Math-3A

Lesson 4-1

Add and Subtract Rational Expressions

What are "like terms" ?

Like variables: $2x + 3y + 4x$

Multiples of the same variable

Like powers: $x^2 + y^2 + x^3 + x^2$

same 'base' and same exponent.

Like radicals: $\sqrt{2} + \sqrt{3} + 3\sqrt{2}$

same 'radicand' and same index number.

Like fractions: $\frac{2}{3}, \frac{4}{3}, \frac{3}{4}$

same denominator.

Adding Fractions

We can add “like fractions”

$$\frac{2}{3} + \frac{1}{3} + \frac{4}{3}$$

Combine the numerator over a <u>common</u> denominator.

$$= \frac{2+1+4}{3} \qquad = \frac{7}{3}$$

Identity Property of Multiplication

The numeral “one” multiplied by any number does not change the “value” of the number (the product will have an “equivalent” value).

Used for:

Obtaining Common Denominators:

$$\frac{2}{3}+\frac{3}{5}=\frac{3}{5}*\frac{3}{3}+\frac{2}{3}*\frac{5}{5}=\frac{10}{15}+\frac{9}{15}$$

Rationalizing Denominators:

$$\frac{2}{\sqrt{3}}*\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$$

<u>Least Common Multiple</u>: The smallest number that two other numbers divide evenly.

<u>Used for</u>:

<u>Obtaining Least Common Denominators</u>:

$$\frac{1}{30}+\frac{1}{42}=\frac{1}{6*5}+\frac{1}{6*7}=\frac{1}{6*5}*\frac{7}{7}+\frac{1}{6*7}*\frac{5}{5}$$

Both denominators have a common factor of 6.

Left denominator has an uncommon factor of 5

Multiply $\frac{1}{42}$ by "one" in the form of $\frac{5}{5}$

right denominator has an uncommon factor of 7

Multiply $\frac{1}{30}$ by "one" in the form of $\frac{7}{7}$

Find the LCD

$$\frac{2}{15}+\frac{3}{20}$$

$$15 = 3 * 5$$

$$20 = 4 * 5$$

$$LCD = 3 * 4 * 5$$

$$LCD = 60$$

$$\frac{1}{60}+\frac{1}{48}$$

$$60 = 2 * 6 * 5$$

$$48 = 6 * 8 = 6 * 2 * 4$$

$$LCD = 2 * 6 * 4 * 5$$

$$LCD = 240$$

Vocabulary

A <u>rational number</u> can be written as a ratio of integers. $\frac{2}{5}, \frac{3}{1}$

A <u>rational expression</u> can be written as a ratio of expressions.

$$\frac{x}{(x+1)}$$

We will be looking at <u>ratios of polynomial expressions</u>.

<u>Excluded Value</u>: the value for 'x' that results in division by zero

$$\frac{x}{(x+1)}, \quad x \neq -1$$

What type of rational expressions can you combine together using addition or subtraction?

"like rational expressions"

Which of these expressions are "like expressions"?

1. $\dfrac{x}{(x+1)}$ 2. $\dfrac{3x}{(x-1)}$ 3. $\dfrac{4}{(x-5)}$ 4. $\dfrac{2x^2}{(x+1)}$

Which of these expressions are "like expressions"?

5. $\dfrac{x}{(x+1)(x-1)}$ 7. $\dfrac{2x^2}{x^2-2x+1}$

6. $\dfrac{3x}{(x-1)^2}$ 8. $\dfrac{4}{(x^2-1)}$

What is the excluded value for each expression?

1. $\dfrac{x}{(x+1)}$

$x \neq -1$

2. $\dfrac{3x}{(x-1)}$

$x \neq 1$

3. $\dfrac{4}{(x-5)}$

$x \neq 5$

4. $\dfrac{2x^2}{(x+1)}$

$x \neq -1$

5. $\dfrac{x}{(x+1)(x-1)}$

$x \neq -1, +1$

6. $\dfrac{3x}{(x-1)^2}$

$x \neq 1$

7. $\dfrac{2x^2}{x^2 - 2x + 1}$

$x \neq 1$

8. $\dfrac{4}{(x^2 - 1)}$

$x \neq -1, +1$

Simplifying Fractions

$$\frac{32}{44} = \frac{32 \div 4}{44 \div 4} = \frac{32 * \frac{1}{4}}{44 * \frac{1}{4}} = \frac{8}{11}$$

What property are you applying?

