Math-3A
Lesson 3-6
Find Zeroes Using “Long Division”

Our goal is to find the x-intercepts of polynomials.

We've learned how to factor:

1) Quadratic form $y = x^4 + 4x^2 + 3$

2) 3rd degree polynomials with a common factor of 'x'

$$y = x^3 + 4x^2 + 3x$$

3) 3rd degree polynomials that have a "nice pattern"

$$y = x^3 + 2x^2 + 3x + 6$$

4) Sum and Difference of 2 "perfect cubes"

$y = x^3 + 8$ $y = x^3 - 27$

Now we learn how to factor Polynomials that *don't* have a "nice pattern".

Long Division

$$\frac{541}{12} = 12\overline{)541}$$

Look at the left numbers

$$= 12\overline{)541}$$

12 divides 54

4 times 12 = 48

Subtract 48 from 54

Bring the “1” down.

$$\begin{array}{r} 45 \\ = 12\overline{)541} \\ -48 \\ \hline =6 \\ =61 \\ -60 \\ \hline =1 \end{array}$$

5 times 12 = 60;
subtract 60 from 61

$$\frac{541}{12} = 45 + \frac{1}{12}$$

Remainder

Divisor

Your turn:

$$\frac{243}{7} = 7\overline{)243}$$

Look at the left numbers

$$= 7\overline{)241}$$

7 divides 44

3 times 7 = 21

$$\begin{array}{r} 34 \\ = 7\overline{)243} \\ -21 \\ \hline =3 \\ =33 \\ -28 \\ \hline =5 \end{array}$$

Subtract 21 from 24

Bring the “3” down.

4 times 7 = 28;
subtract 28 from 33

$$\frac{242}{7} = 24 + \frac{5}{7}$$

Remainder

Divisor

Steps

$$\frac{243}{7} = 7\overline{)243}$$

$$= 7\overline{)241}$$

$$\begin{array}{r} 3 \\ 7\overline{)243} \\ -21 \\ \hline =3 \\ =33 \\ -28 \\ =5 \end{array}$$

1) Look at left-most numbers

2) What # times “left” = “left”?

3) Multiply

4) Subtract

5) Bring down.

6) Repeat steps 1-5.

$$\frac{242}{7} = 34 + \frac{5}{7}$$

Remainder

Divisor

Vocabulary

Polynomial Long division: One method used to divide polynomials similar to long division for numbers.

$$\frac{x^3 + 3x^2 + 14x - 18}{(x-1)} = ax^2 + bx + c$$

Divide Evenly: A divisor divides evenly if there is a zero for the remainder.

Polynomial Long Division

$$x - 1 \overline{\smash{)}\, x^3 + 3x^2 + 14x - 18}^{\;x^2}$$

1) Look at left-most numbers

2) What # times "left" = "left"?

$$x^3 / x = ? = x^2$$

3) Multiply

$$x^2 (x - 1) = x^3 - x^2$$

4) Subtract

$$-(x^3 - x^2)$$

Polynomial Long Division

$$
\begin{array}{r l}
 & x^2 \\
x-1 & \overline{\big)\, x^3 + 3x^2 + 14x - 18} \\
 & -(x^3 - x^2) \\
\hline
 & 4x^2 \;\; +14x - 18
\end{array}
$$

4) Subtract

Careful with the negatives!

5) Bring down.

Polynomial Long Division

$$x-1 \overline{\big) x^3+3x^2+14x-18}$$

Quotient: $x^2 + 4x$

$$-(x^3-x^2)$$

$$4x^2+14x-18$$

$$-(4x^2-4x)$$

$$18x$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

$$\frac{4x^2}{x} = ? \quad = 4x$$

3) Multiply

$$4x(x-1) = 4x^2-4x$$

4) Subtract

$$-(4x^2-4x)$$

Polynomial Long Division

$$
\begin{array}{r l}
 & x^2 \quad +4x \\
\cline{2-2}
x-1 & \big) \; x^3+3x^2+14x-18 \\
 & -(x^3-x^2) \\
\cline{2-2}
 & \qquad 4x^2+14x-18 \\
 & \quad -(4x^2-4x) \\
\cline{2-2}
 & \qquad\qquad 18x \; -18
\end{array}
$$

4) Subtract

Careful of the negatives

5) Bring down.

Polynomial Long Division

$$
\begin{array}{r}
x^2 + 4x + 18 \\
x - 1 \,\overline{)\, x^3 + 3x^2 + 14x - 18} \\
-(x^3 - x^2) \\
\hline
4x^2 + 14x - 18 \\
-(4x^2 - 4x) \\
\hline
18x - 18 \\
-(18x - 18) \\
\hline
0
\end{array}
$$

6) Repeat steps 1-5.

1) Look at left-most numbers

2) What # times "left" = "left"?

$$\frac{18x}{x} = 18$$

3) Multiply

$$18(x - 1)$$

$$18x - 18$$

4) Subtract

$$-(18x - 18)$$

$$
\begin{array}{r|l}
 & x^2 + 4x - 18 \\
\hline
x-1 & x^3 + 3x^2 + 14x - 18 \\
 & -(x^3 - x^2) \\
\hline
 & 4x^2 + 14x - 18 \\
 & -(4x^2 - 4x) \\
\hline
 & 18x - 18 \\
 & -(18x - 18) \\
\hline
 & 0
\end{array}
$$

$$x^3 + 3x^2 + 14x - 18 = (x-1)(x^2 + 4x - 18)$$

How do we find the zeroes of the unfactorable quadratic factor?

Convert to vertex form and take square roots.

$$x - 2 \, \overline{\big) \, x^4 + x^3 - 4x^2 + 2x - 12}$$

$$2x - 3 \overline{\big) \, 2x^4 + 9x^3 - 8x^2 - 15x}$$

Where are we going?

$$f(x) = x^4 - 11x^3 + 29x^2 + 11x - 30$$

$$f(x) = (x-6)(x^3 - 5x^2 - x + 5)$$

$$f(x) = (x-6)(x-1)(x^2 - 6x + 5)$$

$$f(x) = (x-6)(x+1)(x-5)(x-1)$$

Factoring Polynomials: Each linear factor you factor out, results in the quotient being one degree smaller.

$$\begin{array}{r} 2x^2 + 8x - 6 \\ x - 3 \enclose{longdiv}{2x^3 + 2x^2 - 30x + 19} \\ -(2x^3 - 6x^2) \\ \hline 8x^2 - 30x + 19 \\ -(8x^2 - 24x) \\ \hline -6x + 19 \\ -(-6x + 18) \\ \hline 1 \end{array}$$

<u>Remainder</u>: There is a remainder of ‘1’. (x – 3) does not divide the polynomial evenly. Therefor (x – 1) is not a factor and x = 1 is NOT a zero. $(2x^3 + 2x^2 - 30x + 19) \div (x - 1) =$

$$2x^2 + 8x - 6 + \frac{1}{x - 3}$$