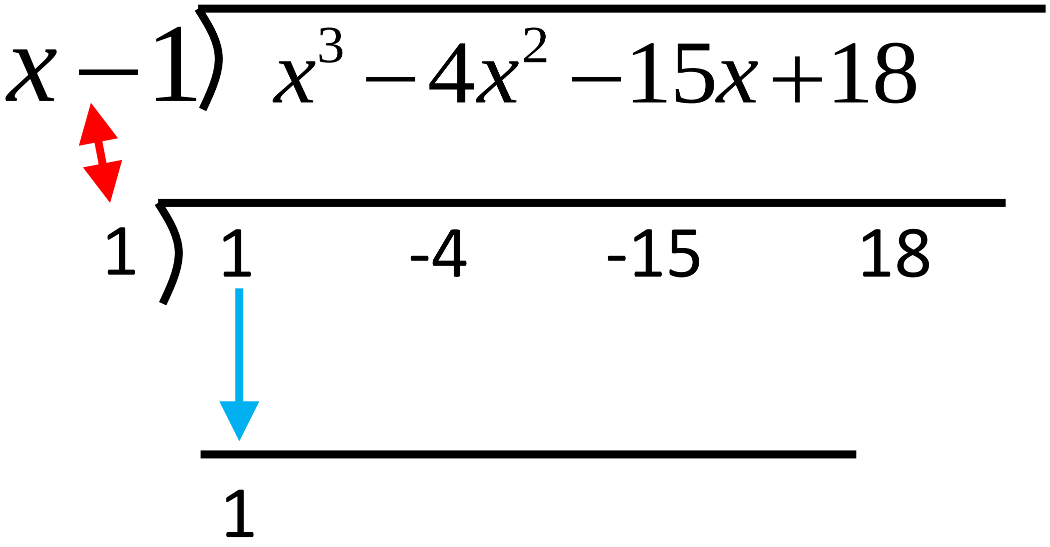
SM3-A HANDOUT 3-8 (Synthetic Division)

Is there an easier way to do this? Yes!



$$x-1\overline{)\,x^3-4x^2-15x+18}$$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ \hline & 1 & & & \end{array}$$

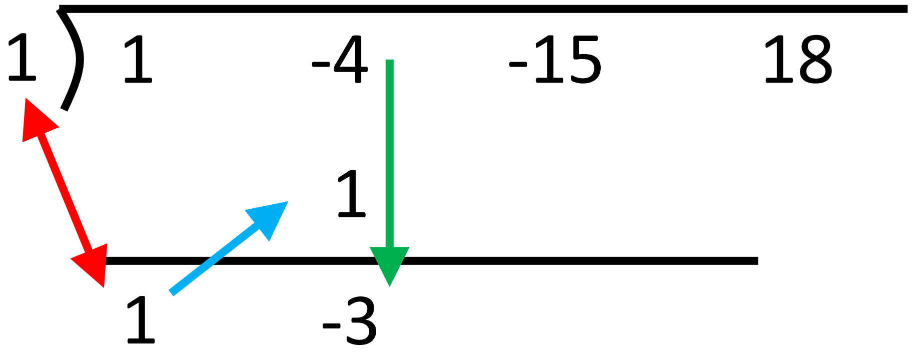
1st step: Write the polynomial with only its coefficients.

2nd step: Write the “zero” of the linear factor.

3rd step: Bring down the lead coefficient

Is there an easier way to do this? Yes!

$$x-1\overline{\smash{)}\,x^3-4x^2-15x+18}$$



1	1	-4	-15	18
		1		
	1	-3		

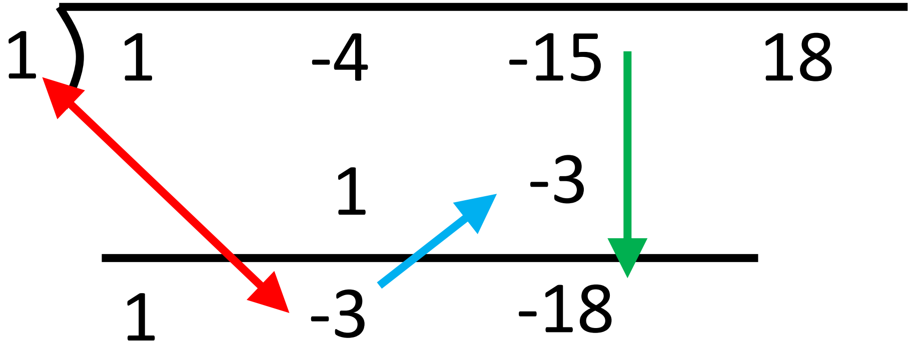
4th step: Multiply the “zero” by the lead coefficient.

