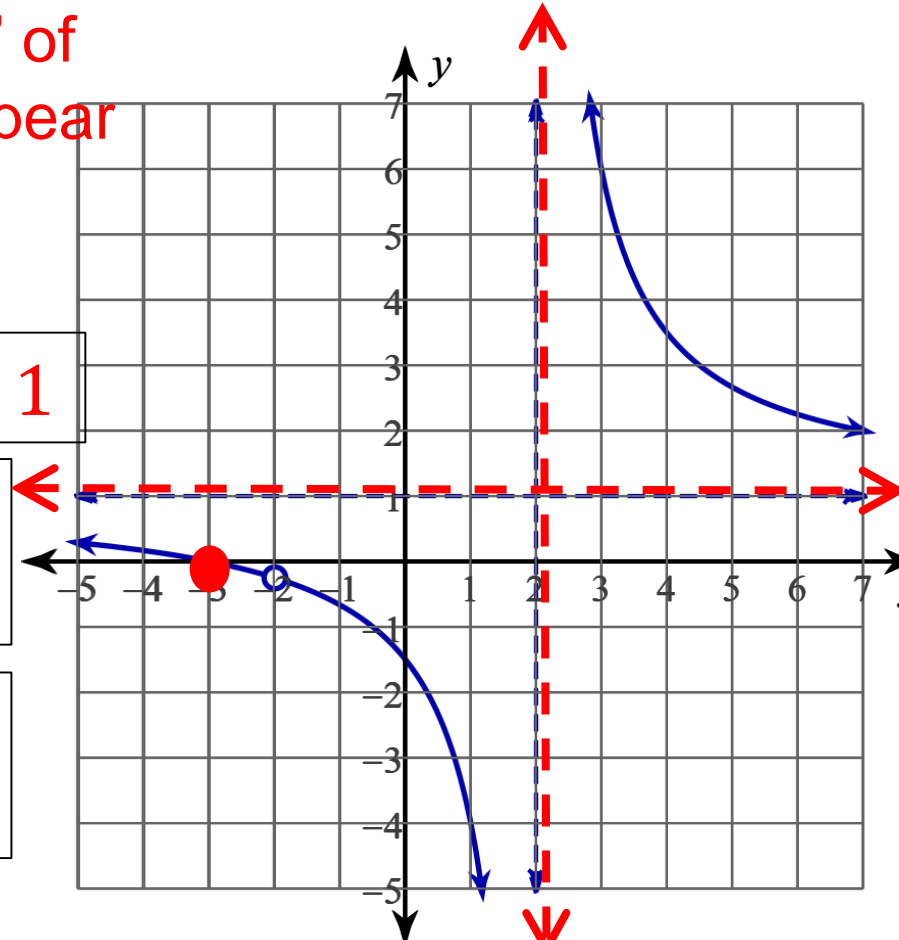
→ Is the “quotient” of division $y = 1$

	1	R
x	x	5
-2	-2	

$$y = 1 + \frac{5}{x-2}$$

$$f(x) = \frac{5}{x-2} + 1$$

3



$$y = \frac{x^2 + 5x + 6}{x^2 - 4}$$

$$y = \frac{(x + 3)}{(x - 2)}$$

Domain=?

$$x \neq 2, -2$$

$$x = 2$$

$$f(-2) = ?$$

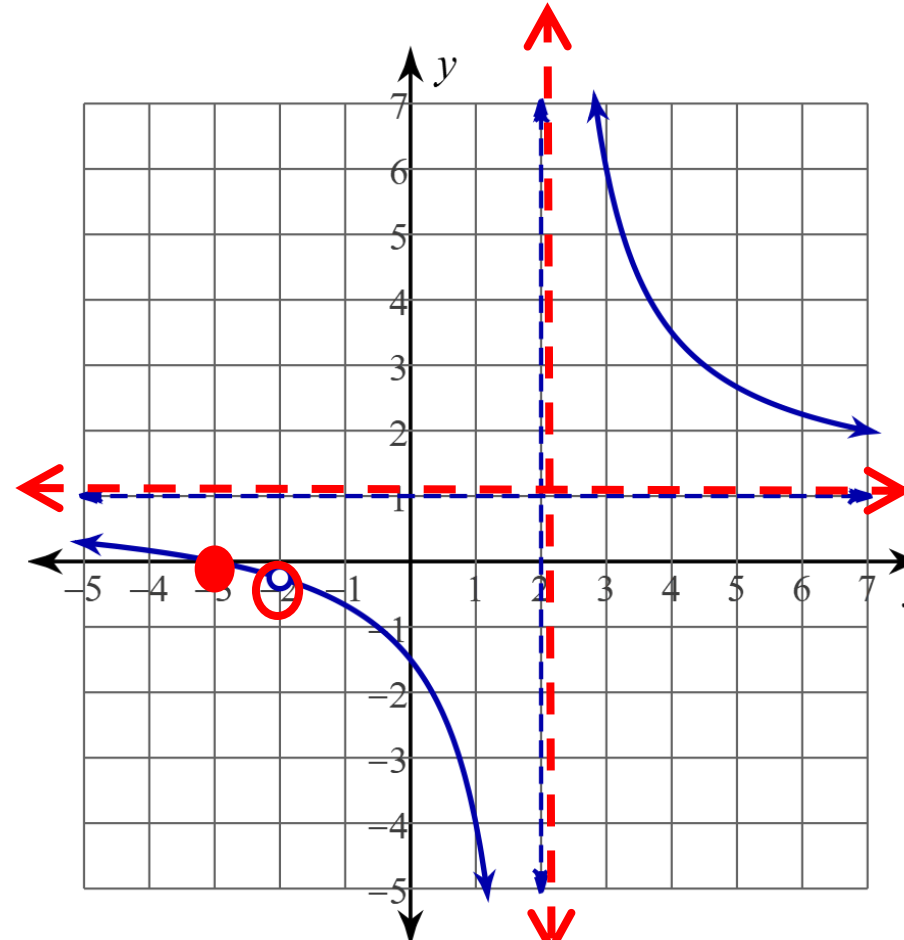
vertical asymptote? → are “zeroes” of the denominator (that do not disappear due to simplification)