Identity Property Of Multiplication

"multiplying by '1' in the form of ¼ over ¼ "

<u>Unfortunately</u> this method <u>*WILL NOT WORK*</u> for simplification of ratios of polynomials.

$$\frac{x^2 - 4}{x^2 - 3x + 2}$$

Simplifying Fractions

<u>You must FACTOR the fractions.</u>

$$\frac{32}{44} = \frac{4*8}{4*11}$$

<u>Break them apart into the product of fractions.</u>

$$= \frac{4}{4} * \frac{8}{11}$$

<u>Notice the fractions that equal ‘1’</u>

$$= 1 * \frac{8}{11} \qquad = \frac{8}{11}$$

Simplifying Fractions

You must FACTOR the fractions.

$$\frac{32}{44} = \frac{4*8}{4*11} \qquad \frac{x^2-4}{x^2-3x+2} = \frac{(x-2)(x+2)}{(x-2)(x-1)}$$

Break them apart into the product of fractions.

$$= \frac{4}{4}*\frac{8}{11} \qquad = \frac{(x-2)}{(x-2)}*\frac{(x+2)}{(x-1)}$$

Notice the fractions that equal '1'

$$=1*\frac{8}{11} \qquad = \frac{8}{11} \qquad =1*\frac{(x+2)}{(x-1)} = \frac{(x+2)}{(x-1)}$$

Adding/Subtraction Rational Expressions

The <u>easy</u> problem: $$\frac{2}{7}+\frac{3}{7}=\frac{2+3}{7}=\frac{5}{7}$$

Combine the numerator over a <u>common</u> denominator.

The <u>easy</u> problem: $$\frac{4}{(x-5)}+\frac{3x}{(x-5)}=\frac{4+3x}{(x-5)}$$

Combine the numerator over a <u>common</u> denominator.

Can you combine
4 and 3x ?

Why not?

Your turn: add/subtract

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$= \frac{\cancel{2}(x-1)}{\cancel{2}*x^2} = \frac{(x-1)}{x^2}$$

What property are we using?

Inverse Property of Multiplication.

Can you do it this way?

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{\cancel{x}+\cancel{2}}{\cancel{2}*\cancel{x}*x}+\frac{\cancel{x}-(\cancel{2}*2)}{\cancel{2}*\cancel{x}*x} = \frac{1}{x}+\frac{-2}{x}$$

Why not?

You CANNOT use the Inverse Property of Multiplication on <u>addends.</u>

Your turn: add/subtract

$$\frac{x+2}{2x^2}+\frac{x-4}{2x^2} = \frac{x+2+x-4}{2x^2} = \frac{2x-2}{2x^2}$$

$$= \frac{2(x-1)}{2*x^2}$$

I <u>will not</u> allow you to simplify using the Inverse Property of Multiplication until you have factored it into two fractions.

$$= \frac{\cancel{2}}{\cancel{2}}*\frac{(x-1)}{x^2} = \frac{(x-1)}{x^2}$$

Only then will you be able to see how the Inverse Property of Multiplication changes the rational expression into <u>multiplication by one.</u>

Your turn: add/subtract

Which one is correct?

$$\frac{(2x-7)}{x^2+2} - \frac{(x-4)}{x^2+2}$$

$$= \frac{2x-7-x-4}{x^2+2}$$

$$= \frac{2x-7-x+4}{x^2+2}$$

$$\frac{(2x-7)-(x-4)}{x^2+2} = \frac{2x-7-x-(-4)}{x^2+2} = \frac{x-3}{x^2+2}$$

Subtract <u>every term</u> in the right side numerator! (Half of you will make this mistake on the HW and on the Test).

Can you factor this into two fractions multiplied together?

<u>One third of you will miss this on the test.</u>