5th step: Write the product under the next term to the right.

6th step: add the second column downward

Is there an easier way to do this? Yes!

$$x-1\overline{\smash{)}\,x^3-4x^2-15x+18}$$



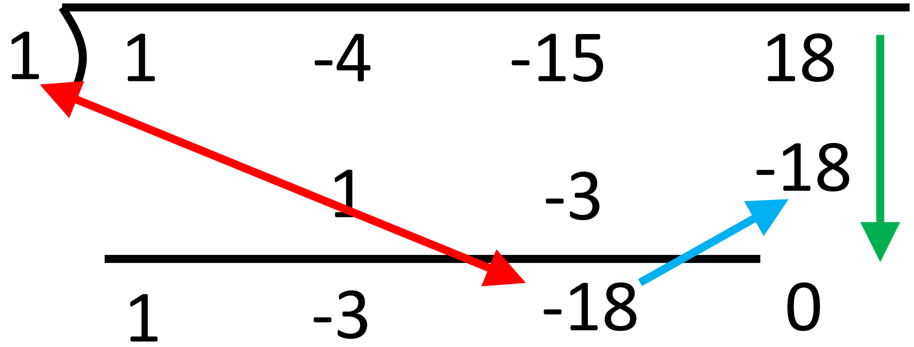
7th step: Multiply the “zero” by the second number

8th step: Write the product under the next term to the right.

9th step: add the next column downward

Is there an easier way to do this? Yes!

$$x-1\overline{\smash{)}\,x^3-4x^2-15x+18}$$



$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

10th step: Multiply the “zero” by the 3rd number

11th step: Write the product under the next term to the right

12th step: add the next column downward

Is there an easier way to do this? Yes!

$$x-1\overline{\smash{)}\,x^3-4x^2-15x+18} = x^2-3x-18$$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & \boxed{0} \end{array}$$

This last number is the remainder when you divide:

$$x^3-4x^2-15x+18$$

by

$$x-1$$

Because the remainder = 0, then (x – 1) is a factor AND x = 1 is a zero of the original polynomial!

The Remainder Theorem:

When dividing a polynomial expression by a lower degree polynomial expression,

→ If the remainder is zero,

→ then the divisor is a factor of the original polynomial.

Can you follow this?

What are the x-intercepts?

$$0 = (2x - 5)(3x + 7)$$

$$x = \frac{5}{2}, \quad \frac{-7}{3}$$

Multiply the two binomials (convert to standard form)

$$0 = 6x^2 + \ldots\ldots\ldots\ldots - 35$$

What do you notice about the first and last terms and the zeroes?

$$0 = (2x - 5)(3x + 7)$$

$$x = \frac{5}{2}, \frac{-7}{3}$$

$$0 = 6x^2 + \ldots\ldots\ldots\ldots - 35$$

The <u>denominators</u> of the solutions are <u>factors</u> of the <u>lead coefficient.</u>

The <u>numerators</u> of the solutions are <u>factors</u> of the <u>constant term.</u>

“root” = “zero” of a polynomial = value of ‘x’ that causes y = 0

The Rational Roots Theorem: the possible rational roots of a polynomial are factors of the constant divided by factors of the lead coefficient.

$$0 = (2x-5)(3x+7)$$

$$0 = 6x^2 - x - 35$$

$$x = \pm\frac{1, 5, 7, 35}{1, 2, 3, 6}$$

$$x = \pm 1, 5, 7, 35, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{5}{2}, \frac{7}{2}, \ldots, \frac{35}{3}, \frac{35}{6}$$

Normally, we would try the easy integers first BUT, for this equation, we already know that the solutions are ugly fractions.