Hole (in the graph) → are “zeroes” of the denominator (that do disappear due to simplification)

$$f(-2) = \frac{(-2) + 3}{(-2) - 2}$$

$$f(-2) = -\frac{1}{4}$$

There is a hole in the graph at $(0, -1/4)$



$$y = \frac{2x^2 - 8x - 24}{x^2 - 2x - 8}$$

$$y = \frac{2(x^2 - 3x - 12)}{(x - 4)(x + 2)}$$

$$y = \frac{2(x + 2)(x - 6)}{(x + 2)(x - 4)}$$

$$y = \frac{2x - 12}{(x - 4)}$$

Domain=? $x \neq -2, 4$

x-intercept? (6,0)

vertical asymptote? $x = 4$

Horizontal asymptote?

→ Is the “quotient” of division

	2	R
x	$2x$	-5
-4	-8	

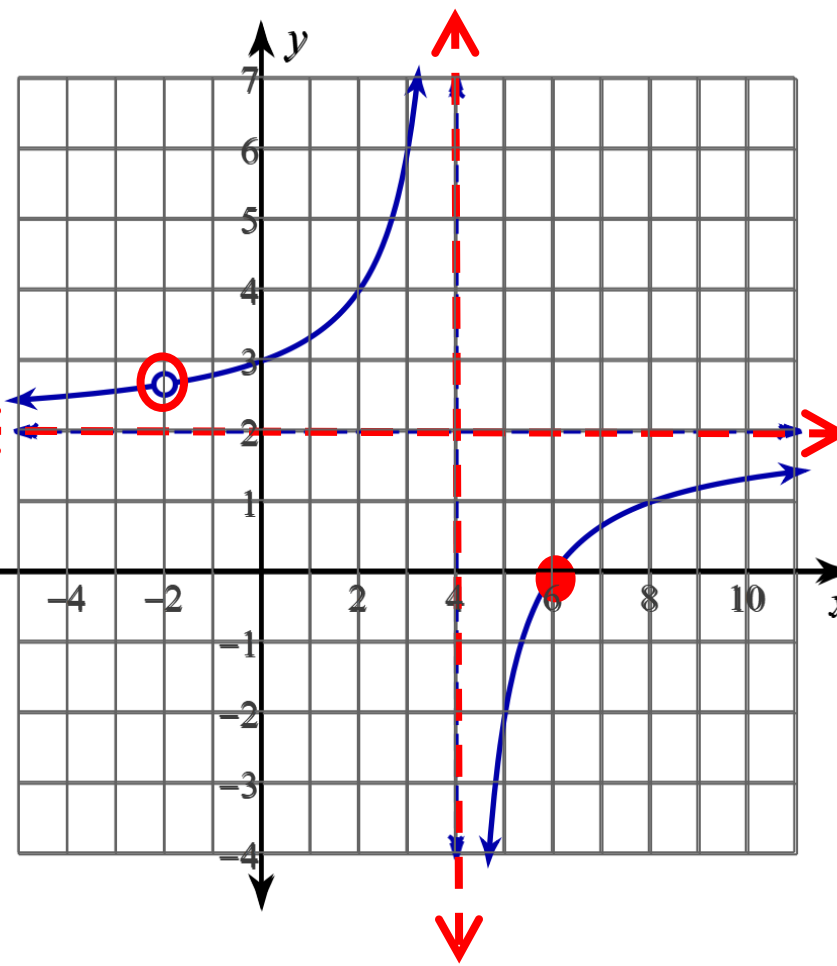
$$y = 2$$

$$y = 2 - \frac{5}{x - 4}$$

$$y = \frac{-5}{x - 4} + 2$$

-12 hole =?

$$f(-2) = 2\frac{5}{6} \quad f(-2) = \frac{-5}{-2 - 4} + 2$$



Your turn: Find the domain, x-intercepts, holes, vertical asymptotes, and the horizontal asymptote: (HINT: simplify first..)

$$g(x) = \frac{x^2 - 1}{x^2 - 6x - 7}$$

x-intercept: $x = 1$

hole: $x = -1$

$$g(x) = \frac{(x+1)(x-1)}{(x+1)(x-7)}$$

Vertical asymptote: $x = 7$

Horizontal asymptote: $y = 1$

Domain: $x \neq \pm 1$

$$g(x) = \frac{(x-1)}{(x-7)}$$

$$g(x) = \frac{x^2 - 1}{x^2 - 6x - 7} = \frac{(x-1)}{(x-7)}$$

Domain: $x \neq \pm 1$

hole: $x = -1 \rightarrow (-1, \frac{1}{4})$

x-intercept: $x = 1$

Vertical asymptote: $x = 7$

Horizontal asymptote: $y = 1$