We call this synthetic division.

$$x-1\overline{)\,x^3-4x^2-15x+18}$$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & \boxed{0} \end{array}$$

Look at the numbers at the bottom. Are these familiar?

$$x^3-4x^2-15x+18=(x-1)(x^2-3x-18)$$

They are the coefficients of the quadratic factor! (wow)

$$x-1\overline{\smash{)}\,x^3-4x^2-15x+18} \qquad = x^2-3x-18$$

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & \boxed{0} \end{array}$$

$$x^3-4x^2-15x+18=(x-1)(x^2-3x-18)$$

$$x^3-4x^2-15x+18=(x-1)(x-6)(x+3)$$

Synthetic Division

$$\begin{array}{rl} & x^3 \\ x+1 & \overline{\big)\, x^4+3x^3+5x^2+9x+6} \end{array}$$

$$\begin{array}{r|rrrrr} -1 & 1 & 3 & 5 & 9 & 6 \\ & & -1 & -2 & -3 & -6 \\ \hline & 1 & 2 & 3 & 6 & \boxed{0} \end{array}$$

$$x^4+3x^3+5x^2+9x+6=(x+1)(x^3+2x^2+3x+6)$$

4th degree poly = (1st degree poly)(3rd degree poly)

Is the 3rd degree polynomial a “nice one”?

Synthetic Substitution

$$f(x) = -x^4 - 5x^3 - 2x^2 + 4x + 1$$

$(x+3)?$

$$\begin{array}{r|rrrrr} -3 & -1 & -5 & -2 & 4 & 1 \\ & & & & & \\ \hline & -1 & & & & \end{array}$$

Synthetic Division & Substitution

$$f(x) = -x^4 - 5x^3 - 2x^2 + 4x + 1$$

$$\begin{array}{r|rrrrr} -3 & -1 & -5 & -2 & 4 & 1 \\ & & 3 & 6 & -12 & 24 \\ \hline & -1 & -2 & 4 & -8 & \boxed{25} \end{array}$$

$$\frac{-x^4 - 5x^3 - 2x^2 + 4x + 1}{x+3} =$$

$$-1x^3 - 2x^2 + 4x - 8 + \frac{25}{x+3}$$

$$\frac{2x^3 + 8x^2 - 2x - 8}{x + 3}$$

$$f(x) = 2x^3 + 8x^2 - 2x - 8$$

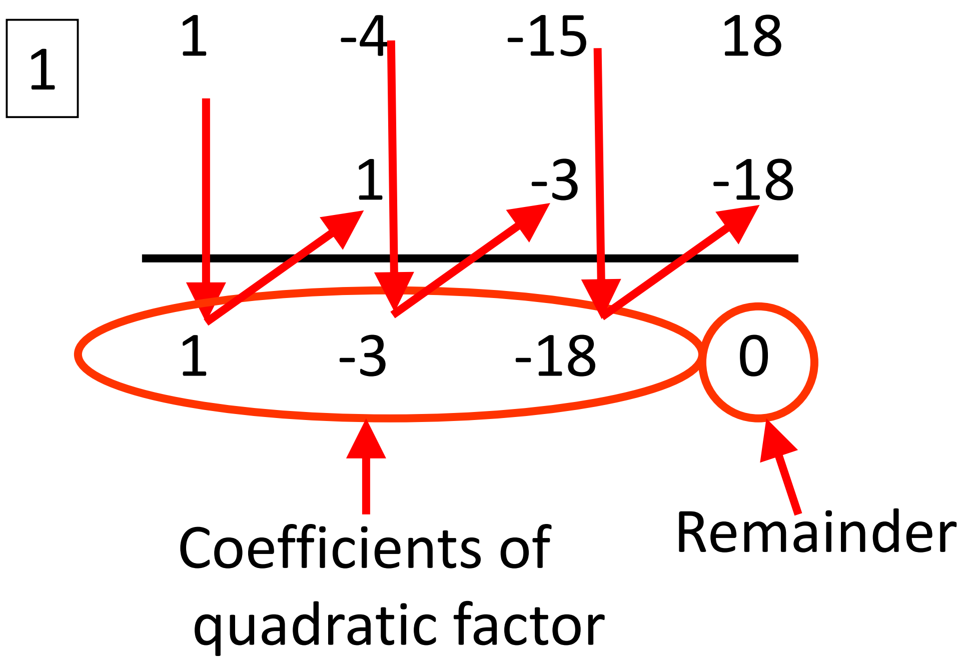
$$f(-3) = ?$$

Possible Rational Zeroes

$$f(x) = 1x^3 - 4x^2 - 15x + 18$$

$$\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$$

Factors of the last term divided by the factors of the first term



$$\begin{array}{c|cccc} \boxed{1} & 1 & -4 & -15 & 18 \\ & & 1 & -3 & -18 \\ \hline & 1 & -3 & -18 & 0 \end{array}$$

Coefficients of quadratic factor

Remainder

x = 1 is a zero of f(x).

(x – 1) is a factor of f(x).

$$(x^3 - 4x^2 - 15x + 18) = (x - 1)(x^2 - 3x - 18)$$